Experimental Test of Compatibility-Loophole-Free Contextuality with Spatially Separated Entangled Qutrits

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The physical impact and the testability of the Kochen-Specker (KS) theorem is debated because of the fact that perfect compatibility in a single quantum system cannot be achieved in practical experiments with finite precision. Here, we follow the proposal of A. Cabello and M. T. Cunha [Phys. Rev. Lett. **106**, 190401 (2011)], and present a compatibility-loophole-free experimental violation of an inequality of noncontextual theories by two spatially separated entangled qutrits. A maximally entangled qutrit-qutrit state with a fidelity as high as 0.975 ± 0.001 is prepared and distributed to separated spaces, and these two photons are then measured locally, providing the compatibility requirement. The results show that the inequality for noncontextual theory is violated by 31 standard deviations. Our experiments pave the way to close the debate about the testability of the KS theorem. In addition, the method to generate high-fidelity and high-dimension entangled states will provide significant advantages in high-dimension quantum encoding and quantum communication.

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The Kochen-Specker (KS) theorem states that a theory with hidden variables assigned independently of the measurement context cannot reproduce quantum mechanics in a spin-1 particle [1]. This reveals that contextuality is a fundamental property of quantum theory. Contextuality refers to the fact that the outcomes of a measurement are dependent on the other compatible measurements performed on the same d-level ($d \geq 3$) quantum system [1–3]. Here, two or more measurements are called compatible if they can be measured (simultaneously or sequentially) on the same individual system without altering their results [4]. Contextuality is further proved to be a critical resource for fault-tolerant universal quantum computation [5].

Recently, some inequalities [1,6–9], which hold for all noncontextual models but are violated in quantum systems, have been proposed to test the KS theorem [1]. However, the experimental testability of the KS theorem is currently under debate [10–13] since Meyer [14] and Kent [15] pointed out that finite precision in practical experiments can be interpreted as a failure of the compatible condition. As a result, the experimental violation of the inequalities can be explained by the violation of the compatibility condition. Therefore, the violations of a series of experimental tests with ions [16], neutrons [17], photons [18–20], and nuclear magnetic resonance systems [21] have been called into question in the context of the compatibility loophole [14,15].

To address this problem, Gühne *et al.* [4,22] proposed extended KS inequalities, in which additional "error" terms were introduced to compensate for imperfect compatibility and experiments have been performed to test the extended KS theorem [23]. However, the extended inequalities contain some experimentally inaccessible terms or require some additional assumptions [4]. Another approach for definitely closing the compatibility loophole and the debate about the experimental testability of the KS theorem was proposed by Cabello *et al.* [24]. In this approach, two compatible measurements are performed on two different spatially separated qutrits. The spatially separated measurements are to ensure no-signaling between measurement devices and provide the physical basis to assume that the measurements are perfectly compatible. No additional assumptions are required and this system is experimentally accessible. Recently, a certain contextuality (not in a KS sense) has been demonstrated in qubit systems without a compatibility loophole [25,26]; however, to demonstrate the strict KS theorem, the use of qutrits is necessary [1] instead of the qubits used in Refs. [25,26]. In this Letter, we, for the first time, experimentally realize Cabello's proposal with high-fidelity spatially separated entangled qutrits and demonstrate the KS theorem without a compatibility loophole.

According to the proposal in Ref. [24], a prepared entangled qutrit-qutrit state, $|\Phi\rangle = (|00\rangle - |11\rangle + |22\rangle)/\sqrt{3}$ is distributed to spatially separated Alice and Bob (as shown in Fig. 1): qutrit 1 belongs to Alice and qutrit 2 belongs to Bob. To derive some types of KS inequalities, two dichotomic observables, D_j^A (D_j^B, $j = 0, 1$) with possible outcomes 0 and 1, and two trichotomic observables, T_j^A (T_j^B, $j = 0, 1$) with possible outcomes a_j, b_j and c_j (with $j = 0, 1$), are introduced to Alice (Bob) to perform on qutrit 1 (qutrit 2). Inspired by the 8-vertex building block of

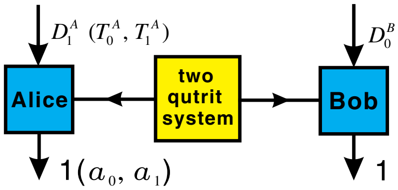

FIG. 1. Scheme of the experiment. Two maximally entangled qutrits are distributed to spatially separated Alice and Bob. In each context, there is only one measurement performed by Alice (D_1^A or T_0^A, T_1^A) and Bob (D_0^B) simultaneously on the qutrit that belongs to them. Spatial separation ensures the compatibility of the measurements on different qutrits.

the KS proof of quantum contextuality [1,24], the observables are defined as

$$\begin{aligned}
D_0^A &= D_1^B = |i\rangle\langle i|,\\
D_1^A &= D_0^B = |f\rangle\langle f|,\\
T_0^A &= a_0|a_0\rangle\langle a_0| + b_0|b_0\rangle\langle b_0| + c_0|c_0\rangle\langle c_0|,\\
T_1^A &= a_1|a_1\rangle\langle a_1| + b_1|b_1\rangle\langle b_1| + c_1|c_1\rangle\langle c_1|,\\
T_0^B &= a_0|b_1\rangle\langle b_1| + b_0|a_1\rangle\langle a_1| + c_0|c_1\rangle\langle c_1|,\\
T_1^B &= a_1|b_0\rangle\langle b_0| + b_1|a_0\rangle\langle a_0| + c_1|c_0\rangle\langle c_0|, \qquad (1)
\end{aligned}$$

where

$$\begin{aligned}
|a_0\rangle &= (|1\rangle - |2\rangle)/\sqrt{2},\\
|b_0\rangle &= (|1\rangle + |2\rangle)/\sqrt{2},\\
|c_0\rangle &= |0\rangle, \qquad (2)
\end{aligned}$$

and

$$\begin{aligned}
|a_1\rangle &= (|0\rangle - |1\rangle)/\sqrt{2},\\
|b_1\rangle &= (|0\rangle + |1\rangle)/\sqrt{2},\\
|c_1\rangle &= |2\rangle, \qquad (3)
\end{aligned}$$

and

$$\begin{aligned}
|i\rangle &= (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3},\\
|f\rangle &= (|0\rangle - |1\rangle + |2\rangle)/\sqrt{3}. \qquad (4)
\end{aligned}$$

Under the noncontextual hypothesis (only the noncontextual hidden variable states are considered), an inequality with some conditional probabilities of the measurements can be derived as

$$\begin{aligned}
P(D_1^A = 1|D_0^B = 1) &- P(T_0^A = a_0|D_0^B = 1)\\
&- P(T_1^A = a_1|D_0^B = 1) \leq 0, \qquad (5)
\end{aligned}$$

where $P(D_1^A = 1|D_0^B = 1)$ denotes the conditional probability of obtaining result 1 for D_1^A when Bob obtains result 1 for D_0^B. The other conditional probabilities are defined in the same way.

However, in quantum mechanics, the previous conditional probabilities are predicted as $P(D_1^A = 1|D_0^B = 1) = 1/9$, $P(T_0^A = a_0|D_0^B = 1) = 0$, and $P(T_1^A = a_1|D_0^B = 1) = 0$ for the ideal maximally entangled qutrits. As a result, the left side of inequality (5) is equal to 1/9, violating the inequality. To experimentally test this inequality, we need a practical high-fidelity, qutrit-qutrit entangled source to achieve the rather demanding requirement in the previous inequality. In addition, there has been only one measurement (D_0^B) with outcome 1 involved on Bob's side and three measurements (D_1^A, T_0^A, and T_1^A) on Alice's side. Only one measurement is performed on Alice's and Bob's sides for each context, and these are measured simultaneously. The spatial separation provides the compatibility requirement and there is no disturbance between these two measurements. Using the high-fidelity entangled qutrit-qutrit state and coincidence counts between Alice's and Bob's sides, we can experimentally test the KS theorem.

The settings of inequality (5) of two spatially separated entangled qutrits and local measurements on these two qutrits are similar to the Bell inequalities [27]. However, they also show some differences. Their violations rule out different hidden-variable models: violation of the Bell inequalities rules out the local hidden variables and violation of the current inequality rules out the noncontextual hidden variables. Because of the local character of the contextuality, the spatial separation of Alice and Bob is not necessarily spacelike, differing from the Bell test [28]. On the other hand, this situation connects these two concepts in a natural way and shows that they are closely related [29,30].

The experimental setup is shown in Fig. 2. First, a cw laser (the wavelength is 404 nm and the power is 100 mW) is separated to three paths to pump a 2 mm-thick type-I cut β-barium borate (BBO) crystal to generate a two-photon state $(|00\rangle - |11\rangle + |22\rangle)/\sqrt{3}$ [31,32]. Then, this two-photon state is distributed to Alice and Bob and is measured by them locally. Here, we use beam displacers to construct the phase-stable interferometers [33]. Beam displacers (BDs) operating at 404 nm (808 nm) are approximately 36.41 mm (39.70 mm) long, introducing 4.21 mm separation between the horizontally and vertically polarized photons at 404 nm (808 nm). The quantum state generated by this scheme can be characterized via the quantum state tomography process. A graphical representation of the reconstructed density matrix of the photon source is presented in Fig. 3. We use a set of 81 measurements to perform the state tomography [34,35]. The reconstructed density matrix of the state is nearly identical to the state $(|00\rangle - |11\rangle + |22\rangle)/\sqrt{3}$. The fidelity

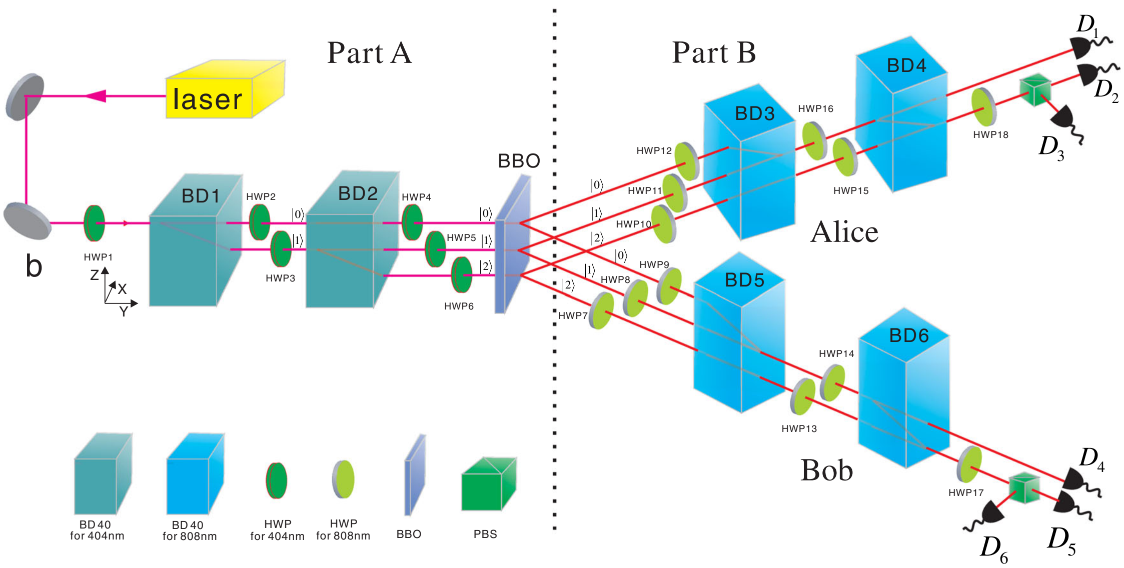

FIG. 2. Experimental setup. Part A is state preparation. A cw laser at 404 nm serves as the pump source. To generate qutrit-qutrit entanglement, the pump laser is separated into three paths by six half wave plates and two beam displacers, and then is directed into a 2 mm-thick β-barium borate (BBO) crystal. At BD1 the pump beam is split into two paths, which the vertically polarized (V) component is refracted in the Z direction while the horizontally polarized (H) component is transmitted directly. After BD1 the pump state is $1/\sqrt{3}|H_0\rangle + \sqrt{2}/\sqrt{3}|V_1\rangle$. HWP3 in the down path rotates the polarization of the pump from V polarized to the state $(|H\rangle + |V\rangle)/\sqrt{2}$. HWP2 at 0° is inserted into the up path to compensate the optical path difference between the up path and the down path. Then, the two paths pass through BD2 and are separated into three paths. HWP4 and HWP5 are used to rotate H polarized to V polarized, HWP6 is used as optical path difference compensator. After the three HWPs, the pump state is $(|V_0\rangle + |V_1\rangle + |V_2\rangle)/\sqrt{3}$. The pump is then focused on three spots of the crystal to generate spatially entangled two photon state [31]. Thus the state $(|00\rangle - |11\rangle + |22\rangle)/\sqrt{3}$ in the spatial mode is prepared if we encode the spatial mode of photons from the top to the bottom path as $|0\rangle$, $|1\rangle$, and $|2\rangle$. Part B is the measurement D_1^A and D_0^B on Alice's and Bob's sides. Six fiber-coupled single-photon detectors $D_1 - D_6$ are used to detect photons according to its coincidence counts $C_{i,j}$.

of the state is 0.975 ± 0.001 (here we only count the statistical error), and to the best of our knowledge, this is the highest fidelity of the qutrit-qutrit entangled state reported to date [36–39]. Our beam displacer based system is phase stable, and after observing the phase stability for over 2 h, we find that the Allan phase deviation is less than 0.3°, which is sufficient for our experiment. Detailed information is provided in the Supplemental Material [40].

To test inequality (5), after the reliable generation of the separated qutrit-qutrit entangled photon pairs, we need to construct the observables D_1^A, T_0^A, T_1^A on Alice's side and D_0^B on Bob's side. The observables are measured with half wave plates (HWPs), BDs, and polarizing beam splitters (PBSs) as shown in Fig. 4 (D_1^A and D_0^B are the same). The angles of the HWPs are chosen to project the state to the eigenstates of the corresponding observable. Six fiber-coupled single-photon detectors D_1-D_6 are used to detect the photons. Interference filters with a bandwidth of 3 nm are used before each detector to remove the background photon noise. The coincidence counts $C_{i,j}$ between D_i (i = 1, 2, 3) and D_j ($j = 4$, 5, 6) are recorded as the experimental results. Then, the conditional probabilities in inequality (5), such as $P(D_1^A = 1|D_0^B = 1)$, can be obtained through the joint probabilities $P(D_1^A = 1, D_0^B = 1)$ and conditional probability $P(D_0^B = 1|D_1^A, D_0^B)$ (denotes the probability of Bob obtaining result 1 when he measured with D_0^B and Alice measured with D_1^A) as

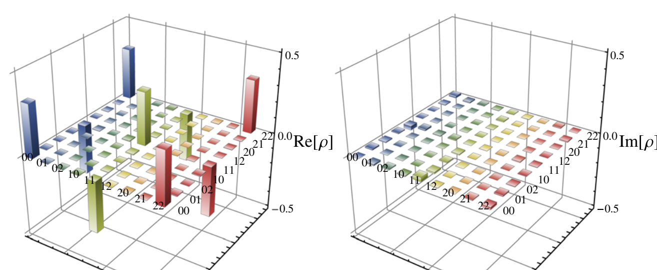

FIG. 3. Graphical representation of the reconstructed density matrix of the two-photon state. Density matrix of the two qutrits is reconstructed from a set of 81 measurements represented by operators $u_i \otimes u_j$ (with $i, j = 1, 2, \ldots, 9$) and $u_k = |\Psi_k\rangle\langle\Psi_k|$. Kets $|\Psi_k\rangle$ for both idler photons and signal photons are selected from the following set: $|0\rangle, |1\rangle, |2\rangle, (|0\rangle + |1\rangle)/\sqrt{2}$, $(|1\rangle + |2\rangle)/\sqrt{2}, (|1\rangle + i|0\rangle)/\sqrt{2}, (|1\rangle - i|2\rangle)/\sqrt{2}, (|0\rangle + |2\rangle)/\sqrt{2}$, $(|0\rangle + i|2\rangle)/\sqrt{2}$. Detailed descriptions of tomographic measurements are presented in Refs. [34,35].

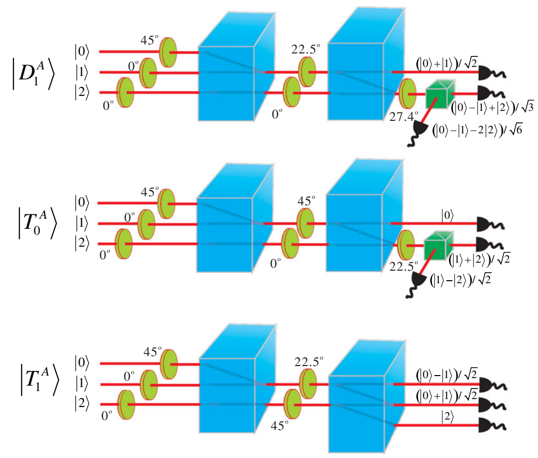

FIG. 4. Typical single-observable measurement devices. Experimental setups for measuring D_1^A, T_0^A, T_1^A from top to bottom. Polarizing beam splitters, half wave plates, and beam displacers are used to construct the observables. Here we only explain how the setup for T_0^A works. Consider that its eigenstate $(|1\rangle + |2\rangle)/\sqrt{2}$ is the input and that this photon state passes through the first BD directly. Then, the up path is rotated from H polarized to V polarized by a HWP set at 45°, while the down path is still H polarized. After the second BD, this photon state is $(|H\rangle + |V\rangle)/\sqrt{2}$ and is rotated to H polarized and passes through the PBS. Similarly, one can deduce the other two eigenstates as shown in the figure. D_1^A, T_1^A can be analyzed similarly.

$$P(D_1^A = 1, D_0^B = 1) = \frac{C_{2,5}}{N},$$
$$P(D_0^B = 1|D_1^A, D_0^B) = \frac{(C_{1,5} + C_{2,5} + C_{3,5})}{N},$$
$$P(D_1^{A)} = 1|D_0^B = 1) = \frac{P(D_1^A = 1, D_0^B = 1)}{P(D_0^B = 1|D_1^A, D_0^B)}, \qquad (6)$$

where $N = C_{1,4} + C_{1,5} + C_{1,6} + C_{2,4} + C_{2,5} + C_{2,6} + C_{3,4} + C_{3,5} + C_{3,6}$ is the total quantity of the photons. The other conditional probabilities can be obtained through the coincidence count in the same way.

We collect all results for the conditional probabilities in Table I (here we only calculate the statistical error).

For the ideal prepared state $|\Phi\rangle$, the theory predicts $P(D_1^A = 1|D_0^B = 1) = 1/9$, $P(T_0^A = a_0|D_0^B = 1) = 0$, and $P(T_1^A = a_1|D_0^B = 1) = 0$. However, because of the imperfection of the entangled source (fidelity 0.975 ± 0.001 here), the theoretical results deviate slightly from the prediction of the maximally entangled state. Detailed information is provided in the Supplemental Material [40].

Our experimental results show that the left side of inequality (5) is 0.095 ± 0.003, close to the quantum mechanics' prediction of $1/9$ and obviously violating the noncontextual hidden variables theories bound of 0 by 31 standard deviations. In our experiment, the measurements within one context are carried out by Alice and Bob at the same time to avoid disturbing each other and ensure the compatibility of the measurements. Because the two qutrits are not spacelike separated, we confirm the compatibility assumption between the measurements performed on Alice's and Bob's sides by checking the no-signaling condition; this shows that no disturbance is propagated from one side to another. The related no-signaling conditions are listed in Table II.

TABLE I. Detailed experimental results leading to inequality (5).

Conditional probability	Result	Expected value
$P(D_1^A = 1\|D_0^B = 1)$	0.114 ± 0.003	0.111
$P(T_0^A = 1\|D_0^B = 1)$	0.009 ± 0.001	0
$P(T_1^A = 1\|D_0^B = 1)$	0.010 ± 0.001	0

The conditional probabilities deviate slightly from zero due to the experimental imperfections; however, they are still in the error bar of our experiment, which is consistent with the compatibility assumption.

In summary, we use spatially separated entangled qutrits to present an experimental violation of the inequality for noncontextual theories without the compatibility loophole. Our experiment is a significant step towards definitely closing the debate on the physical relevance and experimental testability of the KS theorem and will induce a new generation of loophole-free experiments. In our experiment, the compatible condition is checked by the no-signaling condition, strictly, the no-signaling checking can only give an upper bound of the disturbance between the measurements due to the finite precision. To perform a perfect loophole-free test of the KS theorem, it would be better for the entangled qutrits to be spacelike separated so that the compatibility loophole can be automatically closed. The overall detection efficiency of our detectors is approximately 15.8%, which is not particularly efficient and, thus, we need to employ the assumption of fair sampling [43]. To close the detection loophole, one can use ppKTP crystal (down conversion photon spectrum is narrower and the filter loss is lower) and superconductor detectors (detection efficiency is more than 90%) as recently reported in Refs. [44,45]. The threshold of the overall detection efficiency can be estimated by requiring that inequality (5) is violated, even assuming that no detection in one experiment is interpreted as a noncontextual outcome.

TABLE II. No-signaling between Alice and Bob.

Probability	Result
$\|P(D_0^B = 1\|D_1^A, D_0^B) - P(D_0^B = 1\|T_0^A, D_0^B)\|$	0.001 ± 0.003
$\|P(D_0^B = 1\|D_1^A, D_0^B) - P(D_0^B = 1\|T_1^A, D_0^B)\|$	0.002 ± 0.003
$\|P(D_0^B = 1\|T_0^A, D_0^B) - P(D_0^B = 1\|T_1^A, D_0^B)\|$	0.002 ± 0.003

In addition, we use a beam displacer-based interferometer to generate a phase-stable maximally entangled qutrit-qutrit state in the spatial mode, with the fidelity of the states of 0.975 ± 0.001. This method can be used to generate higher-dimensional entangled states or to prepare hyperentanglement if one uses two type-I cut nonlinear crystals [36]. The advantages of our method are the high fidelity and high stability (at least 2 h). The presented results and the new high-fidelity entangled source will stimulate new experiments to test quantum mechanics [46–48], such as the free will theorem where the contextuality and the entanglement will appear simultaneously and are connected in the same experiment [29,30].

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