$\mathbb{Z}_2$ and Chiral Anomalies in Topological Dirac Semimetals

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We demonstrate that topological Dirac semimetals, which possess two Dirac nodes, separated in momentum space along a rotation axis and protected by rotational symmetry, exhibit an additional quantum anomaly, distinct from the chiral anomaly. This anomaly, which we call the $\mathbb{Z}_2$ anomaly, is a consequence of the fact that the Dirac nodes in topological Dirac semimetals carry a $\mathbb{Z}_2$ topological charge. The $\mathbb{Z}_2$ anomaly refers to nonconservation of this charge in the presence of external fields due to quantum effects and has observable consequences due to its interplay with the chiral anomaly. We discuss possible implications of this for the interpretation of magnetotransport experiments on topological Dirac semimetals. We also provide a possible explanation for the magnetic field dependent angular narrowing of the negative longitudinal magnetoresistance, observed in a recent experiment on Na_3Bi.

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The recent theoretical [1–9] and experimental [10–15] discoveries of Weyl and Dirac semimetals have extended the growing family of materials with topologically nontrivial electronic structure. They have also highlighted beautiful analogies and connections that exist between the physics of materials with a topologically nontrivial electronic structure and the physics of relativistic fermions, described by the Dirac equation, which were anticipated some time ago by Volovik and others [16–19].

The chiral anomaly, which refers to nonconservation of the chiral charge in the presence of collinear external electric and magnetic fields, is a particularly striking and important example. First discovered theoretically by Adler [20] and by Bell and Jackiw [21], it provided the explanation for the observed decay of a neutral pion into two photons, prohibited by chiral charge conservation, or chiral symmetry. Very recently, a condensed matter manifestation of the chiral anomaly was observed in a Dirac semimetal Na_3Bi [22], and possibly also in the Weyl semimetal TaAs [23,24] and in $ZrTe_5$ [25], which is proposed to be a Dirac semimetal.

However, despite analogies, condensed matter systems with topologically nontrivial electronic structure are certainly significantly richer than the relativistic Dirac equation. In particular, there in fact exist two distinct classes of Dirac semimetals [26,27]. One, in which the Dirac points occur at time reversal invariant momenta in the first Brillouin zone (BZ) [7], and the second, in which the Dirac points occur in pairs, separated in momentum space along a rotation axis [8,9]. The Dirac semimetals, that have currently been realized experimentally [10,11], are of the second kind. It is now understood [26–30] that the Dirac points in such semimetals possess a nontrivial $\mathbb{Z}_2$ topological invariant, which protects the nodes and leads to the appearance of Fermi arc surface states, which connect projections of the node locations on the surface BZ, much like in Weyl semimetals.

A natural question to ask in this regard is whether the existence of such a $\mathbb{Z}_2$ topological charge manifests in any way in transport, as the chiral charge of Weyl nodes manifests in negative longitudinal magnetoresistance, attributed to the chiral anomaly [22,31–33]. In this Letter, we answer this question in the affirmative. We demonstrate that the Dirac semimetals with two Dirac nodes, carrying the $\mathbb{Z}_2$ topological charge, such as Na_3Bi and Cd_2As_3, exhibit the corresponding $\mathbb{Z}_2$ quantum anomaly, in addition to the chiral anomaly. We further demonstrate that the interplay of the two anomalies leads to observable manifestations in magnetotransport experiments. We also discuss possible relevance of our results to the recent magnetotransport measurements on Na_3Bi [22], which have provided the first strong experimental evidence for the chiral anomaly in condensed matter. In particular, we give a possible explanation for the magnetic field dependent angular narrowing of the negative longitudinal magnetoresistance due to the chiral anomaly, observed in this experiment.

While in this work we will specifically focus on the case of Na_3Bi, our results are equally applicable to Cd_2As_3. We start from the low energy Hamiltonian of Na_3Bi in momentum space, derived in [8]

$$H = v_F(\sigma^x s^z k_x - \sigma^y k_y) + m(\mathbf{k})\sigma^z + \frac{\gamma}{2}\sigma^x k_z(s^+ k_-^2 + s^- k_+^2). \tag{1}$$

The Pauli matrices $\mathbf{s}$ and $\boldsymbol{\sigma}$ act on the spin and the orbital parity degrees of freedom correspondingly and we will be using $\hbar = c = 1$ units throughout. The first two terms in Eq. (1) describe coupling between states of opposite parity, which forces them to be linear in the crystal momentum, measured from the Γ point in the first BZ. Since σ^z is the parity operator, the "mass" term $m(\mathbf{k})$ is parity even and has the following low energy form:

$$m(\mathbf{k}) = -m_0 + m_1 k_z^2, \tag{2}$$

which gives a pair of Dirac points at $\mathbf{k}_\pm = (0, 0, \pm\sqrt{m_0/m_1} \equiv \pm k_0)$. The last term in Eq. (1) is third order in the crystal momentum as a consequence of threefold rotational symmetry of the crystal structure of Na_3Bi, where the rotation axis is the z axis in Eq. (1). This term is much smaller than the other terms in the Hamiltonian in the vicinity of the Dirac points and we will thus ignore it henceforth. This omission has at most a quantitative effect on our results, but makes the presentation more transparent.

We now make the following important observation. In the absence of the last term in Eq. (1), the z component of the spin is a strictly conserved quantity. With the last term included, it will no longer be strictly conserved, but will have a long relaxation time due to the smallness of this term, which is what ultimately justifies ignoring it physically. Let $s = \pm 1$ be the eigenvalues of s^z. Then the Hamiltonian separates into two independent 2×2 blocks, each describing a Weyl semimetal with a single pair of Weyl nodes, separated along the z axis in momentum space

$$H_s = v_F(\sigma^x s k_x - \sigma^y k_y) + m(\mathbf{k})\sigma^z. \tag{3}$$

The two Weyl semimetals are related to each other by the time reversal operation and thus each of the two Dirac band touching points at $\mathbf{k}_\pm$ contains two Weyl nodes of opposite chirality and opposite eigenvalue s. It is convenient to expand the Weyl Hamiltonians H_s near the nodes. To linear order, one obtains

$$H_s = v_F s\sigma^x k_x - v_F \sigma^y k_y + \tilde{v}_F \tau^z \sigma^z (k_z - \tau^z k_0), \tag{4}$$

where the two eigenvalues of the Pauli matrix τ^z, $\tau = \pm 1$ refer to the two nodes and $\tilde{v}_F = 2\sqrt{m_0 m_1}$. We now introduce Hermitian 4×4 gamma matrices as

$$\Gamma_s^1 \equiv \gamma_s^0 \gamma_s^1 = s\sigma^x, \qquad \Gamma_s^2 \equiv \gamma_s^0 \gamma_s^2 = -\sigma^y,$$
$$\Gamma_s^3 \equiv \gamma_s^0 \gamma_s^3 = \tau^z \sigma^z, \tag{5}$$

where γ_s^μ are the relativistic Dirac gamma matrices. Note that we do not need an explicit representation for the Dirac matrices γ_s^μ, all we really need to know are the Hermitian gamma matrices in Eq. (5), the Dirac matrices are introduced only as a convenient notation. We may now define the chiral charge operator

$$\gamma_s^5 = i\gamma_s^0 \gamma_s^1 \gamma_s^2 \gamma_s^3 = -i\Gamma_s^1 \Gamma_s^2 \Gamma_s^3 = -s\tau^z, \tag{6}$$

and the $\mathbb{Z}_2$ charge operator

$$s\gamma_s^5 = -\tau^z. \tag{7}$$

The physical meaning of the chiral charge operator is clear: the eigenvalues of γ_s^5 are the chiralities of the four Weyl fermions, which make up the two Dirac fermions. To clarify the meaning of the $\mathbb{Z}_2$ charge operator, we note that Eq. (7) is equivalent to $C_{\mathbb{Z}_2} = (C_\uparrow - C_\downarrow)/2$, where $C_{\mathbb{Z}_2}$ is the $\mathbb{Z}_2$ charge and $C_{\uparrow,\downarrow}$ are the chiral charges of the spin-up and spin-down Weyl fermions. This definition is closely analogous to the definition of the $\mathbb{Z}_2$ invariant for a two-dimensional quantum spin Hall insulator with conserved spin in terms of the spin Chern numbers [34]. Once the spin-conservation-violating terms in Eq. (1) and impurity scattering are included, only the modulo 2 part of the $\mathbb{Z}_2$ charge retains its meaning. Note that both the chiral charge and the $\mathbb{Z}_2$ charge operators commute with the linearized Hamiltonian, expressing the approximate chiral and $\mathbb{Z}_2$ charge conservation.

What is particularly interesting is that the existence of the conserved $\mathbb{Z}_2$ charge has experimentally observable manifestations, which may be regarded as manifestations of the $\mathbb{Z}_2$ anomaly in analogy to the chiral anomaly. To see this, let us couple gauge fields to the fermions. Since we have two conserved quantities: charge and the z component of the spin, we may introduce, for now purely formally, both the charge and the spin gauge fields. The real time Lagrangian, written in the relativistic notation, has the form

$$\begin{aligned}\mathcal{L} &= \psi_s^\dagger i\partial_t \psi_s - H \\ &= \bar{\psi}_s i\gamma_s^\mu [\partial_\mu + ie(A_\mu + s\tilde{A}_\mu) + i\gamma_s^5(b_\mu + s\tilde{b}_\mu)]\psi_s,\end{aligned} \tag{8}$$

where summation over s is implicit, $\bar{\psi}_s = \psi_s^\dagger \gamma_s^0$, and we have absorbed the Fermi velocities into the definition of the corresponding coordinates. A_μ in Eq. (8) are electromagnetic gauge fields, which couple symmetrically to the fermions with different spin eigenvalue s, while $\tilde{A}_\mu$ are the fictitious spin gauge fields, which couple antisymmetrically. What gives them physical meaning is that the functional derivative of the action with respect to $\tilde{A}_\mu$ produces the corresponding component of the spin current, which is well defined since the spin is conserved. b_μ are the chiral gauge fields, which couple antisymmetrically to Weyl fermions of opposite chirality and thus shift them in opposite directions in momentum space or in energy. Finally, $\tilde{b}_\mu$ couple antisymmetrically to fermions with different $\mathbb{Z}_2$ charge and thus may be called, with a slight abuse of terminology, the $\mathbb{Z}_2$ gauge fields. Specifically, we have

$$b_\mu = (\mu_5, 0, 0, 0), \qquad \tilde{b}_\mu = (\tilde{\mu}_5, 0, 0, b), \tag{9}$$

where $b \equiv \tilde{v}_F k_0$. Here μ_5 is the chiral chemical potential, which is conjugate to the chiral charge operator and shifts Weyl fermions of opposite chirality in opposite direction in energy. It is equal to zero in equilibrium, but will in general

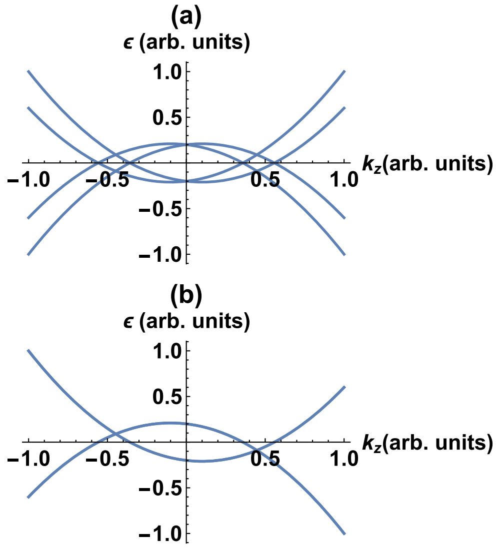

FIG. 1. (a) Effect of a nonzero chiral chemical potential μ_5 on the double Dirac electronic structure. (b) Effect of a nonzero $\mathbb{Z}_2$ chemical potential $\tilde{\mu}_5$ on the electronic structure.

be nonzero away from equilibrium in the presence of charge currents and external electromagnetic field, see Fig. 1. Similarly, $\tilde{\mu}_5$ is the $\mathbb{Z}_2$ chemical potential, conjugate to the $\mathbb{Z}_2$ charge operator. It shifts the two Dirac points in opposite directions in energy. Finally, the z component of $\tilde{b}_\mu$ is the only component that is present in equilibrium and simply determines the distance between the two Dirac points in momentum space.

Integrating out fermions in Eq. (8) using, e.g., the Fujikawa's method [35], we obtain [36,37]

$$S = -\frac{e^2}{2\pi^2}\int dtd^3 r b_\mu \epsilon^{\mu\nu\alpha\beta}(A_\nu \partial_\alpha A_\beta + \tilde{A}_\nu \partial_\alpha \tilde{A}_\beta) - \frac{e^2}{2\pi^2}\int dtd^3 r \tilde{b}_\mu \epsilon^{\mu\nu\alpha\beta}(A_\nu \partial_\alpha \tilde{A}_\beta + \tilde{A}_\nu \partial_\alpha A_\beta). \quad (10)$$

As mentioned above, the functional derivative of S with respect to the gauge field A_μ gives the charge current, while the functional derivative with respect to $\tilde{A}_\mu$ gives the spin current

$$j^\nu = -\frac{\delta S}{\delta A_\nu} = \frac{e^2}{\pi^2} b_\mu \epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta + \frac{e^2}{\pi^2}\tilde{b}_\mu \epsilon^{\mu\nu\alpha\beta}\partial_\alpha \tilde{A}_\beta,$$
$$\tilde{j}^\nu = -\frac{\delta S}{\delta \tilde{A}_\nu} = \frac{e^2}{\pi^2} b_\mu \epsilon^{\mu\nu\alpha\beta}\partial_\alpha \tilde{A}_\beta + \frac{e^2}{\pi^2}\tilde{b}_\mu \epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta. \quad (11)$$

On the other hand, since b_μ and $\tilde{b}_\mu$ act as chiral and $\mathbb{Z}_2$ gauge fields correspondingly, functional derivatives of the action with respect to these give the chiral and the $\mathbb{Z}_2$ currents. Upon taking the divergence, we then obtain the following anomalous chiral and $\mathbb{Z}_2$ charge conservation laws

$$\partial_\mu j_5^\mu = \frac{e^2}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}(F_{\mu\nu}F_{\alpha\beta} + \tilde{F}_{\mu\nu}\tilde{F}_{\alpha\beta}),$$
$$\partial_\mu \tilde{j}_5^\mu = \frac{e^2}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}(F_{\mu\nu}\tilde{F}_{\alpha\beta} + \tilde{F}_{\mu\nu}F_{\alpha\beta}), \quad (12)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ are the field strengths. We may pick a gauge for the spin gauge fields $\tilde{A}_\mu$ in which only the temporal component $\tilde{A}_0$, which is conjugate to the spin density, is nonzero, as we do not expect spin analogs of magnetic fields to arise in our context. Henceforth, we will thus take $\tilde{A}_\mu = (\tilde{A}_0, 0, 0, 0)$. The physical meaning of $\tilde{A}_0$ is defined by the relation $\tilde{j}^0 = g(\epsilon_F)\tilde{A}_0$, where $\tilde{j}_0$ is the spin density and $g(\epsilon_F)$ is the density of states at Fermi energy, which we assume to be small but finite.

Let us now concentrate on the situation of interest to us, that describes Na_3Bi. Substituting Eq. (9) into Eq. (11), we obtain

$$\mathbf{j} = \frac{e^2}{\pi^2}\mu_5 \mathbf{B} + \tilde{\sigma}_{xy}(\hat{z} \times \tilde{\mathbf{E}}), \quad (13)$$

where $\tilde{\sigma}_{xy} = e^2 b/\pi^2$, and $\tilde{\mathbf{E}} = -\boldsymbol{\nabla}\tilde{A}_0$. Analogously,

$$\tilde{\mathbf{j}} = \frac{e^2}{\pi^2}\tilde{\mu}_5 \mathbf{B} + \tilde{\sigma}_{xy}(\hat{z} \times \mathbf{E}). \quad (14)$$

The physical meaning of Eqs. (13) and (14) is straightforward to understand. The first term on the right-hand side in Eq. (13) describes the chiral magnetic effect [38,39], i.e., a contribution to the charge current, proportional to the applied magnetic field and the chiral chemical potential. Analogously, the first term in Eq. (14) describes the $\mathbb{Z}_2$ magnetic effect, which generates a contribution to the spin current, proportional to the $\mathbb{Z}_2$ chemical potential and the external magnetic field. The second term in Eq. (14) describes the spin Hall effect, i.e., the generation of a spin current, transverse to the applied external electric field, and $\tilde{\sigma}_{xy}$ is the corresponding spin Hall conductivity. Note that, in exact analogy to Weyl semimetals [2,3], this spin Hall conductivity may be associated with the Fermi arc edge states: each value of the momentum k_z between the Dirac points contributes e^2/π to the total spin Hall conductivity. Finally, the second term in Eq. (13) describes the inverse spin Hall effect, i.e., the generation of a charge current by a gradient of the spin density.

Similarly, the chiral and the $\mathbb{Z}_2$ charge conservation laws take the following form

$$\frac{\partial n_5}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{j}_5 = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \frac{n_5}{\tau_5},$$
$$\frac{\partial \tilde{n}_5}{\partial t} + \boldsymbol{\nabla}\cdot \tilde{\mathbf{j}}_5 = \frac{e^2}{2\pi^2}\tilde{\mathbf{E}}\cdot\mathbf{B} - \frac{\tilde{n}_5}{\tau_5}, \quad (15)$$

where we have introduced a finite relaxation time τ_5 to account for the fact that the chiral n_5 and the $\mathbb{Z}_2$ $\tilde{n}_5$ charges are not in reality strictly conserved, when terms, nonlinear in momentum, are included in the Hamiltonian [33]. We have taken the relaxation times for both charges to be the same for simplicity. Note that the coefficient of $\mathbf{E}\cdot\mathbf{B}$ on the right hand side of the first of Eq. (15) is twice as large as it would be for a single Dirac point (or single pair of Weyl points). This is because we have two Dirac points (two pairs of Weyl points) and their chiral anomalies add up, which is not obvious in advance, the anomalies may cancel instead. The reason they add in our case is that there exists a conserved quantity (spin) that distinguishes the two pairs of Weyl fermions and, as a result, the chiral chemical potential couples symmetrically to them.

To account for regular transport processes, not related to the anomalies, we also need to add ordinary Drude conductivity terms to the right hand side of Eqs. (13) and (14), which we do by hand [40]. Then we finally obtain

$$\mathbf{j} = \sigma_0\mathbf{E} + \frac{e^2}{\pi^2}\mu_5\mathbf{B} + \tilde{\sigma}_{xy}(\hat{z}\times\tilde{\mathbf{E}}),$$
$$\tilde{\mathbf{j}} = \sigma_0\tilde{\mathbf{E}} + \frac{e^2}{\pi^2}\tilde{\mu}_5\mathbf{B} + \tilde{\sigma}_{xy}(\hat{z}\times\mathbf{E}), \quad (16)$$

where σ_0 is the Drude conductivity. We take σ_0 to be the same for both charge and spin, which is true in our approximation of an exactly conserved spin.

Equations (15) and (16) must be solved simultaneously to obtain the magnetoresistance. Suppose a uniform dc charge current is injected into the sample along the x direction, while voltage leads are attached to the edges, perpendicular to the y direction. In this case, we have from Eq. (15)

$$\mu_5 = \frac{n_5}{g(\epsilon_F)} = \frac{e^2\tau_5}{2\pi^2 g(\epsilon_F)}\mathbf{E}\cdot\mathbf{B},$$
$$\tilde{\mu}_5 = \frac{\tilde{n}_5}{g(\epsilon_F)} = \frac{e^2\tau_5}{2\pi^2 g(\epsilon_F)}\tilde{\mathbf{E}}\cdot\mathbf{B}. \quad (17)$$

Substituting these into Eq. (16), we obtain

$$\mathbf{j} = \sigma_0\mathbf{E} + \chi(\mathbf{E}\cdot\mathbf{B})\mathbf{B} + \tilde{\sigma}_{xy}(\hat{z}\times\tilde{\mathbf{E}}),$$
$$\tilde{\mathbf{j}} = \sigma_0\tilde{\mathbf{E}} + \chi(\tilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{B} + \tilde{\sigma}_{xy}(\hat{z}\times\mathbf{E}), \quad (18)$$

where $\chi = e^4\tau_5/2\pi^4 g(\epsilon_F)$. We now solve the equations

$$j_y = j_z = \tilde{j}_x = \tilde{j}_y = \tilde{j}_z = 0, \quad (19)$$

for $E_{y,z}$, $\tilde{E}_{x,y,z}$ and substitute the result into the equation for j_x to obtain the diagonal resistivity ρ_{xx}. The equation $\tilde{j}_x = 0$ follows from the assumption that we have spin-unpolarized leads. We obtain

$$\rho_{xx}^{-1}(\mathbf{B}) = \sigma_0 + \frac{\chi B_x^2 + \tilde{\sigma}_{xy}^2(1+\chi B_z^2/\sigma_0)/\sigma_0}{1+\chi(B_y^2+B_z^2)/\sigma_0}. \quad (20)$$

Equation (20) is our main result.

Several features of Eq. (20) are noteworthy. Setting $\mathbf{B} = 0$ we obtain

$$\rho_{xx}^{-1}(0) = \sigma_0 + \frac{\tilde{\sigma}_{xy}^2}{\sigma_0}. \quad (21)$$

The second term in Eq. (21) represents a reduction of the diagonal resistivity due to the spin Hall effect, which in turn is associated with the Fermi arc surface states. At a nonzero magnetic field, the dependence of $\rho_{xx}^{-1}(\mathbf{B})$ on the angle between the magnetic field and the current demonstrates the narrowing effect, observed in Ref. [22], see Fig. 2. The origin of this effect is the magnetic field dependence of the denominator in Eq. (20). Physically, this follows from the magnetic field dependence of the chiral chemical potential, Eq. (17). Another consequence of the $\mathbb{Z}_2$ anomaly (and the quantum spin Hall effect as one of its manifestations), is the anisotropy between the angular dependences of the magnetoresistance when the magnetic field is rotated in the xy and the xz planes. Let ϕ be the angle between the magnetic field and the x axis when the field is rotated in the xy plane while θ be the same angle when the field is rotated in the xz plane. Then we obtain

$$\rho_{xx}^{-1}(\theta=\phi) - \rho_{xx}^{-1}(\phi) = \frac{\tilde{\sigma}_{xy}^2}{\sigma_0}\frac{\chi B^2\sin^2\phi/\sigma_0}{1+\chi B^2\sin^2\phi/\sigma_0}, \quad (22)$$

where $\rho_{xx}^{-1}(\theta)$ refers to the inverse resistivity for the field rotated in the xz plane, while $\rho_{xx}^{-1}(\phi)$ refers to the inverse resistivity for the field rotated in the xy plane. This anisotropy exists only when $\tilde{\sigma}_{xy}$ is not zero and is a direct consequence of the spin Hall effect and thus the $\mathbb{Z}_2$ anomaly.

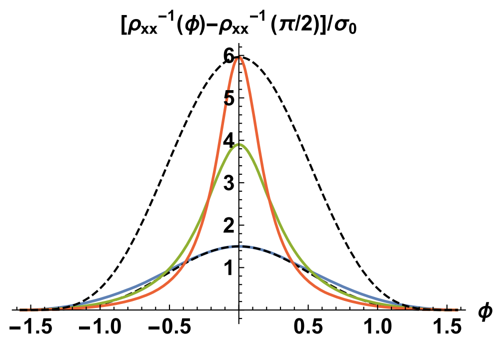

FIG. 2. Plots of $\rho_{xx}^{-1}(\phi) - \rho_{xx}^{-1}(\pi/2)$ for different values of the magnetic field, assuming $\tilde{\sigma}_{xy} = \sigma_0$ and for the field rotated in the xy plane. Dashed lines represent $\cos^4\phi$ dependence, normalized to the same maximum value. The fit to $\cos^4\phi$ is good for the smallest magnitude of the magnetic field (solid blue line), but becomes very poor for the largest (solid red line).

In conclusion, we have demonstrated that in Dirac semimetals with two Dirac nodes, separated in momentum space along a rotation axis and characterized by a nontrivial $\mathbb{Z}_2$ topological charge, there exists the corresponding $\mathbb{Z}_2$ anomaly. This refers to anomalous nonconservation of the $\mathbb{Z}_2$ topological charge in the presence of external electromagnetic fields and gradients of the (nearly) conserved spin density and is closely analogous to the chiral anomaly, which is also present. We have shown that the interplay of the $\mathbb{Z}_2$ and the chiral anomalies leads to observable effects in magnetotransport. We have also provided a possible explanation for the magnetic field dependent narrowing of the dependence of the positive magnetoconductivity on the angle between the current and the applied magnetic field, observed in a recent experiment [22].

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[1] X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B **83**, 205101 (2011).
[2] K.-Y. Yang, Y.-M. Lu, and Y. Ran, Phys. Rev. B **84**, 075129 (2011).
[3] A. A. Burkov and L. Balents, Phys. Rev. Lett. **107**, 127205 (2011).
[4] A. A. Burkov, M. D. Hook, and L. Balents, Phys. Rev. B **84**, 235126 (2011).
[5] G. Xu, H. Weng, Z. Wang, X. Dai, and Z. Fang, Phys. Rev. Lett. **107**, 186806 (2011).
[6] W. Witczak-Krempa and Y. B. Kim, Phys. Rev. B **85**, 045124 (2012).
[7] S. M. Young, S. Zaheer, J. C. Y. Teo, C. L. Kane, E. J. Mele, and A. M. Rappe, Phys. Rev. Lett. **108**, 140405 (2012).
[8] Z. Wang, Y. Sun, X.-Q. Chen, C. Franchini, G. Xu, H. Weng, X. Dai, and Z. Fang, Phys. Rev. B **85**, 195320 (2012).
[9] Z. Wang, H. Weng, Q. Wu, X. Dai, and Z. Fang, Phys. Rev. B **88**, 125427 (2013).
[10] Z. K. Liu, B. Zhou, Y. Zhang, Z. J. Wang, H. M. Weng, D. Prabhakaran, S.-K. Mo, Z. X. Shen, Z. Fang, X. Dai, Z. Hussain, and Y. L. Chen, Science **343**, 864 (2014).
[11] M. Neupane, S.-Y. Xu, R. Sankar, N. Alidoust, G. Bian, C. Liu, I. Belopolski, T.-R. Chang, H.-T. Jeng, H. Lin, A. Bansil, F. Chou, and M. Z. Hasan, Nat. Commun. **5**, 3786 (2014).
[12] S.-Y. Xu *et al.*, Science **349**, 613 (2015).
[13] B. Q. Lv, N. Xu, H. M. Weng, J. Z. Ma, P. Richard, X. C. Huang, L. X. Zhao, G. F. Chen, C. E. Matt, F. Bisti, V. N. Strocov, J. Mesot, Z. Fang, X. Dai, T. Qian, M. Shi, and H. Ding, Nat. Phys. **11**, 724 (2015).
[14] B. Q. Lv, H. M. Weng, B. B. Fu, X. P. Wang, H. Miao, J. Ma, P. Richard, X. C. Huang, L. X. Zhao, G. F. Chen, Z. Fang, X. Dai, T. Qian, and H. Ding, Phys. Rev. X **5**, 031013 (2015).
[15] L. Lu, Z. Wang, D. Ye, L. Ran, L. Fu, J. D. Joannopoulos, and M. Soljačić, Science **349**, 622 (2015).
[16] G. E. Volovik, Sov. Phys. JETP **67**, 1804 (1988).
[17] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford, Clarendon, 2003).
[18] G. E. Volovik, in *Quantum Analogues: From Phase Transitions to Black Holes and Cosmology*, Lecture Notes in Physics Vol. 718, edited by W. Unruh and R. Schtzhold (Springer, Berlin, Heidelberg, 2007).
[19] S. Murakami, New J. Phys. **9**, 356 (2007).
[20] S. L. Adler, Phys. Rev. **177**, 2426 (1969).
[21] J. S. Bell and R. Jackiw, Nuovo Cimento A **60**, 4 (1969).
[22] J. Xiong, S. K. Kushwaha, T. Liang, J. W. Krizan, M. Hirschberger, W. Wang, R. J. Cava, and N. P. Ong, Science **350**, 413 (2015).
[23] C. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, Z. Lin, B. Tong, N. Alidoust, C.-C. Lee, S.-M. Huang, H. Lin, M. Neupane, D. S. Sanchez, H. Zheng, G. Bian, J. Wang, C. Zhang, T. Neupert, M. Z. Hasan, and S. Jia, arXiv:1503.02630.
[24] X. Huang, L. Zhao, Y. Long, P. Wang, D. Chen, Z. Yang, H. Liang, M. Xue, H. Weng, Z. Fang, X. Dai, and G. Chen, Phys. Rev. X **5**, 031023 (2015).
[25] Q. Li, D. E. Kharzeev, C. Zhang, Y. Huang, I. Pletikosic, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu, and T. Valla, arXiv:1412.6543.
[26] B.-J. Yang and N. Nagaosa, Nat. Commun. **5**, 4898 (2014).
[27] B.-J. Yang, T. Morimoto, and A. Furusaki, Phys. Rev. B **92**, 165120 (2015).
[28] E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B **91**, 121101 (2015).
[29] C. Fang, Y. Chen, H.-Y. Kee, and L. Fu, Phys. Rev. B **92**, 081201 (2015).
[30] S. Kobayashi and M. Sato, Phys. Rev. Lett. **115**, 187001 (2015).
[31] D. T. Son and B. Z. Spivak, Phys. Rev. B **88**, 104412 (2013).
[32] A. A. Burkov, Phys. Rev. Lett. **113**, 247203 (2014).
[33] A. A. Burkov, Phys. Rev. B **91**, 245157 (2015).
[34] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[35] K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979).
[36] A. A. Zyuzin and A. A. Burkov, Phys. Rev. B **86**, 115133 (2012).
[37] S. T. Ramamurthy and T. L. Hughes, Phys. Rev. B **92**, 085105 (2015).
[38] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).
[39] D. T. Son and N. Yamamoto, Phys. Rev. Lett. **109**, 181602 (2012).
[40] D. T. Son and P. Surówka, Phys. Rev. Lett. **103**, 191601 (2009).