

Quantum Correlations Are Stronger Than All Nonsignaling Correlations Produced by n -Outcome Measurements

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We show that, for any n , there are m -outcome quantum correlations, with $m > n$, which are stronger than any nonsignaling correlation produced from selecting among n -outcome measurements. As a consequence, for any n , there are m -outcome quantum measurements that cannot be constructed by selecting locally from the set of n -outcome measurements. This is a property of the set of measurements in quantum theory that is not mandatory for general probabilistic theories. We also show that this prediction can be tested through high-precision Bell-type experiments and identify past experiments providing evidence that some of these strong correlations exist in nature. Finally, we provide a modified version of quantum theory restricted to having at most n -outcome quantum measurements.

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Introduction.—The violation of Bell inequalities [1–6] does not only show the impossibility of local realism [7] but also demonstrates (i) the existence of entangled states, i.e., states which cannot be produced by choosing among states produced locally, and (ii) the existence of incompatible measurements, i.e., measurements whose outcomes cannot be obtained from a single joint measurement. Remarkably, this holds not only assuming quantum theory (QT) but also holds for the much broader set of general probabilistic theories (GPTs) [8–11]. GPTs include classical probability theory and QT, and also theories admitting supraquantum nonsignaling correlations, such as, e.g., Popescu-Rohrlich boxes [12].

Svetlichny pointed out that (i) can be refined and that for any number of parties n , there are correlations predicted by QT that cannot be explained by any GPT in which all states are produced by choosing among $(n - 1)$ -partite entangled states [13–15]. Hence, the violation of n -partite Svetlichny inequalities [16–19] demonstrates the existence of genuinely n -partite entangled states and therefore puts strong constraints on which GPTs are suitable to describe nature.

Here, we address the problem of whether there is a sensible way to go beyond (ii) and, assuming that QT is correct, constrain more rigidly the structure of the set of measurements in any GPT describing nature. Our main result is the proof that, according to QT, nature does produce correlations which cannot be generated by shared randomness (e.g., by means of local hidden variables) and nonsignaling correlations for which the number of outcomes is limited to n . In this sense, we show that quantum correlations are not n chotomic, for any $n = 2, 3, \dots$. This implies that the same way Bell inequality experiments exclude all local realistic theories, QT predicts that certain

experiments can exclude all GPTs in which measurements are locally selected from n -outcome measurements. A possible selection mechanism, in which all measurements are produced from two-outcome measurements with the help of hidden variables, is illustrated in Fig. 1.

However, according to our analysis, such experiments require visibilities beyond what is currently feasible. This motivates us to consider a particular subclass of GPTs: those in which measurements are locally selected from n -outcome *quantum* measurements. We identify past experiments

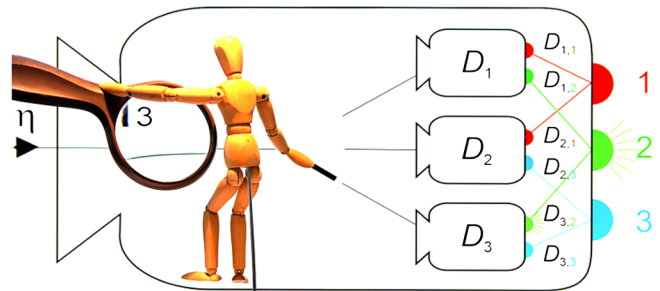


FIG. 1. Illustration of a three-outcome measurement which can be explained as selecting one from three two-outcome measurements. From the outside, the measurement apparatus (represented by the outer box) has three outcomes (represented by three lights of different colors). The state of a physical system tested by the apparatus is described by η_α , where $\alpha = 1, 2, 3$ is a variable that is hidden to the experimenter but can be read off by the measurement apparatus (illustrated by a robot inside the box using a magnifying glass), without disturbing the state of the system. From the inside, the measurement apparatus works as follows: based on the value of α (here, $\alpha = 3$), a corresponding two-outcome measurement D_α is selected (as the robot does by operating the switch selecting the measurement D_3).

which, for $n = 2$ and $n = 3$ and under some assumptions, may be taken as experimental falsifications of this subclass of GPTs. Finally, we take the possibility seriously that QT does not account for correlations in nature and provide a modified version of QT restricted to having at most n -outcome quantum measurements. This theory shows that nonsignaling correlations for which the number of outcomes is limited to n constitute an alternative that should be experimentally tested.

Quantum correlations are not n -chotomic.—For $m > n$, the set of m -outcome measurements in QT is strictly larger than the convex hull of the n -outcome measurements [20]. Hence, there are, e.g., three-outcome quantum measurements which cannot be implemented by choosing one from a set of two-outcome quantum measurements. Here, we present a result which goes beyond this observation. We demonstrate that if QT is correct, then any GPT describing nature needs to share this property. For this, we prove the yet more general result that any GPT not having this property cannot reproduce the correlations predicted by QT. This result only depends on properties of correlations and does not rely on how the preparation and measurement devices work. Therefore, it enables us to exclude all those GPTs in a device-independent way.

Suppose that two parties can perform several measurements on a bipartite system and that each party can independently choose among the measurements settings. For a fixed measurement setting μ on the first party and ν on the second party, we write $P_{\mu,\nu}(k, \ell)$ for the probability to obtain the corresponding outcomes k and ℓ . A set of such correlations is nonsignaling, if $\sum_{\ell} P_{\mu,\nu}(k, \ell) \equiv P_{\mu,-}(k)$ is independent of ν and $\sum_k P_{\mu,\nu}(k, \ell) \equiv P_{-,\nu}(\ell)$ is independent of μ . We are now interested in the case where the number of measurement outcomes is limited to n ; i.e., the measurements are n chotomic. An n -chotomic local measurement obeys $P_{\mu,-}(k) = 0$ for all k , except for a subset of size n , or, similarly, $P_{-,\nu}(\ell) = 0$ for all ℓ , except for a subset of size n . The set of nonsignaling n -chotomic correlations \mathcal{P}_n is then the convex hull of the set of nonsignaling correlations where all measurements are, at most, n chotomic.

We address the question of whether the set of quantum correlations contains correlations that are not in \mathcal{P}_n by considering the combinations of correlations in the Collins-Gisin-Linden-Massar-Popescu inequalities [21] in the formulation of Zohren and Gill [22], namely,

$$I'(\mathbf{P}) = P_{2,2}(k < \ell) + P_{1,2}(k > \ell) + P_{1,1}(k < \ell) + P_{2,1}(k \geq \ell), \quad (1)$$

where $P_{2,2}(k < \ell) = \sum_{k < \ell} P_{2,2}(k, \ell)$, and similarly for the other terms. I' can be evaluated for any set of bipartite correlations \mathbf{P} which features at least two measurement settings per party. We can now state our main result.

Theorem 1: For any n , there is an $m > n$ and quantum correlations $\mathbf{Q} \in \mathcal{P}_m$, such that $I'(\mathbf{Q}) < \inf[I'(\mathcal{P}_n)]$.

Proof.—It has been shown [23] that for any $\varepsilon > 0$, there exists an m and some quantum correlations $\mathbf{Q} \in \mathcal{P}_m$ such that $I'(\mathbf{Q}) < \varepsilon$. In Appendix A, we prove that $q_n \equiv \inf[I'(\mathcal{P}_n)] > 0$ for any n . Therefore, by choosing $\varepsilon = q_n/2$, the assertion follows. ■

This proves that, for any n , there are quantum correlations which are not nonsignaling n chotomic. For example, the hypothetical Popescu-Rohrlich box [12] is a GPT predicting correlations that are impossible according to QT. However, this GPT only contains dichotomic measurements. Hence, Theorem 1 reveals that QT contains correlations that are impossible to achieve for a Popescu-Rohrlich box. **Consequence 2:** QT contains correlations that cannot be explained by dichotomic GPTs, even if we admit supra-quantum correlations, such as Popescu-Rohrlich boxes.

Experiments.—Theorem 1 gives rise to the question: Is it feasible to experimentally demonstrate the existence of correlations which cannot be explained by n -chotomic GPTs with current quantum technology? As shown in Theorem 1, in principle, we could use experiments aiming to violate I' for this purpose. However, in practice, this approach is rather unfeasible since, even for excluding dichotomic GPTs, we would need to observe a value of I' below $\frac{1}{2}$, something that requires quantum measurements with at least ten outcomes [22]. Further investigation is therefore needed in order to identify inequalities with more modest experimental demands.

As a first step in this direction, we explore whether it is possible to experimentally exclude GPTs in which measurements are produced by selecting from, at most, n -outcome quantum measurements. These GPTs constitute interesting variants of QT in which the sets of measurements are arguably simpler than the one of QT, as we discuss below. In addition, unlike most alternatives to QT investigated in the past (e.g., local realistic theories), they share most of the predictions of QT, including the violation of Bell inequalities.

For this purpose, we compute the upper bounds on $1 - I'$ for GPTs for dichotomic and trichotomic quantum measurements when the outcomes k, l take values 1,2,3 (I_3) or values 1,2,3,4 (I_4). We observe that although violating the resulting inequalities is experimentally demanding, there is already experimental evidence [24–26] supporting that there are measurements which cannot be explained choosing from quantum dichotomic or quantum trichotomic measurements. Interestingly, when we compute the upper bounds for the bipartite all-versus-nothing Bell inequality with three four-outcome measurements [27], we observe that the results of a previous experiment show a clear violation of the quantum trichotomic bound [28]. This suggests that this inequality can be a powerful tool to provide conclusive evidence of the existence of genuinely nontrichotomic quantum measurements. We also compute the upper bounds of an inequality due to Vértesi and Bene [29] which, so far, has not been tested experimentally. However, it is *a priori* interesting for our considerations,

TABLE I. Upper bounds on correlations, required visibility, and experimental results. Values with an asterisk have been established in prior work. VB stands for the combination of correlations in the Vértesi-Bene inequality [29], I_n for $1 - I'$ with possible outcomes $k, \ell = 1, 2, \dots, n$, and AN for the correlations in an all-versus-nothing inequality [27]. The rows two-outcome, three-outcome, and quantum list upper bounds when quantum measurements have only two, only three, or an unrestricted number of outcomes, respectively. In the rows two-visibility and three-visibility, the required visibility [absence of white noise, i.e., minimal p if the prepared state is a mixture of the target state and a completely depolarized state $\rho_{\text{prepared}} = p\rho_{\text{target}} + (1-p)\rho_{\text{depolarized}}$] is shown, where the former is for violating the two-outcome bound and the latter for violating the three-outcome bound. In the last rows, observed experimental violations of the two-outcome and three-outcome bounds are shown, in terms of multiples of statistical standard deviations.

	VB [29]	I_3 [21,30]	I_4 [21]	AN [27]
Two-outcome	21.068*	0.20711	0.20711	8.1962
Three-outcome	0.30495	8.1962
Quantum	21.090*	0.30495*	0.36476*	9.0000*
Two-visibility	99.97%	90%	86%	92%
Three-visibility	95%	92%
Experiment	None	Ref. [24]	Ref. [25]	Ref. [28]
Two-violation		5.5σ	16σ	70σ
Three-violation	4.3σ	70σ

since it can be violated by a two-qubit system. Unfortunately, we find that the visibility required to falsify dichotomic quantum measurements using the Vértesi-Bene inequality is too high for current experiments.

We have summarized all our calculations and the significant experimental results in Table I. The methods that we have used for calculating the upper bounds are described in Appendix B. It is important to remark that all mentioned experiments fail to satisfy several of the conditions needed to extract loophole-free conclusions. For example, all of them require the fair sampling assumption due to the low detection efficiency. Furthermore, in all of these experiments, locality is assumed rather than enforced by spacelike separation. Most critically, in all studied cases, the n -outcome measurements are actually implemented using dichotomic measurements due to a limited number of detectors. But, the existing experiments suggest that a loophole-free version of these experiments is within current experimental reach and can demonstrate the existence of genuinely nondichotomic and nontrichotomic quantum measurements.

At this point, the conclusion is that there is already evidence that there are correlations in nature which cannot be explained by GPTs with dichotomic and trichotomic quantum measurements. However, more experimental effort is needed for a loophole-free confirmation of this result, and even more theoretical and experimental effort is needed for demonstrating correlations which cannot be explained by more general GPTs with dichotomic measurements.

Probabilistic theories with n -chotomic measurements.— Our main result, Theorem 1, establishes that nonsignaling n -chotomic correlations $\mathbf{P} \in \mathcal{P}_n$ cannot explain all quantum correlations. In this section, we take the possibility seriously that QT does not account for correlations in nature and we argue how n -chotomic measurements with fixed n may constitute a plausible alternative to the construction used in QT.

The first argument is the observation that, even in the everyday use of QT, we find situations in which the set of actual measurements is only a subset of the set of measurements possible *a priori*. One example is the superselection rules arising from the nonexistence of certain ways of manipulating a system and the constraints on its time evolution [31]. Another example arises when quantum systems can only be manipulated locally. There, the standard paradigm is the paradigm of local operations and classical communication in which several separated parties have access to a shared composite quantum system but there is no quantum interaction between the parts. Consequently, there are outcomes of two-outcome measurements that cannot participate in certain measurements with more than two outcomes [32,33].

The second argument why n -chotomic measurements may be a plausible alternative to QT is its simplicity. From the perspective of GPTs, the fact that a theory includes measurements which cannot be created by choosing from two-outcome measurements is surprising: Any measurement with more than two outcomes can be coarse grained to a two-outcome measurement (k , not k), simply by only distinguishing between the outcome labeled k and any other outcome. Now, consider the converse problem. Suppose that we have the set of all two-outcome measurements of a GPT and we want to construct the set of all measurements with any number of outcomes. Then, the arguably simplest way to do it is as it is illustrated in Fig. 1, i.e., by selecting from two-outcome measurements. The fact that this is not the case in QT tells us that QT is, in this sense, very special. Fortunately, Theorem 1 shows that we can test whether nature is special in this sense.

The third argument is that there is nothing *a priori* problematic in a dichotomic theory. To illustrate this point, we develop a dichotomic theory based on QT. For this purpose, it is enough to consider experiments consisting of two stages, the preparation stage and the measurement stage. In standard QT, a preparation is described by a density operator ρ and a measurement by a family of positive semidefinite operators (E_1, E_2, \dots) summing to $\mathbb{1}$, so that the probability to obtain outcome k is given by $\text{tr}(E_k \rho)$.

A straightforward example where two-outcome measurements are insufficient is a measurement which can perfectly distinguish between more than two states so that $\text{tr}(\rho_\ell E_k) = \delta_{\ell,k}$, where $\delta_{\ell,k}$ denotes the Kronecker delta. However, there is nothing particularly characteristic of QT in this example, as already in our everyday classical experience we can easily distinguish different preparations—for example, the six

distinct outcomes of a die. In order to be able to separate this trivial example from the case we are interested in, we consider a modification of QT. Imagine that the state preparation does not only prepare the quantum state but, in addition, transmits some information, e.g., an integer value α . In turn, the measurement apparatus is sensitive to α and can exhibit different behavior depending on α . This means that α carries some classical information, e.g., which state ρ_k was prepared or which side of the die is up, covering the aforementioned situation; see also Fig. 1. In fact, this scenario is more realistic than it may seem. For example, in a photon experiment, the halfwave plate used to prepare different polarization states may introduce a slight shift in momentum, and it may happen that the analyzing setup is sensible to this shift and gives a different response, depending on the momentum.

A general formalism to capture this situation is to modify the standard formulation of QT by replacing the density operator ρ by positive semidefinite operators $(\eta_1, \eta_2, \dots) \equiv \boldsymbol{\eta}$ obeying $\sum_{\alpha} \text{tr}(\eta_{\alpha}) = 1$ and to substitute each operator E_k by positive semidefinite operators $(D_{1,k}, D_{2,k}, \dots) \equiv \boldsymbol{D}$ such that $\sum_k D_{\alpha,k} = \mathbb{1}$ for each α . If there is no other sensitivity to α , then outcome k will have probability $P(k) = \sum_{\alpha} \text{tr}(\eta_{\alpha} D_{\alpha,k})$. If we restrict the quantum part of the measurements to be trivial, i.e., all $D_{\alpha,k}$ are either $\mathbb{1}$ or 0 , then, effectively, we would have a hidden variable model. If, for each α , at most two operators $D_{\alpha,k}$ are nonzero, then, on a fundamental level, all measurements are dichotomic, and similarly for the n -chotomic case.

Let us now use the above example to illustrate why at least bipartite correlations are required to falsify these GPTs. For a single party, we can always explain *a posteriori* any experiment in which the correlations of certain states $\boldsymbol{\eta}^{(\mu)}$ and measurements $\boldsymbol{D}^{(\nu)}$ are considered. Indeed, we may let $D_{\alpha,k}^{(\nu)} = p_k^{(\alpha,\nu)}$ and $\eta_{\alpha}^{(\mu)} = \delta_{\alpha,\mu}$, where $p_k^{(\mu,\nu)}$ are probability distributions that do not contradict the observations. A way to inhibit such constructions is to move into a setup in which a system is distributed between two parties and each of them performs local measurements. Then, instead of preparing states $\boldsymbol{\eta}^{(\mu)}$ and performing measurements $\boldsymbol{D}^{(\nu)}$, both parties perform independent local measurements $\boldsymbol{D}'^{(\mu)}$ and $\boldsymbol{D}^{(\nu)}$, respectively, on a fixed bipartite state $\boldsymbol{\eta}$. The resulting observations are then distributed according to the correlations

$$P_{\mu,\nu}(k, \ell) = \sum_{\alpha, \alpha'} \text{tr}(\eta_{\alpha', \alpha} D_{\alpha', k}^{(\mu)} \otimes D_{\alpha, \ell}^{(\nu)}). \quad (2)$$

When all local measurements are at most n chotomic, then, by construction, these correlations are nonsignaling n chotomic and are therefore subject to Theorem 1.

Conclusions.—Quantum theory is in agreement with all existing experimental evidence. Therefore, when exploring alternative theories to QT, it is sensible to focus on those giving similar predictions. In this Letter, we have studied a

large class of such alternative theories. We have considered a class of general probabilistic theories in which the set of measurements is constructed by selecting from measurements with n outcomes. For any n , these theories satisfy Bell-type inequalities which are violated by QT. Testing this prediction is a fundamental challenge for the future, as it would demonstrate that correlations in nature are stronger than those allowed by theories which, in other experiments, produce correlations exceeding those of QT, e.g., as it is the case for Popescu-Rohrlich boxes. However, this challenge is difficult and will require further efforts both in theory and experiments.

Meanwhile, as an example of the kind of tools that will be needed, we have considered theories with the same set of n -outcome measurements than QT for a fixed n , but such that any measurement with more outcomes is constructed by selecting measurements with only n outcomes. These theories share many features with QT and can, e.g., explain the violation of Bell inequalities. However, we have shown that these alternative theories satisfy certain Bell-type inequalities which are violated by QT. The violations predicted by QT are very small, and testing them requires high-precision experiments. We have identified previous experiments which, up to some assumptions, seem to rule out these theories for $n = 2$ and $n = 3$.

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APPENDIX A: PROOF OF THEOREM 1

For the remaining step in the proof of Theorem 1, we assume without loss of generality that all measurement outcomes are labeled $k, \ell = 1, 2, \dots$, and we define $\mathcal{P}_{n,r}$ as the subset of \mathcal{P}_n for which the maximal index k or ℓ is at most r . We show that (a) $\inf[I'(\mathcal{P}_{n,r})] \geq 2^{1-r}$ for any r and (b) $I'(\mathcal{P}_{n,r'}) = I'(\mathcal{P}_n)$ for some r' . It follows that $\inf[I'(\mathcal{P}_n)] = \inf[I'(\mathcal{P}_{n,r'})] \geq 2^{1-r'} > 0$ holds, which is the statement needed in order to complete the proof in the main text. (a) For arbitrary correlations $\boldsymbol{P} \in \mathcal{P}_{n,r}$, we denote by $\boldsymbol{P}' \in \mathcal{P}_{n,r-1}$ the correlations where in \boldsymbol{P} the r th outcomes are merged with the first outcomes. This implies $P_{\mu,\nu}'(k \geq \ell) = P_{\mu,\nu}(k \geq \ell) + P_{-\nu}(r) - P_{\mu,-}(r) + P_{\mu,\nu}(r, 1)$, and therefore, $I'(\boldsymbol{P}') = I'(\boldsymbol{P}) - [P_{2,2}(r, 1) + P_{1,2}(1, r) + P_{1,1}(r, 1) - P_{2,1}(r, 1)] \leq I'(\boldsymbol{P}) + P_{2,1}(r, 1) \leq 2I'(\boldsymbol{P})$. By induction and

due to $I'(\mathcal{P}_{n,1}) = \{1\}$, we have $2^{1-r} \leq I'(\mathcal{P})$. (b) We consider the set $\tilde{\mathcal{P}}_n$ of those correlations which can be created from the correlations in $\mathcal{P}_{n,n}$ by applying all changes of the labels of the outcomes $\lambda'_\mu: 1, \dots, n \rightarrow \mathbb{N}$, and similarly λ_ν , via $P_{\mu,\nu}(k, \ell) \mapsto P_{\mu,\nu}(\lambda'_\mu(k), \lambda_\nu(\ell))$, while all other correlation terms are 0. This does not yield more than $4n^2$ logical relations like $\lambda_\mu(k) < \lambda_\nu(l)$ in I' , and hence, at most 2^{4n^2} different labelings are needed to reach all logical relations. Since this is a finite set, there is an integer r' denoting the maximal resulting index in the labelings, and therefore, $I'(\mathcal{P}_{n,r'}) \supseteq I'(\tilde{\mathcal{P}}_n)$. By definition, \mathcal{P}_n is the convex hull of $\tilde{\mathcal{P}}_n$, so that $I'(\tilde{\mathcal{P}}_n) = I'(\mathcal{P}_n)$ follows from the fact that I' is affine. Therefore, $I'(\mathcal{P}_{n,r'}) = I'(\mathcal{P}_n)$ holds due to $\mathcal{P}_{n,r'} \subseteq \mathcal{P}_n$.

APPENDIX B: QUANTUM n -CHOTOMIC BOUNDS IN TABLE I

The maximal quantum value is known for some inequalities, or it can be numerically approximated by a hierarchy of semidefinite programs suggested by Navascués, Pironio, and Acín [34]. For n -chotomic quantum measurements, one can proceed similarly, since it is enough to maximize the value of the inequality, but with the additional assumption that at most n measurement outcomes are nontrivial. By exploring all possible combinations with n nontrivial outcomes and calculating the maximal bound for each of these cases, we obtain the n -chotomic bounds provided in Table I.

We used the third level of the hierarchy for the values in the rows labeled two-outcome and three-outcome. Since this is an upper approximation on the true value, these values are at most too pessimistic. For the values in the row labeled quantum, the given values are for certain quantum states and measurements. This value is optimal for AN, and the value coincides with the bound from the second level of the hierarchy for I_3 and I_4 . Only for VB does the third level of the hierarchy give a slightly larger value $21.092 > 21.090$.

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