Rotating and Precessing Dissipative-Optical-Topological-3D Solitons

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We predict and study a new type of three-dimensional soliton: asymmetric rotating and precessing stable topological-dissipative-optical localized structures in homogeneous media with saturable amplification and absorption. The crucial factor determining their dynamics is the ratio of the diffusion coefficients characterizing the frequency dispersion and angular selectivity (dichroism) of the scheme. These vortex solitons exist and are stable for overcritical values of the selectivity coefficients and can be realized in lasers of large sizes with saturable absorption.

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An important class of essentially nonlinear wave phenomena is presented by solitons, stable field structures localized in nonlinear homogeneous media or media with periodic variation of their characteristics due to the balance between linear spreading and nonlinear focusing. The limiting case of nonlinear localization—over all three spatial dimensions—corresponds to three-dimensional (3D) solitons, or bullets. Solitons are known in many fields of physics, hydrodynamics, chemistry, and biology, among others [1]. However, investigating them is easier in optics due to recent progress in laser and optical materials technologies, as well as because of the relative simplicity of nonlinear optical and laser schemes [2]. Nevertheless, even in optics, the demonstration of 3D solitons, especially topological ones, presents a challenge and needs development of new approaches.

Solitons are divided into two large subclasses: conservative (in schemes without significant dissipative factors) and dissipative (with essential energy inflow and outflow). Fundamental 3D-conservative solitons are possible in homogeneous media with saturable [3] or nonlocal [4] nonlinearity. In optics, they are known as "light bullets" [5]. Experimentally and numerically, discrete fundamental and vortex weakly unstable conservative localized structures were found in arrays of coupled optical waveguides [6,7]. However, their topological counterparts, like knotted solitons [8,9], are known only for multicomponent fields [10,11]. Stable localized conservative structures can also exist in nonlinear media with localized inhomogeneity [12–17], but such structures are not, strictly speaking, self-localized. Therefore classification of these structures as solitons is under question; the term "nonlinear defect modes" is more appropriate.

A natural way to construct topological 3D solitons is provided by the use of dissipative factors to ensure extra robustness to dissipative solitons. In fact, 3D fundamental-dissipative solitons and their complexes were found in [18–24], and vortex solitons with axially symmetric intensity distributions were reported in [25,26]. Knotted dissipative discrete solitons of arbitrary high complexity were presented in [27]. The dissipative solitons are "calibrated," with sizes forming a discrete set of values; this is contrary to conservative solitons, which have a corresponding continuous spectrum. Additionally, these solitons obey a nontrivial internal structure determined by complex field of energy flows. The topology of these flows allows one to distinguish between weak and strong couplings of solitons and to reveal the connection between the symmetry and type of motion of soliton complexes [28], which can be curvilinear even in homogeneous environment [29,30]. Another consequence of the spectrum discreteness is that dissipative solitons of classical fields can mimic elementary particles in quantum field theory with the discrete spectrum of their characteristics [31]. The requirement of energy inflow and outflow balance can then correlate with the hypothesis of the existence of "positive" and "negative" dark energies.

The goal of this Letter is to present a new type of soliton —asymmetric rotating and precessing 3D-dissipative-vortex-optical solitons in a homogeneous active or passive (with nonlinear amplification and absorption) medium.

We consider the propagation of long and large (as compared to the main optical period and wavelength, i.e., the slowly varying envelope approximation) optical radiation through a continuous medium with saturable amplification (laser gain) and absorption. We assume also that the radiation pulse duration exceeds the relaxation times (medium with fast nonlinearity) and radiation is nearly linearly polarized. Then, the dimensionless form of the governing equation for the electric field envelope E is

$$\frac{\partial E}{\partial z} = \left((i + d_\perp)\nabla_\perp^2 + (i + d_\parallel)\frac{\partial^2}{\partial \tau^2}\right)E + f_{\rm nl}(|E|^2)E. \quad (1)$$

Here z is the Cartesian coordinate along the direction of the predominant radiation propagation, $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, x and y are the transverse Cartesian coordinates, and $\tau = t - z/v_g$ is the time in the system of coordinates moving along the z axis with the group velocity v_g. The coefficient $d_\parallel$ describes the frequency dispersion of medium gain or loss. The medium dichroism, i.e., the angular selectivity of amplification or absorption, is represented by the coefficient $d_\perp$. "Diffusion" coefficients $d_\perp$ and $d_\parallel$ are positive (otherwise, the propagation of plane monochromatic waves would be unstable even in linear media). We suppose that these coefficients are small, $0 < d_\perp,\ d_\parallel \ll 1$. Finally, the nonlinear function f_{nl} of radiation intensity $I = |E|^2$ corresponds to intensity-dependent medium amplification and absorption. For a two-level scheme of active (laser gain) and passive (saturable absorber) centers doped in the medium, this function, for the case of exact frequency tuning, is [32]

$$f_{\mathrm{nl}}(|E|^2) = -1 + \frac{g_0}{1 + |E|^2/\beta} - \frac{a_0}{1 + |E|^2}. \qquad (2)$$

Here g_0 (a_0) is small-signal gain (absorption) coefficient, the intensity I is normalized on the intensity of absorption saturation, β is the ratio of intensities of saturation for gain and absorption, and the term -1 describes nonresonant linear absorption (after the normalization of the longitudinal coordinate z). Bright localized structures can exist only if the linear absorption overcomes the small-signal gain: $f_{\mathrm{nl}}(0) = g_0 - 1 - a_0 < 0$; then, the trivial solution of Eq. (1), $E = 0$, is stable. Another restriction on the coefficients comes from the condition of bistability: a nontrivial solution $I_h > 0$ of the equation $f_{\mathrm{nl}}(I_h) = 0$ should exist corresponding to the homogeneous balance of gain and loss [28]. For f_{nl} given by Eq. (2), $I_h = 0.5\beta[(u^2 - v)^{1/2} - u]$, where $u = 1 - g_0 + (1 + a_0)/\beta$ and $\beta v = 4(1 + a_0 - g_0)$.

Equation (1) has translational symmetry, i.e., symmetry with respect to shifts of coordinates x, y, z and time τ, and with respect to rotation around the axis τ. It is also invariant to the inversions $x \to -x$, $y \to -y$, and $\tau \to -\tau$. Correspondingly, the medium is homogeneous, but Eq. (1) has no Galilean symmetry due to effective friction (for $d_\perp$, $d_\parallel > 0$). In the space $\mathbf{r} = (x, y, \tau)$, Eq. (1) has spherical symmetry if $d_\perp = d_\parallel$. Specifying various "initial conditions" $E_0 = E(\mathbf{r}, z = 0)$, one can obtain different field structures formed with increasing z. The structures are characterized by the following integral quantities: energy $W(z) = \int |E(\mathbf{r}, z)|^2 d\mathbf{r}$, vector of coordinates of the structure energetic center $\mathbf{R}_c(z) = \int \mathbf{r}|E(\mathbf{r}, z)|^2 d\mathbf{r}/W$, torque $\mathbf{M}(z) = \int \mathbf{r} \times \mathrm{Im}(E^* \nabla_\mathbf{r} E) d\mathbf{r}$, and inertia tensor $\hat{\mathbf{J}}(z)$, $J_{ij} = \int (\delta_{ij} r_i^2 - r_i r_j)|E(\mathbf{r}, z)|^2 d\mathbf{r}$ ($\mathbf{M}$ and $\hat{\mathbf{J}}$ are calculated with respect to the structure center $\mathbf{R}_c$, and δ_{ij} is the Kronecker symbol). Three mutually orthogonal principal axes of tensor $\hat{\mathbf{J}}$ (its eigenvectors) form a trihedron characterizing the orientation of the intensity distribution. The trihedron orientation allows one to introduce a z-dependent vector of angular velocity Ω. A different definition of the angular velocity $\Omega_S = \hat{\mathbf{J}}^{-1}\mathbf{M}$ characterizes the rotation of energy flows; these two definitions do not coincide in the general case. Additional information on the structure orientation is provided by the angle θ between the axis τ and the vector of angular velocity Ω.

For dissipative structures, the distribution of energy flow, or averaged Poynting vector in space $\mathbf{r}$, is important. This is $\mathbf{S}(\mathbf{r}) = \mathrm{Im}(E^* \nabla_\mathbf{r} E) = I \nabla_\mathbf{r} \Phi$, where $\Phi = \arg E$ is the radiation phase; its divergence $\nabla_\mathbf{r} \mathbf{S}$ in point $\mathbf{r}$ indicates whether this point is an energy source or sink [33]. Similar to the 2D case [28], the type of dissipative-localized-structure motion depends on the symmetry of the intensity and energy flow distributions. For example, if these stable distributions are invariant to rotation along the axis τ on an angle $2\pi/N$ with integer $N = 2, 3, \ldots$, then the center's velocity, $\mathbf{V}_c = d\mathbf{R}_c/dz$, is directed along axis τ. Correspondingly, in the degenerate case $d_\perp = d_\parallel$, the structure with two or more such axes of rotation is motionless (in the moving coordinate system).

Below we present the results of numerical solution of Eq. (1) with initial toroidal intensity and phase distribution corresponding to a vorticity line with topological charge $m = 1$. The parameters used ($\beta = 10$, $a_0 = 2$) are typical for fundamental symmetric 3D [32] and vortex 2D [29,33] laser solitons.

In the case $d_\perp = d_\parallel = d$, we vary the diffusion coefficient in the wide interval $0.004 < d < 0.1$ and use, as the initial condition, the axially symmetric vortex toroidal distribution with the vorticity line (of phase dislocations, $E = 0$) as the straight line. At the first stage, $z \sim 10^2$, a metastable structure is established, but for larger propagation distances the symmetry is broken and the vorticity line curves [34]. Finally, a solidlike localized structure is established with azimuthal asymmetry, an inversion center, and three different principal moments of inertia (see Fig. 1). The vector of angular velocity Ω preserves its absolute value and direction, $\Omega = \mathrm{const}$, while the direction of the principal axis of inertia $\mathbf{J}_1$ circumscribes periodically a cone with axis Ω; the period of this "fast" rotation is $z_{\mathrm{rot}} \sim 70$ (see insets in Fig. 3). Correspondingly, the structure orientation and angle θ can be arbitrary, $0 < \theta < \pi/2$. The inversion of coordinates, as illustrated in Figs. 1(c) and 1(d), gives structures rotating in the opposite direction. Solidlike 3D rotating solitons are stable inside the parameter domain shown in Fig. 2(a); the stability is confirmed by Figs. 2(c) and 2(d). Outside the stability domain, to the left from its boundary, the structure collapses. Then, as a rule, two metastable antiphase bullets arise that disappear for longer propagation distances [38]. Near the right boundary, the structure remains localized, but begins to oscillate. At larger deviations from the

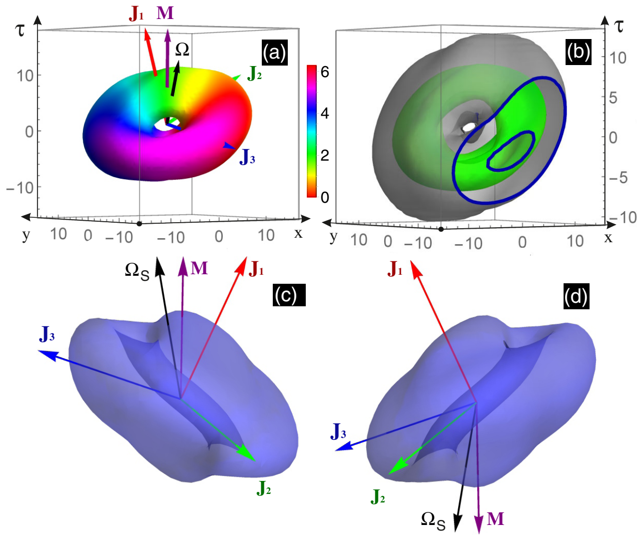

FIG. 1. (a) Instantaneous (at fixed propagation distance z) toroidal isointensity surface (where $I/I_{\max} = 0.27$) of a solidlike asymmetric rotating soliton. The angular velocity of rotation of intensity distribution is constant, $\Omega = |\Omega| = 0.092$, with a corresponding rotation period $T_{\mathrm{rot}} = 68$, while the value of the integral angular velocity is $\Omega_S = |\Omega_S| = 0.0081$ and the rotation period is $T_S = 2\pi/\Omega_S = 772$. The principal axes $\mathbf{J}_{1,2,3}$ and the torque $\mathbf{M}$ rotate around Ω; the principal values of the inertia tensor are $J_1 = 33$, $J_2 = 22.1$, $J_3 = 21.7$, and $M = |\mathbf{M}| = 0.244$. The scale on the right shows the field phase. (b) Two toroidal surfaces, one embedded in the other, of vanishing divergence of the Poynting vector, separating domains of energy sources and sinks. (c) One half of the soliton at (a). Panel (d) is obtained from (c) by the inversion of coordinate $x \to -x$. The parameters are $g_0 = 2.135$ and $d = 0.06$.

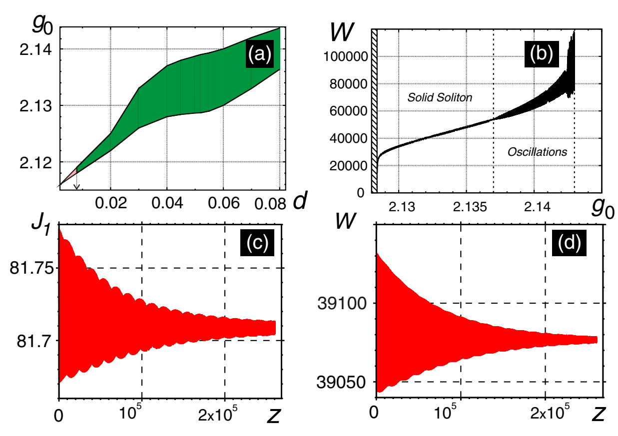

FIG. 2. (a) Stability domain of solidlike 3D solitons (shaded). Localized structures disappear to the left of the domain and they oscillate to the right. (b) Soliton energy vs small-signal gain g_0 for $d = 0.06$; in the left shaded zone there are no stable localized structures. (c),(d) Transient process for (c) the maximum principal momentum of inertia J_1 and (d) energy W of the localized structure; the parameters are $g_0 = 2.135$ and $d = 0.06$.

stability domain, modulation depth increases, the structure widens, its boundaries move away. Finally, the intensity in the infinite domain of the space $\mathbf{r}$ is switched up to the level corresponding to the intensity of homogeneous lasing I_h [39]; this is similar to the behavior near the right boundary of stability in Fig. 2(b) with an increase of g_0. In the limit of zero diffusion coefficient $d \to 0$, no stable 3D rotating solitons were found.

Next, for $d_\perp \neq d_\parallel$, there is the preferred axis τ. Then stationary, or solidlike rotating localized, structures are possible only if their angular velocity Ω is directed along the axis τ. The resulting structure type depends crucially on the ratio of the diffusion coefficients $d_\perp$ and $d_\parallel$. For $d_\perp < d_\parallel$, similarly to the previous case, a solidlike vortex structure rotating with a constant angular velocity is established. Contrary to the degenerate case, its angular velocity is directed definitely, along the axis τ; therefore, the angle $\theta = 0$, see the right inset in Fig. 3(a) and [39]. Because of the inversion symmetry, an antiparallel

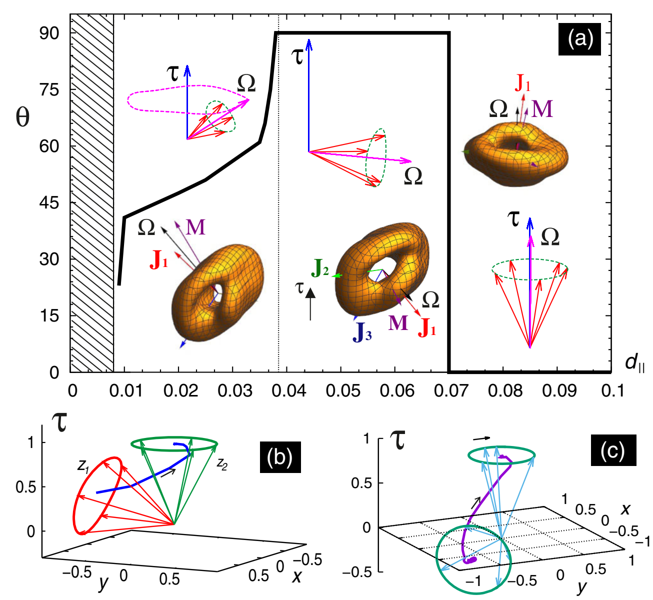

FIG. 3. (a) Dependence of the angle θ on the diffusion coefficient $d_\parallel$ for fixed $d_\perp = 0.07$ and $g_0 = 2.137$. The vertical straight line corresponds to the angular velocity arbitrary orientation in the degenerate case $d_\perp = d_\parallel$. The curved closed dashed (rose) line in the left inset illustrates the slow precession and nutation of the angular velocity direction. For small diffusion coefficients $d_\parallel < 0.008$ (the shaded zone), there are no quasistationary localized structures, see Fig. 4(a). Three cones in the insets show the schematically fast rotation of the principal axis of inertia around the direction of the angular velocity Ω; the corresponding dynamics of isointensity surfaces is presented in [35–37]. (b) Reorientation of the soliton: the cones are similar to those in the insets to panel (a). The left (red) cone is for the initial orientation at $z_1 = 18\,000$ and the right (green) cone represents $z_2 = 40\,000$; the (blue) curve with arrows shows the slow evolution of the angular velocity direction, with $d_\parallel = 0.08$. (c) Evolution of the direction of angular velocity for $d_\parallel = 0.065$.

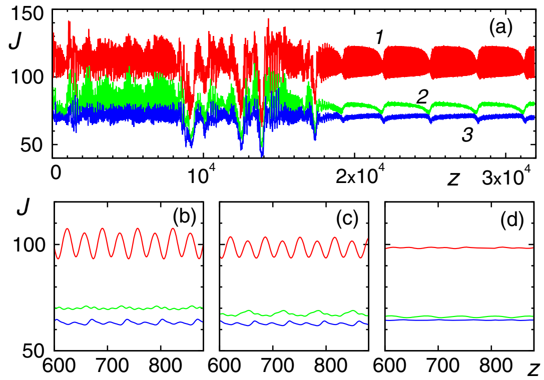

FIG. 4. Dependence of the three principal moments of inertia on propagation distance, with $g_0 = 2.137$, $d_\perp = 0.07$, and (a) $d_\parallel = 0.01$ (chaoticlike transient and strongly modulated structures), (b) $d_\parallel = 0.03$ (precessing soliton), (c) $d_\parallel = 0.04$ (periodically oscillating soliton), and (d) $d_\parallel = 0.075$ (rotating soliton).

orientation of the angular velocity is also possible. Such an orientation is restored even for large initial deviations, see Fig. 3(b). In our simulations, a parallel or antiparallel orientation of Ω and axis τ is established depending on initial conditions. The transient length $z_{tr} \sim 20000$.

For the opposite ratio of the diffusion coefficients $d_\perp > d_\parallel$, the orientation of angular velocity Ω along the axis τ is energetically unfavorable and found to be unstable. Then, there are no possible stationary structures featuring no variation in their shape, energy, and other characteristics with propagation (Fig. 4). However, the modulation depth can be small for small diffusion coefficients and such structures are quasistationary. As illustrated in Fig. 3(a), with decreasing $d_\parallel$, the angular velocity direction first turns to the plane (x, y) (orientation within this plane is indifferent). The transient process is shown in Fig. 3(c), where the initial orientation is along the axis τ and the final orientation is orthogonal to the initial one.

A further decrease in $d_\parallel$ results in the instability of this orientation as well. Instead, in the range $0.02 < d_\parallel < 0.035$, the regime of slow precession arises with precession period $z_{prec} \sim 40\,000$. The angular velocity averaged over the precession period is directed along the axis τ. During the period, the angular velocity forms a coniclike surface. In the narrower range $0.035 < d_\parallel < 0.04$, the orientation of the structure angular velocity is intermediate, see Fig. 3(a). For smaller $d_\parallel$, more complex dynamics with prolonged chaoticlike transient process for still localized vortex structure is observed [Fig. 4(a) and [40]].

In conclusion, we have revealed—to our knowledge, for the first time—the existence of stable asymmetric rotating and precessing dissipative-optical-vortex-3D solitons, i.e., "dissipative precessons." These structures arise naturally as a result of the instability of symmetric vortex solitons and have a nontrivial internal structure presented by the divergence of the Poynting vector. The results underline the close connection between the type of motion and symmetry of localized-dissipative structures that can be useful in other fields of physics, chemistry, and biology, where the dissipative dynamics is important.

The unusual solitons presented above can be formed in lasers with saturable absorption and long ring cavities. The transverse size of the laser should exceed the soliton width (about 10 μm) and the soliton round-trip time for the case considered, and the pulse duration should be longer than medium relaxation times. Multimode fiber lasers are promising for such experiments [41,42]. The transition to shorter pulses and cavities results in the possibility of the generation of new types of self-localized structures as shown by the example of two-dimensional laser schemes [28]. Of special interest is the dynamics of asymmetric solitons in dynamic cavities [43,44], with a rich variety of scenarios of orientation-dependent reflections of asymmetric solitons from oscillating cavity mirrors.

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[37] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.117.183901 for Fig. S6.
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