Chaos in AdS_2 Holography

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We revisit two-dimensional holography with the Sachdev-Ye-Kitaev models in mind. Our main result is to rewrite a generic theory of gravity near a two-dimensional anti-de Sitter spacetime throat as a novel hydrodynamics coupled to the correlation functions of a conformal quantum mechanics. This gives a prescription for the computation of n-point functions in the dual quantum mechanics. We thereby find that the dual is maximally chaotic.

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Introduction.—The Sachdev-Ye-Kitaev (SYK) models [1,2] are quantum mechanical systems with random all-to-all interactions. It has recently been conjectured that they have a gravity dual in two dimensions.

The basic SYK model is a theory of $2N$ Majorana fermions ψ^a ($a = 1, \ldots, 2N$) perturbed by quenched disorder. The Hamiltonian is

$$H = \sum_{a,b,c,d} \frac{J_{abcd}}{4!} \psi^a \psi^b \psi^c \psi^d, \tag{1}$$

where $\overline{J_{abcd}} = 0$ and $\overline{J_{abcd} J^{abcd}} = 3! J^2/(2N)^3$. At a temperature T, there is a single dimensionless coupling J/T. The high-temperature theory has $2N$ weakly interacting fermions, while the low-temperature theory is strongly correlated. Crucially, the theory is soluble at large N (see, e.g., Refs. [3,4]).

There are two main pieces of evidence that indicate that the SYK models have a gravity dual. The first is an emergent conformal symmetry at low energies, together with a large N extremal entropy. The second is much more nontrivial. The SYK models saturate the "chaos bound" of Ref. [5] on the Lyapunov exponent, which characterizes the rate of growth of certain out-of-time-ordered four-point functions [6,7]. This bound, which exists in any quantum system, is $2\pi T$. Conformal field theories with an Einstein gravity dual also saturate the chaos bound [7], which led Kitaev [2] to conjecture that the SYK model gives a toy model for quantum gravity in two dimensions (see also Refs. [8,9]).

This prospect brings us back to the AdS_2/CFT_1 correspondence, along with all of its baggage. The AdS_2/CFT_1 correspondence has never been satisfactorily developed, largely due to problems on both sides of a putative duality. In one dimension, field theories are ordinary quantum mechanics, so we will refer hereafter to a one-dimensional conformal field theory (CFT_1) as a conformal quantum mechanics (CQM).

On the CQM side, one runs into a paradox due to Polchinski [10]. Let $\rho(E)$ be the density of states. Scale invariance implies that

$$\rho(E) = e^{S_0}\delta(E) + \frac{e^{S_1}}{E}. \tag{2}$$

If the second term is nonzero, then there must be an infrared cutoff Λ_{IR}, but if it vanishes, then a CQM is a topological theory with no dynamics.

This CQM paradox is dual to the fact that two-dimensional anti–de Sitter spacetimes (AdS_2) cannot support finite-energy excitations. Injecting a lump of energy into an AdS_2 throat leads to strong backreaction, which cannot be consistently analyzed within the throat.

These two paradoxes are dual to each other in that they reflect modest UV-IR mixing. On the CQM side, "irrelevant deformations" to the density of states allow for consistent time evolution and nontopological correlators, while, on the gravity side, AdS_2 throats do not admit a decoupling limit. A consistent study of scattering requires the flow to the throat.

There is a connection to large N limits here, in that these paradoxes arise at a finite N. In the strict $N \to \infty$ limit, there is nothing wrong with a generalized free CQM [11], while backreaction disappears on the gravitational side. However, at a finite N, there is no such thing as an interacting CQM or AdS_2 holography. A large N theory may be only approximately conformally invariant, with conformal invariance broken at $O(1/N)$, as advocated in Ref. [12].

The point of this Letter is twofold. First, we assess the viability of a SYK/AdS_2 correspondence. Second, we revisit AdS_2 holography. For theories dual to dilaton gravity with an AdS_2 near horizon, we derive an effective hydrodynamic action for the near-AdS_2 physics from which we see that they saturate the chaos bound.

The SYK models.—We begin with a brief review of the SYK models. The theory of $2N$ Majorana fermions ψ^a ($a = 1, \ldots, 2N$) [13] is an exact CQM:

$$S_\psi = \sum_a \int dt \psi_a \partial_t \psi^a. \tag{3}$$

It is merely a system of 2^N zero-energy states and so has a large extremal entropy $S = N \ln 2$. The two-point function of ψ is topological,

$$\langle \psi^a(t) \psi^b(0) \rangle = \frac{\delta^{ab}}{2} \mathrm{sgn}(t). \tag{4}$$

This theory admits N relevant deformations built from fermion monomials. The SYK model is the theory of $2N$ fermions perturbed by quenched disorder for the quartic monomial,

$$S_{\mathrm{SYK}} = \int dt \left(\sum_a \psi_a \partial_t \psi^a - \sum_{a,b,c,d} \frac{J_{abcd}}{4!} \psi^a \psi^b \psi^c \psi^d \right). \quad (5)$$

Note that if the source J_{abcd} were not disordered, then the four-Fermi interaction would break the global symmetry. However, the quenched disorder preserves the full $SO(2N)$ flavor symmetry of the free-field fixed point.

The SYK model realizes an emergent conformal symmetry at low energies and large N. At $T = 0$, the solution to the leading large N Schwinger-Dyson equation for the two-point function of ψ^a is

$$\langle \psi^a(t) \psi^b(0) \rangle = \left(\frac{1}{4\pi J^2} \right)^{1/4} \frac{\mathrm{sgn}(t) \delta^{ab}}{|t|^{1/2}}, \qquad t \gg 1/J, \quad (6)$$

so that ψ^a has dimension $1/4$ in the infrared.

There is a generalization of the SYK model characterized by two integers, the number of fermions $2N$, and the degree q of the disordered interaction:

$$S_q = S_\psi - \int dt \sum_{a_1,\ldots,a_q} \frac{J_{a_1,\ldots,a_q}}{q!} \psi^{a_1}, \ldots, \psi^{a_q}. \quad (7)$$

This theory also hosts an emergent conformal symmetry at low energies and large N (with $q \ll N$), where ψ^a behaves like a dimension-$1/q$ operator in the IR.

The SYK model exhibits another hallmark of emergent conformal symmetry in one dimension: it has a large N extremal entropy. Standard large N power counting shows that the leading contribution to the low-temperature, large N thermal partition function is the one-loop determinant of the inverse, resummed fermion propagator (6). Conformally mapping to the thermal circle, the thermal Euclidean two-point function of ψ is

$$G(\omega_n) \propto \frac{\Gamma(\Delta - n + \frac{1}{2})}{\Gamma(1 - \Delta - n + \frac{1}{2})}, \qquad \Delta = \frac{1}{q}, \quad (8)$$

where $\omega_n = 2\pi(n - 1/2)T$ is the nth Matsubara frequency. The extremal entropy is given by

$$\frac{S}{N} = \sum_n \ln |G^{-1}(\omega_n)| + O(N^{-1}). \quad (9)$$

This sum cannot be done explicitly. Following Ref. [2], we differentiate with respect to Δ (dropping the $1/N$ corrections):

$$\frac{1}{N} \frac{dS}{d\Delta} = \pi(2\Delta - 1) \tan(\pi\Delta). \quad (10)$$

Integrating with respect to Δ and using that the entropy at $\Delta = 0$ is $N \ln 2$ gives

$$\frac{S}{N} = (1 - 2\Delta) \ln [2 \cos(\pi\Delta)] - \frac{\mathrm{Li}_2(-e^{2\pi i\Delta}) - \mathrm{Li}_2(-e^{-2\pi i\Delta})}{2\pi i}. \quad (11)$$

(This result was first obtained numerically in Ref. [14], but, to our knowledge, this is the first time it has been computed analytically.) For $\Delta = 1/4$, this gives $S/N = G/\pi + (\ln 2)/4 \approx 0.464848$, where G is Catalan's constant.

With all of this in mind, we find two simple reasons why the SYK models cannot have a conventional (weakly curved, weakly coupled) gravity dual. (These reasons were also mentioned in Ref. [4].) 1. The entropy of the SYK models is $O(N)$, so that the Newton's constant of the putative dual would be $O(1/N)$. The $2N$ fermions ψ^a would be dual to $2N$ degenerate, bulk fermions Ψ^a. However, the existence of so many light fields invalidates the saddle-point approximation: the one-loop correction to the bulk partition function from the Ψ^a would be comparable to the classical saddle. More simply, the SYK spectrum is not sparse. 2. Theories with a conventional gravity dual exhibit large N factorization. Consequently, given an operator O of dimension Δ dual to a bulk field, there are necessarily "double-trace" operators, e.g., $\sim O(\partial^2)^n O$ of dimension $2\Delta + 2n + O(1/N)$. Computation of the four-point function of the ψ^a in the SYK models [3,4] reveals no such operators.

These ills might be cured by gauging a large subgroup of the flavor symmetry. That is, there may yet be a *gauged* SYK/AdS correspondence. This would be immensely satisfying if true. We cannot help but mention that this would be consistent with arguments that bulk locality is tied to "large" gauge symmetries in a field theory dual (see, e.g., Refs. [11,15]).

Simpler still, perhaps the singlet sector of the SYK models is dual to a two-dimensional higher-spin theory in the spirit of Ref. [16].

Dilaton gravity.—Two-dimensional gravity is rather different from its higher-dimensional cousins. Compactification to two dimensions generally leads to a dilaton gravity characterized by a two derivative action,

$$S_{\mathrm{bulk}} = \frac{1}{2\kappa^2} \int d^2x \sqrt{-g} (\varphi R + U[\varphi]) + S_{\mathrm{matter}}, \quad (12)$$

where φ is the dilaton and U its potential. The equations of motion are

$$T_{\mu\nu} = -D_\mu D_\nu \varphi + g_{\mu\nu} \Box \varphi - \frac{g_{\mu\nu}}{2} U, \qquad \Phi = R + U', \quad (13)$$

with $T_{\mu\nu}$ and Φ being the stress tensor and the dilaton source,

$$\delta S_{\mathrm{matter}} = \frac{1}{2\kappa^2} \int d^2x \sqrt{-g} (T^{\mu\nu} \delta g_{\mu\nu} - \Phi \delta\varphi). \quad (14)$$

Dilaton gravities have AdS_2 vacua at the roots of the dilaton potential, $U[\varphi_0] = 0$ with matter fields vanishing,

$$\varphi = \varphi_0, \quad g = L^2(-r^2dt^2 + 2dtdr), \quad L^2 = \frac{2}{U'[\varphi_0]}. \quad (15)$$

We take $U'[\varphi_0] = 2$ hereafter. Observe that we are using infalling Eddington-Finkelstein coordinates. Holographically renormalizing in the AdS_2 throat [17] shows that (i) the dilaton is not dual to an operator, (ii) the metric is not either, in that the dual stress tensor vanishes, and (iii) the dual theory is invariant under a Virasoro symmetry with $c = 0$ [18]. The boundary theory lives at $r \to \infty$ with the metric $h = -dt^2$.

The vanishing of the boundary stress tensor is another way of stating the usual result that AdS_2 does not support finite-energy excitations [19].

Conformal symmetry is infinite dimensional in one dimension. Any reparametrization of time $t = t(w)$ can be compensated for by a Weyl rescaling of the metric $h_{\mu\nu} \to e^{2\Omega} h_{\mu\nu}$ so as to leave the metric invariant. On the gravity side, conformal transformations correspond to diffeomorphisms which preserve the radial gauge in Eq. (15) and fix the boundary metric. Under the conformal transformation $t(w)$, the AdS_2 vacuum (15) becomes

$$\varphi = \varphi_0, \qquad g = -[r^2 + 2\{t(w), w\}]dw^2 + 2dwdr, \quad (16)$$

with $\{t(w), w\}$ being the Schwarzian derivative

$$\{t(w), w\} = \frac{t'''(w)}{t'(w)} - \frac{3}{2}\frac{[t''(w)]^2}{[t'(w)]^2}. \quad (17)$$

The conformal transformation $t(w) = \tanh(\pi w T)$ has constant Schwarzian $\{t(w), w\} = -2\pi^2 T^2$ and maps the AdS_2 vacuum to an AdS_2 black hole

$$\varphi = \varphi_0, \qquad g = -(r^2 - r_h^2)dw^2 + 2dwdr, \quad (18)$$

with $r_h = 2\pi T$ and T being the Hawking temperature. The thermal entropy is $S = 2\pi\varphi(r_h)/\kappa^2 = 2\pi\varphi_0/\kappa^2$.

Now consider a holographic renormalization group (RG) flow terminating in an AdS_2 throat. To get the basic idea, we turn off the matter fields $T_{\mu\nu} = 0$, $\Phi = 0$ and try to glue the AdS_2 near horizon (16) to a RG flow at large r. Enforcing the rr component of Einstein's equations (13), the near-AdS_2 geometry is given by the perturbative solution

$$\varphi = \varphi_0 + \ell[r\varphi_1(w) + \varphi_2(w)] + O(\ell^2 r^2),$$
$$g = -[r^2 + 2\{t(w), w\}]dw^2 + 2dwdr + O(\ell r), \quad (19)$$

where ℓ is a length scale satisfying $\ell r \ll 1$. The dilaton formally behaves as if it is dual to a dimension-2 operator, with a source $\ell\varphi_1(w)$. We work in the same spirit as Refs. [20,21] and take the dual QM to "live" on a constant-r slice at large $r \to \infty$, and we fix $\varphi_1 = 1$ as a boundary condition. The rw component of Einstein's equations fixes $\varphi_2 = 0$, and the ww component gives

$$\partial_w(\{t(w), w\}) = 0. \quad (20)$$

So, in the absence of matter, the RG flow must terminate in an AdS_2 black hole (18). The flow corrects the near-extremal entropy,

$$S = \frac{2\pi}{\kappa^2}[\varphi_0 + 2\pi\ell T + O(\ell^2 T^2)]. \quad (21)$$

We now send in matter. For simplicity, consider a small amount of infalling null dust described by a stress tensor $T_{ww}(w) \sim \ell$. The rr component of Einstein's equations is unmodified, so the perturbative solution (19) still holds. We again impose $\varphi_1 = 1$, and the rw component fixes $\varphi_2 = 0$. The ww component gives (to first order in ℓ)

$$\ell\partial_w[\{t(w), w\}] = -T_{ww}(w). \quad (22)$$

This relation is familiar: the horizon grows as matter falls in. Let us translate it into an equation in the boundary quantum mechanics. Holographically renormalizing to first order in ℓ, we find that the boundary energy $E = -h_{\mu\nu}\langle t^{\mu\nu}\rangle$ (with $t^{\mu\nu}$ being the boundary stress tensor) is

$$E = -\frac{\ell}{\kappa^2}\{t(w), w\}. \quad (23)$$

A microscopic model for the dust is a massless scalar field,

$$S_{\mathrm{matter}} = -\frac{1}{2}\int d^2x\sqrt{-g}Z_0[\varphi](\partial\chi)^2, \quad (24)$$

dual to a dimension-1 operator O (we normalize $Z[\varphi_0] = 1$). The infalling solutions are $\chi = \lambda(w)$ on which $T_{ww} = \kappa^2\dot{\lambda}^2$ (with $\dot{f} = \partial_w f$). The source for O is $\lambda(w)$, and its one-point function is $\langle O\rangle = \dot{\lambda}$. Putting the pieces together, Eq. (22) becomes

$$\dot{E} = \dot{\lambda}\langle O\rangle, \quad (25)$$

which is the diffeomorphism Ward identity.

We consider a general matter action in the Supplemental Material [22]. For a single bulk field χ dual to a dimension Δ operator O_Δ with source λ, the Einstein's equations boil down to Eq. (25) with the energy given by

$$E = -\frac{\ell}{\kappa^2}\{t(w), w\} + (1-\Delta)\lambda\langle O_\Delta\rangle, \quad (26)$$

and the extension to multiple fields is obvious.

We can do better and obtain the effective action for dilaton gravity near the throat. It is

$$S_{\mathrm{eff}} = -\frac{\ell}{\kappa^2}\int dw\{t(w), w\} + W_{\mathrm{CQM}}[\lambda; t(w)], \quad (27)$$

where W_{CQM} is the "generating functional" obtained by integrating out the matter in the fixed AdS_2 background (16). Equivalently, W_{CQM} comes from integrating out matter in the pure AdS_2 geometry (15), followed by a conformal transformation $t(w)$. Here, $t(w)$ is the fundamental field and its Euler-Lagrange equation is Eq. (25).

Hydrodynamics.—This result evokes the fluid-gravity correspondence [28] in that we have rewritten the gravitational dynamics as the (non)conservation of energy in the boundary quantum mechanics with a "constitutive relation" (26) for the energy. Unlike the fluid-gravity correspondence, this rewriting does not rely on a gradient expansion or even a black hole to start with.

Let us take this connection to hydrodynamics seriously.

Haehl, Loganayagam, and Rangamani (HLR) have classified [29] the most general hydrodynamics consistent with the second law of thermodynamics, building upon earlier results in hydrostatic equilibrium [30,31]. HLR also obtained Schwinger-Keldysh effective actions [32] for hydrodynamics (see also Ref. [33]). A subset of allowed transport (which they dub class L, for Lagrangian) admits an ordinary action via a sigma model, where the fundamental fields are maps from a "reference manifold" to the physical spacetime [29].

The effective action (27) for dilaton gravity is just such a class L action. Recall that $t(w)$ is the conformal transformation from the AdS_2 vacuum to the state of the system. It is useful to redefine $t(w) = \tanh[\pi\sigma(w)/\beta]$ so that $\sigma(w)$ is the fundamental field, which represents a conformal transformation starting from the thermal state with temperature $1/\beta$. In terms of $\sigma(w)$ and after an integration by parts, the effective action (27) becomes

$$S_{\mathrm{eff}} = \frac{\ell}{2\kappa^2}\int dw\left(\frac{\sigma''(w)^2}{\sigma'(w)^2} + \frac{4\pi^2}{\beta^2}\sigma'(w)^2\right) + W_{\mathrm{CQM}}. \tag{28}$$

We take w to be the coordinate on the physical spacetime $\mathcal{M}$, and σ parametrizes the reference manifold M. The metric on M is $\mathsf{h} = -w'(\sigma)^2 d\sigma^2$, and on M we define the fixed vector field $\boldsymbol{\beta}^\sigma = \beta$. From this data we define a time-dependent temperature and velocity

$$\mathsf{T} = \frac{1}{\sqrt{-\mathsf{h}_{ab}\beta^a\beta^b}}, \qquad \mathsf{u}^a = \frac{\beta^a}{\sqrt{-\mathsf{h}_{bc}\beta^b\beta^c}}, \tag{29}$$

and $\dot{f} = \mathsf{u}^a\partial_a f$. Then,

$$S_{\mathrm{eff}} = \int d\sigma\sqrt{-\mathsf{h}}\left\{\mathsf{P}(\mathsf{T}) + \frac{\ell}{2\kappa^2}\frac{\dot{\mathsf{T}}^2}{\mathsf{T}^2}\right\} + W_{\mathrm{CQM}}, \tag{30}$$

where $\mathsf{P}(\mathsf{T})$ is the pressure

$$\mathsf{P}(\mathsf{T}) = -E_0 + \frac{2\pi}{\kappa^2}(\varphi_0\mathsf{T} + \pi\ell\mathsf{T}^2), \tag{31}$$

and E_0 is the ground state energy. [Strictly speaking, neither the ground state energy nor the linear term was present in Eq. (27), but neither affects the equation of motion, so we lose nothing by adding them.] Reparametrization invariance guarantees that the equation of motion for $w(\sigma)$, keeping h and β^a fixed, is precisely Eq. (25).

A few comments are in order. (1) The hydrodynamic action also computes the low-temperature free energy. Wick rotating to Euclidean signature, the action evaluated on the solution $w(\sigma) = \sigma$ (so that $\mathsf{T} = T$) gives

$$\ln \mathcal{Z}_E = iS_E = -\beta E_0 + \frac{2\pi}{\kappa^2}[\varphi_0 + \pi\ell T + O(\ell^2 T^2)].$$

(2) The $\dot{\mathsf{T}}^2$ and T^2 terms in the hydrodynamic action are linked: they arise from the Schwarzian action (27) after conformally transforming from the vacuum. In this way, the low-temperature correction to the entropy (equivalently a low-energy correction to the density of states) determines the dynamics. In principle, there are higher derivative corrections to the $O(\ell)$ hydrodynamic action (30), e.g., $(\ell/\kappa^2)(\ddot{\mathsf{T}}^2/\mathsf{T}^4)$. However, as far as we can tell, all such terms are forbidden by demanding regularity in the vacuum [as long as $\sigma'(w) > 0$]. In this sense, the $O(\ell)$ hydrodynamic action seems to be unique. (3) At $O(\ell^2)$, however, we expect there to be additional terms in S_{eff}, like $\ell^2\mathsf{T}^3$. (4) It would be interesting to go beyond the classical limit and compute quantum corrections to the free energy, correlators, etc., arising from the hydrodynamic mode $w(\sigma)$.

Chaos.—A basic entry in the holographic dictionary is the computation of CFT correlation functions via Witten diagrams in AdS. In the tree-level approximation to dilaton gravity near AdS_2, the computation of two- and three-point functions of boundary operators is straightforward, and the result is the usual one dictated by conformal invariance. The four-point function is much richer. It has two parts. The first is a conformally invariant contribution involving a sum over conformal blocks, dual to tree-level contact and exchange Witten diagrams. The second breaks conformal invariance, dominates the first, and is due to the hydrodynamics (30). What happens is this. Quadratic fluctuations of the source λ for the operator O inject energy: they source the "Goldstone mode" $w(\sigma)$. Plugging the fluctuation $\delta w(\sigma) \sim \lambda^2$ back into the matter action W_{CQM} leads to an $O(\lambda^4)$ contribution to the on-shell action.

We stress that this "hydrodynamic backreaction" and the concomitant conformal symmetry breaking was anticipated by Almheiri and Polchinski [12], who studied a soluble toy model of two-dimensional holography.

We illustrate the importance of this hydrodynamic contribution by computing the Lyapunov exponent. Consider an out-of-time-ordered thermal four-point function [6,7] of two operators, W and V,

$$F(w) \equiv \langle W(w)V(0)W(w)V(0)\rangle_\beta. \tag{32}$$

The Lyapunov exponent λ_L characterizes the growth of $F(w) \sim e^{\lambda_L w}$. We obtain $F(w)$ from the Euclidean vacuum four-point function by the same method as in Refs. [3,34].

We begin on the Euclidean line $\bar{\tau}$ and turn on a source λ for O_Δ, normalized as $\langle O_\Delta(\tau)O_\Delta(0)\rangle = 1/|\bar{\tau}|^{2\Delta}$. The conformally transformed W_{CQM} is

$$W_{\mathrm{CQM}} = \frac{1}{2}\int\frac{d\tau_1 d\tau_2[\bar{\tau}'(\tau_1)\bar{\tau}'(\tau_2)]^\Delta}{|\bar{\tau}(\tau_1) - \bar{\tau}(\tau_2)|^{2\Delta}}\lambda(\tau_1)\lambda(\tau_2) + O(\lambda^3). \tag{33}$$

With $\bar{\tau}(\tau) = \tau + \varepsilon(\tau)$, the equation of motion (25) gives

$$\varepsilon(\tau) = \frac{\kappa^2\Delta}{12\ell}\int\frac{d\tau_1 d\tau_2}{|\tau_1 - \tau_2|^{2\Delta}}|\tau - \tau_1|^3 \times\left\{\frac{3}{\tau - \tau_1} + \frac{2}{\tau_1 - \tau_2}\right\}\lambda(\tau_1)\lambda(\tau_2) + O(\lambda^3). \tag{34}$$

Feeding this back into $S_{\rm eff}$ leads to an $O(\lambda^4)$ term:

$$\delta_\varepsilon S_{\rm eff} = \int d\tau \left(\frac{\ell}{2\kappa^2} \ddot{\varepsilon}^2 - \Delta \dot{\varepsilon} \lambda \langle O_\Delta \rangle - \varepsilon \lambda \langle \dot{O}_\Delta \rangle \right). \tag{35}$$

This gives the connected, Euclidean, hydrodynamic four-point function for two different operators, W and V:

$$\begin{aligned} &\frac{\langle W(\tau_1)W(\tau_2)V(\tau_3)V(\tau_4)\rangle}{\langle W(\tau_1)W(\tau_2)\rangle\langle V(\tau_3)V(\tau_4)\rangle} \\ &= \frac{\kappa^2}{\ell}\Delta_W\Delta_V\left\{|\tau_{13}|^3\left(\frac{2}{3\tau_{12}\tau_{34}} + \frac{1}{\tau_{12}\tau_{13}} + \frac{1}{\tau_{13}\tau_{34}}\right) \right. \\ &\quad \left. - |\tau_{13}| + (\text{permutations})\right\}, \end{aligned} \tag{36}$$

with $\tau_{ij} = \tau_i - \tau_j$. For $1/\kappa^2 \sim N$, this contribution is $1/(N\ell)$, which is $O(1/N)$ and breaks conformal invariance, as advertised. It well approximates the full four-point function at late times (but with $|\tau_{ij}|/\ell \ll N$). We conformally map to the thermal state $\tau = \tanh(\pi w/\beta)$ and take the "second sheet" analytic continuation

$$w_1 = w + 2i\epsilon, \quad w_2 = w - i\epsilon, \quad w_3 = i\epsilon, \quad w_4 = -2i\epsilon. \tag{37}$$

The terms in brackets cancel against their permutations so that the four-point function grows as $\tau_1 \sim \tanh(\pi w T) \sim \exp(2\pi w T)$. We thereby extract

$$\lambda_L = 2\pi T. \tag{38}$$

Conclusions.—We have found two main results. First, the SYK models do not have a conventional gravity dual, although perhaps there is a gauged SYK/AdS correspondence. Second, with the prospect of such a correspondence in mind, we unraveled various thorny issues in AdS_2 holography. Our central result was to rewrite the gravitational dynamics near an AdS_2 throat in terms of an effective quantum mechanical action (27). This action was that of a novel hydrodynamics (30) coupled to CQM correlators. Unlike ordinary hydrodynamics, it describes the dynamics all the way down to extremality.

This hydrodynamics is intimately tied up with diffeomorphism invariance, ensuring that the diffeomorphism Ward identity is satisfied in the infrared. The hydrodynamic description plays a similar role in two-dimensional holography as the Virasoro identity block with $c \gg 1$ does in AdS_3: both approximate the leading contribution to four-point functions in their respective field theory duals.

Maldacena and Stanford [4] have recently obtained the same Schwarzian effective action (27) from the large N solution of the SYK models. The emergence of the same description in two rather different systems raises the question of its universality.

In the main text, we suggested that the leading low-energy part of the hydrodynamic action was unique, in the sense that the coefficients of the $\dot{\mathsf{T}}^2$ and T^2 terms were linked as in Eq. (30) and that there are no gradient corrections.

If this is the case, then it seems reasonable that the hydrodynamic description universally describes (diffeomorphism-invariant) large N systems with an emergent conformal invariance, and consequently any such system will be maximally chaotic [22].

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Note added.—Recently, Maldacena and Stanford posted a very interesting paper [4] which displays some overlap with this Letter.

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