Gendelman *et al.* Reply: The determination of the normal and tangential forces between frictional disks from visual data was considered insoluble for three main reasons: (i) the tangential forces that accumulate at contacts are history dependent and were believed not to be obtainable from a visual [1], (ii) the number of mechanical constraints, i.e., the vanishing of the net force and the torque on each disk, is much smaller than the number of interdisk normal and tangential forces, and the problem is underdetermined, and (iii) in many realistic granular systems (sand, metallic disks, etc.) the compression is so small that the change in the distances between centers of mass cannot be measured accurately. In the context of an array of disks of diameters σ_i, one can determine the positions of the center of mass $\boldsymbol{r}_i$ relatively easily. But if the disks are highly incompressible, it is not possible to determined accurately the difference between the nominal distance $\sigma_i + \sigma_j$ and the actual distance $|\boldsymbol{r}_i - \boldsymbol{r}_j|$. In Ref. [2] it was shown that given the directions of the vectors connecting the centers of masses of the disks (but not the actual distances between the center of mass) and the external forces on the disks, all the normal and tangential forces can be determined *exactly* provided the normal forces are linear. There is no need to know the tangential force law. The solution of all the aforementioned difficulties is achieved by adding geometric constraints in the form of the minimal polygons that connect the centers of mass of adjacent disks.

In a comment on that paper, DeGiuli and McElwaine showed that if the radii of the disks are not known with proper accuracy, this results in errors in the determined forces [3]. This is obvious; given highly incompressible disks in contact, introducing errors in the their radii changes their positions and the vector distances between the centers of mass. Of course, experimental errors are unavoidable, and care should be taken to diminish them as much as possible. The theoretical solution of the conceptual difficulties (i)–(iii) still requires experimental efforts to achieve the highest possible precision. For example using larger and stiffer disks will automatically reduce the relative error in the radii. The theory in Ref. [2] aimed at finding the forces when provided with good measurement of the disk radii; the comment [3] addresses another problem: the statistics of forces in uncertain configuration. The obtained forces may depend on inaccuracies, but this fact does not make the solution of Ref. [2] "false" in any conceivable way as they claim.

Even with experimental errors one can improve the determination of the interparticle forces by noticing that the predicted forces do not annul the net force on each particle. An iterative procedure to achieve such an improvement was proposed in Ref. [4]. The idea is to move the particles in the direction of the net force, and recompute the interparticle forces. For systems with only normal forces this procedure converges extremely well. An equivalent procedure for frictional assemblies of disks will be provided elsewhere.

Oleg Gendelman, Yoav G. Pollack, Itamar Procaccia, Shiladitya Sengupta and Jacques Zylberg
Deptartment of Chemical Physics
The Weizmann Institute of Science
Rehovot 76100, Israel

Received 22 August 2016; published 6 October 2016
DOI: 10.1103/PhysRevLett.117.159802

[1] Photoelastic disks are the only exception where information about tangential forces may be extracted from experiments.
[2] O. Gendelman, Y. G. Pollack, I. Procaccia, S. Sengupta, and J. Zylberg, Phys. Rev. Lett. **116**, 078001 (2016).
[3] E. DeGiuli and J. N. McElwaine, preceding Comment, Phys. Rev. Lett. **117**, 159801 (2016).
[4] O. Gendelman, Y. G. Pollack, and I. Procaccia, Phys. Rev. E **93**, 060601(R) (2016).

