Quantum Phase Transition in the Finite Jaynes-Cummings Lattice Systems

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Phase transitions are commonly held to occur only in the thermodynamical limit of a large number of system components. Here, we exemplify at the hand of the exactly solvable Jaynes-Cummings (JC) model and its generalization to finite JC lattices that finite component systems of coupled spins and bosons may exhibit quantum phase transitions (QPTs). For the JC model we find a continuous symmetry-breaking QPT, a photonic condensate with a macroscopic occupation as the ground state, and a Goldstone mode as a low-energy excitation. For the two site JC lattice we show analytically that it undergoes a Mott-insulator to superfluid QPT. We identify as the underlying principle of the emergence of finite system QPTs the combination of increasing atomic energy and increasing interaction strength between the atom and the bosonic mode, which allows for the exploration of an increasingly large portion of the infinite dimensional Hilbert space of the bosonic mode. This suggests that finite system phase transitions will be present in a broad range of physical systems.

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Introduction.—Quantum phase transitions (QPTs) and spontaneous symmetry breaking are fundamental concepts in physics that lie at the heart of our understanding of various aspects of nature, e.g., phases of matter such as magnetism and superconductivity [1,2] or the generation of mass [3,4] in high energy physics. A second-order QPT is characterised by a closing spectral gap and degenerate ground states with a spontaneously broken symmetry. A QPT is typically held to occur only in the thermodynamical limit, i.e., a system with a diverging number of constituent particles or lattice sites [1]. A finite system size generally opens the spectral gap, lifts the ground state degeneracy, and restores the symmetry of the ground state [5,6].

A notable exception is a recent finding in Ref. [7] concerning the Rabi model [8–12], which describes a single-mode cavity field coupled to a two-level atom. While the Dicke model, an N-atom generalization of the Rabi model, has long been known for having a QPT for $N \to \infty$ [13,14], Ref. [7] demonstrates that the Rabi model itself undergoes a QPT with the same universal properties when the ratio η of the transition frequency to the cavity frequency diverges [11,15,16], and that the finite-frequency scaling exponents for η are identical to those for N [17,18]. It is then urgent and important task to see if reaching a limit of the QPT for a system of finite components is a principle that is generally applicable to photonic (phononic) systems with different underlying symmetries, phases, and dimensions. If positively answered, it could open up an important possibility of experimentally investigating the critical phenomena in a fully controlled quantum system [19].

In this Letter, we consider the Jaynes-Cummings (JC) model [20], the Rabi model without the so-called counter-rotating terms, which due to its $U(1)$ symmetry is exactly solvable. We first point out that the well-known analytical solution of the JC model exhibits a ground state instability in the $\eta \to \infty$ limit, beyond a critical coupling strength, in the sense that the ground state can lower its energy indefinitely by increasing its photon occupation. In this regime, we derive the analytical solution for the ground state and the excitation spectrum by developing a low-energy effective theory. It shows that the JC model undergoes a second order superradiant QPT. In the broken-symmetry phase, we find that the ground state forms a photon condensate with a macroscopic photon occupation number and that the excitation spectrum is gapless because the Goldstone mode [21] emerges. Note that, unlike in Ref. [22,23], the QPT discussed here occurs in the absence of a driving field.

We develop this further by showing that the JC lattice model with only two lattice sites, the JC dimer, undergoes a Mott-insulating-superfluid QPT in the same $\eta \to \infty$ limit. While the JC lattice model can undergo a Mott-insulating-superfluid QPT in the limit of infinite lattice sites [24–30], here the QPT is supported by the infinite dimensional Hilbert space associated with the harmonic oscillator degree of freedom. Our exact analytical solution shows that (i) the antisymmetric normal mode of the coupled cavities undergoes a transition from an insulating phase to a superfluid phase with a broken global $U(1)$ symmetry, while the symmetric mode gets merely squeezed in the superfluid phase, and (ii) the spectral gap of the antisymmetric mode closes at the critical point, beyond which the excitation is gapless, while the symmetric mode remains gapped for any coupling strength. Our analysis is analytic and fully quantum mechanical, going beyond the mean-field approach that is often used in the studies of the JC lattice model for lack of the exact methods [25,28].

Quantum phase transition in the JC model.—The Jaynes-Cummings Hamiltonian reads

$$H_{\rm JC} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a\sigma_+ + a^\dagger\sigma_-). \qquad (1)$$

Here, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ with $\sigma_{x,y,z}$ the Pauli matrices, and a ($a^\dagger$) is the lowering (raising) operator of a cavity field. The cavity frequency is ω_0, the transition frequency is Ω, and the coupling strength is λ. The conserved total number of excitations $N_{\rm tot} = a^\dagger a + \sigma_+\sigma_-$ leads to a $U(1)$-continuous symmetry. Let us denote $|n\rangle$ as an n-photon Fock state and $|\uparrow(\downarrow)\rangle$ as an eigenstate of σ_z with an eigenvalue $1(-1)$. We introduce a dimensionless coupling strength $g = \lambda/\sqrt{\omega_0\Omega}$ and a frequency ratio $\eta = \Omega/\omega_0$. Typically, the JC model is obtained as an approximation to the Rabi model by neglecting the counterrotating terms $-\lambda(a\sigma_- + a^\dagger\sigma_+)$ [7]. In systems where the atom-field interaction can be engineered, such as in circuit QED [31] or trapped-ion systems [32], such counterrotating terms can be strongly suppressed and, remarkably, for an atomic $\Delta m = \pm 1$ transition in interaction with a circularly polarized light mode the rotating wave approximation is exact such that Eq. (1) becomes a correct description for any g [33].

The vacuum state $|0,\downarrow\rangle$ is an energy eigenstate of the JC model with an eigenvalue $E_{0,\downarrow} = -\Omega/2$. There are two basis states with a total number of excitations n, $|n,\downarrow\rangle$ and $|n-1,\uparrow\rangle$, which span the so-called JC doublet, denoted as $|n,\pm\rangle$, whose energy eigenvalues read

$$E_{n,\pm}(\omega_0,\Omega,g) = \left(n - \frac{1}{2}\right)\omega_0 \pm \frac{\Omega}{2}\sqrt{(1-\eta^{-1})^2 + 4g^2 n\eta^{-1}}. \qquad (2)$$

Regardless of η, for $g < 1$ the ground state of Eq. (1) is always $|0,\downarrow\rangle$, until at $g = 1$ there occurs a level crossing between $|0,\downarrow\rangle$ and $|1,-\rangle$. This is followed by a series of level crossings between the lower-energy states of adjacent JC doublets $|n,-\rangle$ and $|n+1,-\rangle$ [Fig. 1(a)]. Therefore, increasing the atom-cavity coupling strength increases $\langle N_{\rm tot}\rangle$ in the ground state, denoted as n_G, in discrete steps [Fig. 1(c)]; in this sense, the JC-type atom-cavity coupling itself assumes the role of a chemical potential. Moreover, as η increases, the increase of n_G becomes progressively sharper near $g = 1$ [Fig. 1(c)].

We now consider a particular limit of $\eta \to \infty$ and $\lambda/\omega_0 \to \infty$, while g is kept finite. We emphasize that the oscillator frequency ω_0, a unit of energy in our analysis, is considered to be nonzero, as otherwise the spectrum would be unbounded from below for any nonzero g. First, for $\eta \gg 1$, $E_{n,-}$ can be expanded to give

$$E_{n,-}(\omega_0,\Omega,g) = n(1-g^2)\omega_0 + g^2 n(g^2 n - 1)\eta^{-1}\omega_0 - \frac{\Omega}{2} + \mathcal{O}(\eta^{-2}). \qquad (3)$$

Therefore, when $\eta \to \infty$, $\lambda/\omega_0 \to \infty$, and g is finite, the nonlinearity in the spectrum of the JC model

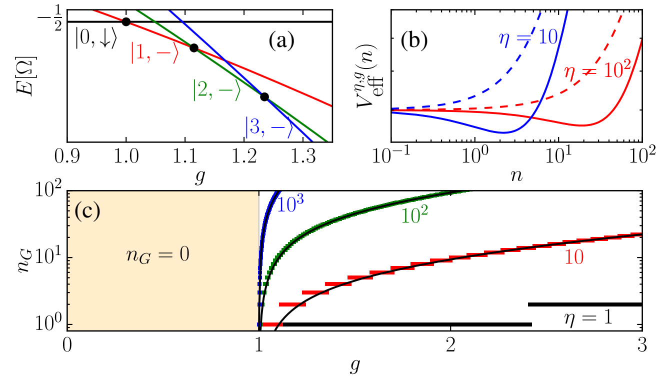

FIG. 1. Analytic solution of the JC model. (a) Level crossings for the ground state for a frequency ratio $\eta = 10$. (b) An effective potential $V_{\rm eff}^{\eta,g}$ for $g = 0.8$ (dashed) and $g = 1.2$ (solid) for different values of $\eta = 10$ and 100. (c) The total number of excitation of the ground state. As η increases, the change near $g = 1$ becomes progressively sharper, which is well described by $n_{\rm sp}(g) = \eta(g^2 - g^{-2})/4$ (solid).

disappears, leading to a harmonic spectrum, i.e., $\lim_{\eta\to\infty}[E_{n,-}(\eta,g) - E_{0,\downarrow}] = \omega_0(1-g^2)n$, which is a valid expression for any finite n. The excitation energy for $g < 1$, a normal phase, is therefore $\epsilon_{\rm np} = \omega_0(1-g^2)$, which becomes zero at $g = 1$, leading to a degeneracy between $|n,-\rangle$ of any finite n and $|0,\downarrow\rangle$. For $g > 1$, Eq. (2) shows a ground state instability (in the limit $\eta \to \infty$) in the sense that the ground state energy can be indefinitely lowered by increasing n. It is insightful to consider the energy spectrum given in Eq. (2) as an effective potential for n, $V_{\rm eff}^{\eta,g}(n) \equiv (n-\frac{1}{2}) - \frac{\eta}{2}\sqrt{(1-\eta^{-1})^2 + 4g^2 n\eta^{-1}}$, where n is approximated to be a real number and η, g are constants. We find the potential minimum at $n=0$ for $g<1$ and for $n>0$ for $g > 1$ and any η [Fig. 1(b)]. For $\eta \gg 1$, the potential minimum is at $n_{\rm sp}(g) \equiv n_G(g>1) = \eta(g^2 - g^{-2})/4 + \mathcal{O}(\eta^0)$, which explains very well the quadratic behavior of n_G shown in Fig. 1(c). Furthermore, in the $\eta \to \infty$ limit and $g > 1$, it is immediately obvious n_G diverges; that is, a ground state superradiance occurs.

The instability of the JC model for $g > 1$ in the $\eta \to \infty$ limit predicted from Eq. (2) and the infinite value of $n_{\rm sp}$ suggests the derivation of a low-energy effective Hamiltonian for the superradiant phase $g > 1$ that is valid around the potential minimum. To this end, we displace the cavity field a in Eq. (1) by a complex number $\alpha = \alpha_g e^{i\theta}$ with $\alpha_g = \sqrt{n_{\rm sp}} = \sqrt{\eta(g^2 - g^{-2})/4}$, i.e., $\bar{H}_{\rm JC}(\alpha_g,\theta) = \mathcal{D}^\dagger[\alpha]H_{\rm JC}\mathcal{D}[\alpha]$, where $\mathcal{D}[\alpha] = e^{\alpha a^\dagger - \alpha^* a}$. By factoring out the phase $e^{-i\theta N_{\rm tot}}\bar{H}_{\rm JC}(\alpha_g,\theta)e^{i\theta N_{\rm tot}}$, we have $\bar{H}_{\rm JC}(\alpha_g) = \omega_0(a^\dagger a + \alpha_g^2) - (\omega_0\sqrt{\eta}/2g)(x\tau_x - g^2 p\tau_y) + (g^2\Omega/2)\tau_z + \omega_0\alpha_g x(\tau_0 + \tau_z)$. Here, we introduce the new spin operators $\tau_z = |\bar{\uparrow}\rangle\langle\bar{\uparrow}| - |\bar{\downarrow}\rangle\langle\bar{\downarrow}| = g^{-2}\sigma_z - \sqrt{1-g^{-4}}\sigma_x$, $\tau_x = |\bar{\uparrow}\rangle\langle\bar{\downarrow}| + |\bar{\downarrow}\rangle\langle\bar{\uparrow}| = \sqrt{1-g^{-4}}\sigma_z + g^{-2}\sigma_x$, and $\tau_y = -i(|\bar{\uparrow}\rangle\langle\bar{\downarrow}| - |\bar{\downarrow}\rangle\langle\bar{\uparrow}|) = \sigma_y$, as well as $x = a + a^\dagger$ and $p = i(a^\dagger - a)$. Note that $\bar{H}_{\rm JC}(\alpha_g)$ no longer possesses the $U(1)$ symmetry, and the analytical solution is not available in general. Then, we apply a unitary transformation $U_{\rm JC} = \exp[(i/2g\sqrt{\eta})(g^{-2}x\tau_y + p\tau_x)]$ to $\bar{H}_{\rm JC}(\alpha_g)$ so that a transformed Hamiltonian $U_{\rm JC}^\dagger \bar{H}_{\rm JC}(\alpha_g) U_{\rm JC}$ is free of coupling terms between spin subspaces $\mathcal{H}_{\bar{\downarrow}}$ and $\mathcal{H}_{\bar{\uparrow}}$. Finally, a projection onto $\mathcal{H}_{\bar{\downarrow}}$, that is, $\langle \bar{\downarrow}|U^\dagger \bar{H}_{\rm JC}(\alpha_g) U|\bar{\downarrow}\rangle$, leads to the low-energy effective Hamiltonian of the JC model in the superradiant phase,

$$\bar{H}_{\rm JC}^{\rm sp} = \frac{\omega_0}{4}(1 - g^{-4})x^2 + E_G^{\rm sp}(g). \tag{4}$$

Here, the ground state energy $E_G^{\rm sp}(g) = -\Omega(g^2 + g^{-2})/4$, leading to a discontinuity in the second derivative of E_G at $g = 1$, locating a second order QPT. The energy scale for the low-energy excitation, $(\omega_0/4)(1 - g^{-4})$, is finite for any $g > 1$, while the ground state energy in unit of ω_0, which is an extensive quantity in η, diverges, as in the normal phase.

Interestingly, the effective Hamiltonian is quadratic only in the x quadrature, while the p quadrature does not appear in the Hamiltonian [34]. The ground state of $\bar{H}_{\rm JC}^{\rm sp}$ is an eigenstate of the x quadrature, which is an infinitely squeezed vacuum, whose major axis is the p quadrature, i.e., $|r \to \infty\rangle = \lim_{r\to\infty} \mathcal{S}[r]|0\rangle$ with $\mathcal{S}[r] = \exp[-(r/2)(a^{\dagger 2} - a^2)]$. Going back to the original basis, the ground state is $|\Psi_G^{\rm sp}(\theta)\rangle = e^{i\theta a^\dagger a}\mathcal{D}[\alpha_g]\mathcal{S}[r \to \infty]|0\rangle$ for $\theta \in [0, 2\pi]$. Since any choices of the phase θ of α lead to an identical spectrum, the ground states are infinitely degenerate. The ground state for the superradiant phase is therefore an infinitely squeezed photon condensate, whose renormalized photon occupation number is $n_G/\eta = (g^2 - g^{-2})/4$. Moreover, the $U(1)$ symmetry is spontaneously broken, as is evident from a nonzero spontaneous coherence $\langle a\rangle/\eta \equiv \langle \Psi_G^{\rm sp}(\theta)|a|\Psi_G^{\rm sp}(\theta)\rangle/\eta = e^{i\theta}\sqrt{(g^2 - g^{-2})/4}$, which is an order parameter of the QPT [Fig. 2].

We note that the critical behaviors described here, the diverging ground state energy, squeezing, and spontaneous coherence, arise only in the limit of $\eta \to \infty$ as the QPT. For any finite η, the ground state has a finite energy with a finite number $\langle N_{\rm tot}\rangle$ for any g; moreover, by the symmetry, the coherence $\langle a\rangle$ and the squeezing of the ground state is always zero. This is exactly analogous with the fact that a model that undergoes a QPT in the $N \to \infty$ limit restores analytical behaviors for any finite values of N [1,5,6].

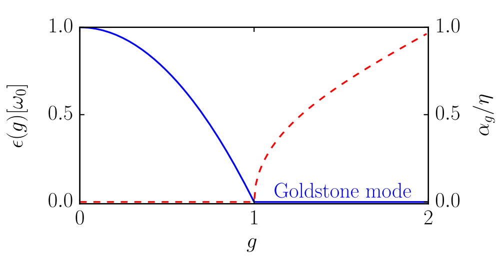

FIG. 2. QPT of the JC model. The excitation energy $\epsilon(g)$ (left, blue solid) and the ground state coherence $\langle a\rangle/\eta$ of the cavity field (right, red dashed) in the $\eta \to \infty$ limit. For $g > 1$, the $U(1)$ symmetry of the JC model is broken, leading to a Goldstone mode and a nonzero coherence.

Because Eq. (4) is quadratic in only one quadrature without the conjugate variable appearing in the Hamiltonian, the excitation spectrum is gapless [Fig. 2]. This gapless excitation is a well known consequence of the spontaneous breaking of continuous $U(1)$ symmetry and is often called a Goldstone mode [21]. The effective photon number potential shown in Fig. 1(b) or the mean-field energy of the JC model [35] assumes the form of the Mexican-hat potential in a phase space of the cavity field a; therefore, the appearance of the Goldstone mode can be intuitively understood from the fact that the excitation along the circle of the potential minima does not cost any energy. Finally, the vanishing spectral gap near the critical point gives rise to a critical exponent, $\epsilon(g) \propto |g - 1|^\alpha$ with $\alpha = 1$, which differs from $\alpha = \frac{1}{2}$ of the Rabi model [7].

We have shown so far that the JC model, one of the most fundamental in quantum optics, exhibits a second-order QPT. Our analysis clearly demonstrates that the atom-cavity coupling controls the ground state photon numbers, and that a large η leads to a divergence in the ground state photon number. We note that η plays precisely the same role in the JC model as the number of atoms in the Tavis-Cummings model [36], an N-atom generalization of the JC model, which undergoes the same kind of QPT [31]. Therefore, the fact that arbitrarily many photons can be created through interaction with another quantum system, regardless of its size, is the origin of the QPTs in a photonic (phononic) system with finite components; this is in contrast to systems with hard-core bosons or spins, which require infinitely many components to achieve a QPT.

Mott-insulator to superfluid transition in a finite JC lattice model.—We now consider a photonic lattice model with a finite lattice size. We demonstrate that this model is capable of exhibiting Mott-superfluid type phase transitions away from the conventional thermodynamic limit of infinite lattice sites. Specifically, we consider the JC lattice model [24–27], which describes a one-dimensional lattice of coupled cavities each containing a two-level atom to realize the JC model, which reads $H_{\rm JCL} = \sum_{i=1}^N H_{{\rm JC},i} + \sum_{i=1}^{N-1} J(a_i a_{i+1}^\dagger + {\rm H.c.})$, where i indicates ith cavity and $H_{{\rm JC},i} = \omega_0 a_i^\dagger a_i + (\Omega/2)\sigma_{iz} - \lambda(a_i\sigma_{i+} + a_i^\dagger\sigma_{i-})$. The model has a global $U(1)$ symmetry due to the conserved total excitation number $N_{\rm tot} = \sum_i(a_i^\dagger a_i + \sigma_{i+}\sigma_{i-})$. In the $N \to \infty$ limit, it is in general not amenable to exact solutions, neither analytically nor numerically; therefore, its phase diagram, showing the Mott-insulating-superfluid transition, is often studied based on the mean-field solution [25,28]. For finite N, the numerically exact calculation shows a crossover from a Mott insulating phase to a superfluidlike phase, due to the finite-size effect, which generally prevents the system undergoing a true QPT [26].

We now choose $N = 2$, thus called a JC dimer, which is the smallest possible number of sites for a lattice system, and show that it undergoes a second-order Mott-insulating-superfluid QPT, in the $\eta \to \infty$ limit. Note that, unlike some of the previous works [25,28], we introduce neither a chemical potential term to fix the number of polaritons nor counterrotating terms, which has been shown to stabilize the chemical potential in Ref. [37]; rather, as witnessed in the previous section, a strong JC-type interaction between the field and the atom itself modulates the number of polaritons of each cavity. The JC dimer Hamiltonian can be written in terms of normal modes $b_{1(2)} = (a_1 \mp a_2)/\sqrt{2}$ and $s_{1+(2+)} = (\sigma_{1+} \mp \sigma_{2+})/\sqrt{2}$, that is,

$$H_{\rm JD} = \sum_{i=1}^{2} \left\{ [\omega_0 + (-1)^i J] b_i^\dagger b_i + \frac{\Omega}{2}\sigma_{iz} - \lambda (b_i s_{i+} + b_i^\dagger s_{i-}) \right\}, \tag{5}$$

where we assume $J/\omega_0 < 1$. In the following we treat the two cases $g < g_c$ and $g > g_c$, which lead to different phases, separately. To treat the $g < g_c$ case, we first apply a unitary transformation to $H_{\rm JD}$, which decouples the normal modes from the atom, $U_{\rm JD} = \exp[(g/\sqrt{\eta})\sum_{i=1,2}(b_i s_{i+} - b_i^\dagger s_{i-})]$, followed by a projection onto the subspace of $|\downarrow\rangle_1|\downarrow\rangle_2$ [35]. The resulting Hamiltonian is

$$H_{\rm JD}^{\rm Mott} = \omega_0 \sum_{i=1}^{2} \left(1 - g^2 + (-1)^i \frac{J}{\omega_0}\right) b_i^\dagger b_i - \Omega + \mathcal{O}\left(\eta^{-\frac{1}{2}}\right), \tag{6}$$

which becomes exact in the $\eta \to \infty$ limit. Note that there is a phase boundary $g_c(J) = \sqrt{1 - J/\omega_0}$ [Fig. 3(a)], on which the spectral gap of the b_1 mode vanishes as $\epsilon \propto [g - g_c(J)]^\mu$ with $\mu = 1$ and beyond which the b_1 mode becomes unstable. As a consequence, Eq. (6) is the valid effective Hamiltonian only for $g < g_c(J)$. In this phase, the ground state in the original cavity field basis is $|0,\downarrow\rangle_1|0,\downarrow\rangle_2$. This corresponds to an $n = 0$ Mott-insulating phase, where each cavity assumes the fixed, same number of excitations. The b_2 mode remains stable for $g < g_c(J)$.

As in the JC model, the fact that the b_1 mode becomes unstable for $g > g_c(J)$ suggests that it gets occupied by a macroscopic number of photons. Therefore, it is insightful to look at the mean-field energy of $H_{\rm JD}$, which we find as $E_{\rm JD}^{\rm MF}(\eta, g, J/\omega_0, \beta)/\Omega = g_c^2(J)\eta^{-1}|\beta|^2 - \sqrt{1 + 2g^2\eta^{-1}|\beta|^2}$ [35]. $E_{\rm JD}^{\rm MF}$ assumes the form of the Mexican-hat potential for $g > g_c(J)$, where the potential minimum occurs at $\beta_1 = e^{\theta_1}|\beta_1|$ with $|\beta_1| = \sqrt{\eta/[2g_c^2(J)]}\sqrt{[g/g_c(J)]^2 - [g/g_c(J)]^{-2}}$. The mean-field solution predicts a spontaneously broken-symmetry phase and an appearance of the Goldstone mode. The second derivative of the ground state energy in g become discontinuous at $g = g_c(J)$, indicating that it is a second order QPT [35].

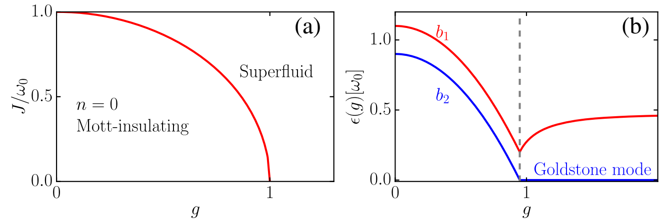

FIG. 3. JC lattice model. (a) Phase diagram in the (g, J) plane. (b) Excitation energy of the antisymmetric (b_1) and symmetric (b_2) normal mode as a function of g for $J/\omega_0 = 0.1$. At the critical point, where the b_1 mode becomes the Goldstone mode, the first derivative of the excitation energy of the b_2 mode becomes discontinuous.

For $g > g_c(J)$, we first displace the b_1 mode by its mean-field amplitude β_1, which leads to a new atomic state for the ground state, and then apply a unitary transformation decoupling the normal modes and atoms, followed by a projection onto the low-energy subspace [35]. The resulting effective Hamiltonian reads

$$\bar{H}_{\rm JD}^{\rm SF} = \frac{\omega_0 g_c^2}{4}\left(1 - \frac{g_c^4}{g^4}\right)x_1^2 + \frac{J}{2}p_2^2 + \frac{\omega_0}{4}\left(1 + \frac{J}{\omega_0} - \frac{g_c^6}{g^4}\right)x_2^2 \tag{7}$$

up to the constant ground state energy and g_c here denotes $g_c(J)$.

The two normal modes are decoupled from each other, and the above Hamiltonian is exactly solvable. First, the p_1 quadrature of the b_1 mode disappears from the effective Hamiltonian, as in Eq. (4). Therefore, it immediately follows that the global $U(1)$ symmetry of the JC lattice model is broken for $g > g_c(J)$. The nonzero coherence of each cavity field $\langle a_i \rangle \neq 0$ marks the onset of the superfluid phase and becomes an order parameter. The excitation spectrum of the b_1 mode is gapless (Goldstone mode) in the broken symmetry phase [Fig. 3(b)]. The Hamiltonian for the b_2 mode in Eq. (7) leads to a harmonic spectrum with an excitation frequency of $\epsilon_2^{\rm SF}(g) = J\sqrt{2[1 + \omega_0/J(1 - g_c^6(J)/g^4)]}$. As shown in Fig. 3(b), the b_2 mode remains gapped for both phases. Interestingly, the first derivative of $\epsilon_2^{\rm SF}(g)$ is discontinuous at $g = g_c(J)$. Such a slope discontinuity of the b_2 mode can be potentially used to detect the presence of the Goldstone mode as suggested in Ref. [31]. The ground state of the b_2 mode is a squeezed vacuum, whose squeezing parameter is given by $\xi = -1/4\ln\{\frac{1}{2}[1 + \omega_0/J(1 - g_c^6(J)/g^4)]\}$, which is zero at $g = g_c$ and gradually increases. The JC dimer may also serve as the testing ground for the physics of phase interfaces in lattice systems [38].

Conclusion.—Unlike massive particles, photons can be created by their interaction with an atom, as the chemical potential of the photon vanishes [39]. We have shown that for an atom with a much larger characteristic frequency than the photon but strongly coupled to it, it is possible to have a macroscopic photon occupation in the ground state. This, as we have demonstrated at the hand of the JC models, leads to the emergence of a QPT in a system composed of finitely many components, photonic modes, and atoms. We note that the required parameter regime can be realized in a trapped ion setup [19,40] where the critical scaling relation due to the finite-system QPT can be observed using the adiabatic preparation of the ground state [19]. Our finding here, together with one presented in Ref. [7], opens up an important possibility to study the critical phenomena of light and sound, such as QPT, universality, and the dynamics of the QPT, in fully controlled quantum systems including superconducting circuits and trapped ions.

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