Charge Redistribution from Anomalous Magnetovorticity Coupling

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We investigate novel transport phenomena in a chiral fluid originated from an interplay between a vorticity and strong magnetic field, which induces a redistribution of vector charges in the system and an axial current along the magnetic field. The corresponding transport coefficients are obtained from an energy-shift argument for the chiral fermions in the lowest Landau level due to a spin-vorticity coupling and also from diagrammatic computations on the basis of the linear response theory. Based on consistent results from both methods, we observe that the transport coefficients are proportional to the anomaly coefficient and are independent of temperature and chemical potential. We therefore speculate that these transport phenomena are connected to quantum anomaly.

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Introduction.—A number of intensive and extensive studies have shown that the dynamics of chiral fermions in various systems manifests itself in anomalous transport phenomena induced by the quantum anomaly. The broad set of such systems includes the primordial electroweak plasma in the early Universe [1], the QCD matter created in the relativistic heavy-ion collisions [2], and newly invented condensed matter systems—Weyl and Dirac semimetals [3,4] (see also Refs. [5–7] for recent reviews).

One prominent example of such anomalous transport phenomena is known as the chiral magnetic effect (CME) [8,9], that is, an induction of a vector (electric) current in response to a magnetic field $\boldsymbol{B}$. In the presence of a chirality imbalance quantified by the axial chemical potential μ_A, the vector current is induced along $\boldsymbol{B}$ as

$$\boldsymbol{j}_{V,\mathrm{CME}} = q_f C_A \mu_A \boldsymbol{B}, \tag{1}$$

where q_f is the electric charge of the chiral fermion and $C_A = 1/2\pi^2$ is the nonrenormalizable coefficient characterizing the chiral anomaly relation

$$\partial_\mu j_A^\mu = q_f^2 C_A \boldsymbol{E}\cdot\boldsymbol{B}. \tag{2}$$

The CME current has been investigated by various theories and methods that consistently confirm Eq. (1) (see Refs. [5,9] for reviews). This indicates the universality of CME attributed to the topological nature of the chiral anomaly.

It is also known that the magnetic field induces not only the vector current but also an axial current. Namely, the chiral separation effect (CSE) [10] emerges in the presence of a vector chemical potential μ_V as

$$\boldsymbol{j}_{A,\mathrm{CSE}} = q_f C_A \mu_V \boldsymbol{B}. \tag{3}$$

A vorticity in a chiral fluid plays a similar role as that of the magnetic field, and hence induces anomalous vector and axial currents—this is referred to as the chiral vortical effect (CVE) [2,11–13]. The CME and CVE have been understood on equal footing within the framework of anomalous hydrodynamics from the second law of thermodynamics [14].

It should be emphasized that the above studies are devoted to the separate effects of the magnetic field $\boldsymbol{B}$ or the vorticity $\boldsymbol{\omega}$. In the pioneering hydrodynamic analysis with the anomaly [14], both vorticity and magnetic field are accounted as the first order in the gradient expansion. Consequently, the coupling between $\boldsymbol{B}$ and $\boldsymbol{\omega}$ is dropped as a higher-order effect in that systematic framework. However, in the context of magnetohydrodynamics, the magnetic field is not screened in a medium, and its strength can be much larger than the gradients, suggesting the importance of going beyond the conventional gradient expansion.

In this Letter, we will show that the interplay between the vorticity and strong magnetic field induces a local vector-charge density

$$\Delta j_V^0 = q_f \frac{C_A}{2}(\boldsymbol{B}\cdot\boldsymbol{\omega}), \tag{4}$$

where the vorticity is defined by $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$. Below, Eq. (4) will be consistently derived both from an analysis of the energy shift by a spin-vorticity coupling in the lowest Landau level (LLL) and from a diagrammatic computation on the basis of the Kubo formula. Remarkably, Δj_V^0 in Eq. (4) is proportional to anomaly coefficient C_A, and does not depend on temperature and chemical potential. This suggests a connection to the underlying quantum anomaly, as discussed below.

It is worth pointing out that Eq. (4) does not create a globe vector charge; i.e., $\int d^3\boldsymbol{x}\Delta j_V^0 = 0$. This can be seen as $\int d^3x \boldsymbol{B}\cdot\boldsymbol{\omega} = \frac{1}{2}\int d^3x \nabla\cdot(\boldsymbol{v}\times\boldsymbol{B}) = \frac{1}{2}\int_{\partial V} d\boldsymbol{S}\cdot(\boldsymbol{v}\times\boldsymbol{B}) = 0$ for a homogenous magnetic field $\boldsymbol{B}$. As usual, we assume that the flow velocity $\boldsymbol{v}$ vanishes sufficiently fast at the asymptotic region. Therefore, Eq. (4) indicates a redistribution of the vector charge in the system. In general, due to the inherent inhomogeneity of the vorticity, Eq. (4) will induce intriguing charge distribution patterns in a chiral fluid.

We will also show that, accompanying the induction of the local vector-charge imbalance Eq. (4), a new contribution to the axial current emerges as

$$\Delta \boldsymbol{j}_A = |q_f| \frac{C_A}{2} (\boldsymbol{B}\cdot\boldsymbol{\omega})\hat{\boldsymbol{B}}, \tag{5}$$

where $\hat{\boldsymbol{B}} = \boldsymbol{B}/|\boldsymbol{B}|$ is the unit vector along the magnetic field. This is an analogue of CSE Eq. (3) induced by the imbalance of vector charge μ_V. Here, it is remarkable that the axial current is dynamically generated without an initial finite value of μ_V.

The generation of the vector-charge density in chiral media is also discussed in condensed matter physics on the basis of the realization of an effective axial gauge field [15,16]. However, to the best of our knowledge, Eqs. (4) and (5) are new in the literature. Since the vorticity is one of the most important dynamical variables in magnetohydrodynamics, its coupling to the strong magnetic field, indicated by Eqs. (4) and (5), should be incorporated in anomalous magnetohydrodynamics [see also Eq. (21)]. Results reported in this Letter clearly open a new avenue for studying the intriguing interplay occurring in a wide variety of chiral media in strong magnetic fields.

Physical picture.—Prior to performing an explicit diagrammatic analysis, we first provide a physical picture as to why the vorticity would induce a local vector-charge density when coupled to a magnetic field.

We shall consider chiral fermions in the presence of a static and homogeneous magnetic field $\boldsymbol{B}$. The energy spectra of chiral fermions are discretized into the Landau levels (LLs). We next turn on a slowly varying velocity field $\boldsymbol{v}$ which leads to a nonzero vorticity, $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$. After a sufficiently long time, each fluid cell reaches a local equilibrium with the single-particle distribution function given by $f(\epsilon,\omega) = f_0(\epsilon')$, where f_0 denotes the equilibrium distribution function. Our key observation is that the vorticity shifts the single-particle energy from ϵ to ϵ' by an amount $\Delta\epsilon \equiv \epsilon' - \epsilon = -\boldsymbol{S}\cdot\boldsymbol{\omega}$. Here, $\boldsymbol{S}$ is the intrinsic angular momentum (spin) carried by fermions. Such an energy shift due to the spin-vorticity coupling can be derived by observing the shift of the single-particle Hamiltonian in a rotating frame [17]. The energy shift also naturally arises in the equilibrium fermion distribution by computing the distribution function which maximizes the entropy [11,18] or by working out a constraint imposed by the detailed balance [19]. In the every higher LL, the spin-vorticity coupling splits the degenerated spin states into the opposite directions, so that these effects cancel at the linear order in $\boldsymbol{\omega}$ when averaging over the spin. We will, therefore, concentrate on the unique grand state, i.e., the lowest Landau level.

In the LLL, the spin directions of both right- and left-handed particles are frozen in the same direction along the magnetic field $\boldsymbol{S}_{R/L} = \frac{1}{2}\mathrm{sgn}(q_f)\hat{\boldsymbol{B}}$, and those of antiparticles are oriented in the opposite direction. Consequently, the energy shift in the LLL has no dependence on the chirality and is given by

$$\Delta\epsilon^{\pm}_{\mathrm{LLL}} = \mp \frac{1}{2}\mathrm{sgn}(q_f)\hat{\boldsymbol{B}}\cdot\boldsymbol{\omega}, \tag{6}$$

where the upper and lower signs refer to a particle and antiparticle, respectively. Below, we take $\boldsymbol{B} = B\hat{e}_3$ without loss of generality.

We are now ready to compute the change of the density of chiral fermions $n_{R/L}$ due to the vorticity. As explained above, we only need to consider the contributions from the LLLs where the fermion dynamics is reduced to the $(1+1)$ dimensional one along $\boldsymbol{B}$. Expanding $f_0(\epsilon')$ up to the linear order in $\Delta\epsilon$, and using the linear dispersion relation of the right-handed LLL fermion, i.e., $\epsilon_{\mathrm{LLL}} = +p^3$, we find

$$\begin{aligned}\Delta n_R &= \left(\frac{|q_f B|}{2\pi}\right)\left(\Delta\epsilon^{+}_{\mathrm{LLL}}\int_0^{\infty}\frac{dp^3}{2\pi}\frac{\partial f_0(p^3)}{\partial p^3}\right.\\ &\qquad\left.+\Delta\epsilon^{-}_{\mathrm{LLL}}\int_{-\infty}^{0}\frac{dp^3}{2\pi}\frac{\partial \bar{f}_0(p^3)}{\partial p^3}\right)\\ &= q_f\frac{C_A}{4}(\boldsymbol{B}\cdot\boldsymbol{\omega})[f_0(0)+\bar{f}_0(0)]\\ &= q_f\frac{C_A}{4}(\boldsymbol{B}\cdot\boldsymbol{\omega}).\end{aligned} \tag{7}$$

Here, the factor of $|q_f B|/2\pi$ is the density of states in the LLL per unit transverse area. The Fermi-Dirac distribution functions of particles and antiparticles are given by $f_0(\epsilon) = 1/[e^{(\epsilon-\mu)/T}+1]$ and $\bar{f}_0(\epsilon) = 1/[e^{-(\epsilon-\mu)/T}+1]$, respectively. We have used the fact that $f_0(\infty) = \bar{f}_0(-\infty) = 0$. Remarkably, one finds an identity $f_0(0) + \bar{f}_0(0) = 1$, which is independent of temperature T and chemical potential μ. Consequently, the last line in Eq. (7) is also independent of T and μ. For the left-handed fermions with $\epsilon_{\mathrm{LLL}} = -p^3$, a similar computation leads to $\Delta n_L = \Delta n_R$. Therefore, we find $\Delta n_V = \Delta n_L + \Delta n_R = q_f C_A(\boldsymbol{B}\cdot\boldsymbol{\omega})/2$. This is the aforementioned result shown in Eq. (4).

Furthermore, since the chiral fermions in the LLL are moving along $\hat{e}_3$ with the speed of light, the generation of $\Delta n_{R,L}$ also induces currents $\Delta j^3_R = \mathrm{sgn}(S_R)\Delta n_R$ and $\Delta j^3_L = -\mathrm{sgn}(S_L)\Delta n_L$. Therefore, from Eq. (7), we find an axial current $\Delta j^3_A = \Delta j^3_R - \Delta j^3_L = |q_f| C_A(\boldsymbol{B}\cdot\boldsymbol{\omega})\hat{\boldsymbol{B}}/2$. This verifies Eq. (5). On the other hand, the vector current vanishes, $\Delta j^3_V = \Delta j^3_R + \Delta j^3_L = 0$. Alternatively, one might also interpret the amount of the energy shift Eq. (6) as an effective chemical potential $\Delta\mu_{R,L} = -\Delta\epsilon^{+}_{\rm LLL}$ (see also Ref. [20] for a discussion on the analogy between rotating and charge density). Plugging the effective vector chemical potential, $\Delta\mu_V = (\Delta\mu_R + \Delta\mu_L)/2 = \mathrm{sgn}(q_f)\boldsymbol{\omega}\cdot\hat{\boldsymbol{B}}$, into the CSE current Eq. (3), we again find the generation of the axial current Eq. (5) along the magnetic field. Note that the sign of the axial current depends only on the direction of the vorticity and is independent of that of the magnetic field.

Importantly, since Eq. (7) and thus Eq. (4) manifestly depend on the anomaly coefficient C_A, but neither T nor μ, it is natural to speculate that the form of Eq. (4) is nonrenormalizable and is tied to the chiral anomaly. Below, we will verify Eqs. (4) and (5) by an explicit field-theoretical computation and provide further evidence on the connection to the quantum anomaly.

Diagrammatical computations.—We now perform the field-theoretical computation. We will consider the response of the chiral medium to the vorticity $\boldsymbol{\omega}$ in the presence of external magnetic field. An inhomogeneous velocity field $\boldsymbol{v}(\boldsymbol{x})$ may be mimicked by turning on a fictitious gravitational field, $ds^2 = dt^2 + 2\boldsymbol{v}d\boldsymbol{x}dt - d\boldsymbol{x}^2$, i.e., $g_{0i}(\boldsymbol{x}) = \delta_{ij}v^j(\boldsymbol{x})$. Therefore, the Fourier representation of Eq. (4) is cast into

$$j^0_V = \frac{\lambda}{2}\epsilon^{ljk}\hat{\boldsymbol{B}}_l(iq_j)g_{0k}, \tag{8}$$

where we used $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$, and λ is the transport coefficient to be computed below. We again take the direction of the magnetic field to be $\boldsymbol{B} = B\hat{e}_3$ and specify an inhomogeneous velocity profile as $\boldsymbol{v} = v(x_1)\hat{e}_2$ or, equivalently, an inhomogeneous perturbation of the metric as $\delta g_{02}(x_1)$. Inverting Eq. (8), we find the Kubo formula,

$$\lambda = (-2i)\lim_{\boldsymbol{q}\to 0}\left[\lim_{\omega\to 0}\frac{\partial}{\partial q^1}G_R^{0,02}(\omega,\boldsymbol{q})\right], \tag{9}$$

with the retarded Green's function (see Fig. 1):

$$G_R^{0,02}(x-x') \equiv \langle j^0_V(x)T^{02}(x')\rangle\theta(t-t'). \tag{10}$$

A similar Kubo formula was used to study the CVE without an external magnetic field in Ref. [21].

We now evaluate the Green's function Eq. (10) in a weak coupling theory. The vector current and energy-momentum tensor of Dirac fermions are given by

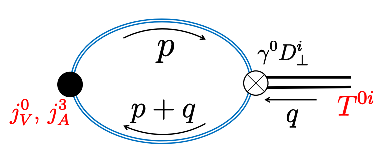

FIG. 1. One-loop diagram for the Kubo formula. The internal double lines represent the fermions in the LLL.

$$j^\mu_V(x) \equiv \bar\Psi(x)\gamma^\mu\Psi(x), \tag{11a}$$

$$T^{0i}(x) \equiv \frac{i}{2}\bar\Psi(x)(\gamma^0 D^i + \gamma^i D^0)\Psi(x), \tag{11b}$$

where $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. The covariant derivative $D^\mu = \partial^\mu + iq_f A^\mu_{\rm ext}(x)$ includes the gauge potential $A^\mu_{\rm ext}(x)$ for the magnetic field $\boldsymbol{B}$.

We consider the one-loop diagram composed of the dressed Fermion propagators in the external magnetic field. Below, we will restrict ourselves to the contributions from the lowest Landau levels. This is because the anomalous currents, such as the CME current, are solely transported by the fermions populated in the LLL. We therefore project the fermion wave function into the LLL: $\Psi = \mathcal{P}_+\psi_{\rm LLL}$, with $\psi_{\rm LLL}$, and $\mathcal{P}_\pm = (1 \pm is_f\gamma^1\gamma^2)/2$ being the LLL wave function and the spin-projection operator with $s_f \equiv \mathrm{sgn}(q_f B)$, respectively.

The coordinate representation of the retarded Green function Eq. (10) is written as (cf. Fig. 1)

$$G_R^{0,02}(x-x') = \frac{1}{2i}\mathrm{tr}\{\gamma^0\mathcal{P}_+S_{\rm LLL}(x,x')\gamma^0[D^2_{x'}S_{\rm LLL}(x',x)]\}, \tag{12}$$

where we have used the fact that the second term of Eq. (11b) vanishes for the transverse components ($i = 1, 2$), when the wave function is projected to the LLL. Here, $S_{\rm LLL}(x',x) = \langle\psi_{\rm LLL}(x')\bar\psi_{\rm LLL}(x)\rangle$ symbolically represents the LLL propagator in the medium and is factorized as [6,22]

$$S_{\rm LLL}(x',x) = e^{i\phi(x',x)}\tilde{S}_{\rm LLL}(x'-x), \tag{13}$$

where the Schwinger phase is given by

$$\phi(x',x) = -q_f\int_x^{x'}dz^\mu\left[A^{\rm ext}_\mu(z) + \frac{1}{2}F^{\rm ext}_{\mu\nu}(z^\nu - x^\nu)\right], \tag{14}$$

with $F^{\rm ext}_{\mu\nu} = \partial_\mu A^{\rm ext}_\nu - \partial_\nu A^{\rm ext}_\mu$. The above integrand is curl free, and, hence, the integral is path independent. Therefore, a straightforward calculation gives $D^\mu_{x'}\phi(x',x) = \phi(x',x)\{\partial^\mu_{x'} - iq_f F^{\mu\nu}_{\rm ext}\Delta x_\nu/2\}$, where $\Delta x^\mu = x'^\mu - x^\mu$. Consequently, the Schwinger phases in Eq. (12) cancel each other as $\phi(x,x') + \phi(x',x) = 0$. The remaining parts then depend only on the difference Δx^μ and are independent of the gauge potential, indicating the manifest translational and gauge invariances.

With these manifest symmetries, we are now ready to transform Eq. (12) into the Fourier space:

$$G_R^{0,02}(q) = \int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\Bigg[\gamma^0\mathcal{P}_+\tilde{S}_{\rm LLL}(p+q) \times\left(p^2 + is_f\frac{|q_f B|}{2}\frac{\partial}{\partial p^1}\right)\gamma^0\tilde{S}_{\rm LLL}(p)\Bigg]. \tag{15}$$

Note that $\tilde{S}_{\rm LLL}$ is completely factorized into the transverse and longitudinal parts as [6,23]

$$\tilde{S}_{\rm LLL}(p_\parallel, p_\perp) = 2e^{-|p_\perp|^2/|q_f B|} S_{1+1}(p_\parallel), \qquad (16)$$

where $p^\mu_\parallel = (p^0, 0, 0, p^3)$ and $p^\mu_\perp = (0, p^1, p^2, 0)$. This of course is anticipated from the dimensional reduction in the LLL. The longitudinal part $S_{1+1}(p_\parallel)$ is the $(1+1)$-dimensional fermion propagator in a medium. At this moment, its explicit form is not important. The integration over the transverse momentum $p_\perp$ in Eq. (15) can be easily performed, and we then arrived at

$$G_R^{0,02}(q) = is_f \frac{|q_f B|}{8\pi}(q^1 + is_f q^2)\Pi_R^{00}(q_\parallel), \qquad (17)$$

where q^1 and q^2 are components of the external momentum q^μ.

Remarkably, we find that the retarded Green's function $G_R^{0,02}$, which determines the medium's response to the vorticity in $(3+1)$ dimension, is connected to the polarization tensor in $(1+1)$ dimension:

$$i\Pi_R^{00}(q_\parallel) \equiv \int \frac{d^2 p_\parallel}{(2\pi)^2} {\rm tr}_{\rm 2D}[\gamma^0 S_{1+1}(p_\parallel + q_\parallel)\gamma^0 S_{1+1}(p_\parallel)]. \qquad (18)$$

Furthermore, since this polarization tensor Π_R^{00} is related to the chiral anomaly in $(1+1)$ dimension, it is one-loop exact and is not subject to any temperature or density correction for the massless fermion [23,24]. Here, the one-loop exact form is given by

$$\Pi_R^{00}(q_\parallel) = -\frac{1}{\pi}\left[\frac{(q_3)^2}{\omega^2 - (q_3)^2}\right], \qquad (19)$$

where $\omega \equiv q^0$. By plugging the result of the Green's function [Eqs. (17) and (19)] into the Kubo formula [Eq. (9)], the transport coefficient λ is finally obtained as

$$\lambda = \frac{C_A}{2} q_f B. \qquad (20)$$

Inserting λ into Eq. (8), we indeed verify Eq. (4), which was also obtained from the physical argument presented in the previous section. We also note that one should take the $\omega \to 0$ limit first in Eq. (9) as in the perturbative computations of other vorticity-induced transport phenomena [21] (see also Ref. [25] for discussions).

The existence of j_V^0 also implies a corresponding term in the axial current, $\boldsymbol{j}_A = \bar{\Psi}(x)\gamma^\mu\gamma^5\Psi(x)$. This is due to the relation between the vector and axial currents in the LLL, $j_A^\mu = -s_f \epsilon_\parallel^{\mu\nu} j_{V\nu}$, with $\epsilon_\parallel^{03} = -\epsilon^{03}_\parallel = +1$. From this relation $j_A^3 = s_f j_V^0$ and the vector charge density Eq. (4), we also verify Eq. (5). Of course, one can reach the same conclusion by starting out from Eq. (8) with the replacement of j_V^μ by j_A^μ.

We have thus far considered a single-flavor and color-neutral fermion. Since the flavor dependence appears only in the overall factor of q_f, extension to multiflavor cases is simply implemented as the sum of fermion charges. The color factor N_c for quarks should be included just as the overall factors in Eqs. (4) and (5).

Summary and applications.—We investigated novel anomalous transport phenomena in a chiral fluid in the presence of both vorticity and magnetic field. Our main results are summarized in Eqs. (4) and (5). Our analyses suggest that the corresponding transport coefficients are, due to the relation to the chiral anomaly in $(1+1)$ dimension, protected from temperature and density corrections. The factorization in Eq. (17) plays a crucial role for establishing the relation to the chiral anomaly. It would be interesting to examine Eqs. (4) and (5) by different approaches, for example, by means of the analytic solution of the Dirac equation in a rotating frame [20], the holographic correspondence [26], and the Wigner function formalism [27].

It is important to implement our findings into the "anomalous magnetohydrodynamics" [28]. Casting Eq. (4) into a covariant form, we propose the following realization of the magnetovorticity coupling in the framework of anomalous magnetohydrodynamics:

$$u_\mu j_V^\mu = n_0(T, \mu; B) + \Delta n, \qquad \Delta n = C_A \omega_\mu B^\mu. \qquad (21)$$

As in the conventional cases, n_0 denotes the local equilibrium density as a function of temperature T and chemical potential μ in the absence of vorticity, and u_μ is the flow velocity. The magnetovorticity coupling is included in Δn, with $\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu \partial_\alpha u_\beta$ and $B^\mu \equiv \tilde{F}^{\mu\nu}u_\nu$. As mentioned in the Introduction, this coupling term becomes comparable in magnitude to the first-order terms in the presence of strong magnetic fields. Therefore, the modification Eq. (21) should be included in anomalous magnetohydrodynamics together with the anomalous terms already considered in Ref. [14].

Finally, turning to phenomenological applications of our work, we call attention to the relativistic heavy-ion collisions where both a strong magnetic field and a rotation of the quark-gluon plasma are created [5]. While effects of the magnetic field and vorticity have been considered separately in the heavy-ion phenomelogy, their interplay has been overlooked up to now. It is also interesting to investigate effects of the coupling between magnetic fields and rotations of compact stars.

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