Detection of 15 dB Squeezed States of Light and their Application for the Absolute Calibration of Photoelectric Quantum Efficiency

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Squeezed states of light belong to the most prominent nonclassical resources. They have compelling applications in metrology, which has been demonstrated by their routine exploitation for improving the sensitivity of a gravitational-wave detector since 2010. Here, we report on the direct measurement of 15 dB squeezed vacuum states of light and their application to calibrate the quantum efficiency of photoelectric detection. The object of calibration is a customized InGaAs positive intrinsic negative $(p-i-n)$ photodiode optimized for high external quantum efficiency. The calibration yields a value of 99.5% with a 0.5% $(k = 2)$ uncertainty for a photon flux of the order 10^{17} s^{−1} at a wavelength of 1064 nm. The calibration neither requires any standard nor knowledge of the incident light power and thus represents a valuable application of squeezed states of light in quantum metrology.

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Squeezed light was first produced by four-wave mixing in sodium atoms in 1985 [\[1\]](#page-3-2) and shortly after by degenerate parametric down-conversion in a nonlinear crystal placed inside an optical resonator, also called optical parametric amplification [\[2\]](#page-3-3). Since then, the latter approach has been used in many squeezed-light experiments, as reported for instance in [3–[11\]](#page-3-4). The down-converted photon pairs exiting the squeezing resonator show quantum correlations that lead to a squeezed photon-counting noise upon detection. The closer the squeezing resonator is operated to its oscillation threshold (thr) the stronger the quantum correlations are and thus the larger the generated squeezing is. In principle, a squeezed-light source can produce an infinite squeezing level; however, the measured squeezing level is usually limited by photon loss during squeezed-light generation, propagation, and detection. Also, phase noise [\[7\]](#page-3-5), excess noise [\[12\]](#page-3-6), and detector dark noise [\[3\]](#page-3-4) have been found to impair the observable squeezing strength. An important milestone regarding squeezing strength was achieved in 2008 when a 10 dB squeezed vacuum state of light was measured for the first time [\[11\]](#page-3-7). In subsequent experiments the detected squeezing levels could be further increased, leading to values of up to 12.7 dB [\[8,9\]](#page-3-8).

Squeezed states of light allow for fundamental research on quantum physics, e.g., on the famous Einstein-Podolsky-Rosen paradox [13–[15\]](#page-3-9) and they have been proposed for compelling applications for quantum enhanced metrology [\[16,17\].](#page-4-0) The advanced LIGO detectors, during the recent detections of gravitational waves from binary black hole mergers [\[18,19\]](#page-4-1), operated at a sensitivity mainly limited by quantum noise. To further increase the sensitivity a reduction of quantum noise is obligatory. Since 2010, squeezed states of light have been routinely used to increase the quantum noise limited sensitivity of the gravitational-wave (GW) detector GEO 600 [\[20,21\]](#page-4-2). This first true application and the demonstration of a squeezing enhanced LIGO detector [\[22\]](#page-4-3) support the plans for future GW detectors, such as the Einstein Telescope [\[23\]](#page-4-4) and upgrades to advanced LIGO, to include the squeezed-light injection technique.

A 10 dB quantum enhancement of future GW detectors seems to be within reach, but requires a total photon loss of less than 10% for the squeezed field. This includes all losses that occur during the generation of the squeezed field, its injection into and propagation through the GW detector, and finally the photoelectric detection. In contrast to the loss of optical components the photoelectric quantum efficiency of a positive intrinsic negative $(p-i-n)$ photodiode is more difficult to determine. To calibrate the quantum efficiency the photon flux of the light beam needs to be known or a calibrated detector is required for comparison. With the exploitation of radiation standards, measurements of absolute external quantum efficiencies with measurement uncertainties down to 0.5×10^{-3} have been achieved at some near-infrared wavelengths for optical radiant powers of up to 400 μ W [\[24,25\]](#page-4-5). For 1050 nm, measurement uncertainties between 0.1% and 0.15% were achieved in an international comparison [\[26\]](#page-4-6), carried out under incoherent irradiation with optical radiant powers between 1 and 100 μ W. For laser power in the mW range, radiation standards provide a measurement uncertainty at 1064 nm of around 0.2% [\[27\]](#page-4-7). For the regime of photon-numberresolving photodetectors quantum correlations of singlephoton pairs generated via parametric down-conversion have been used as an absolute calibration technique, for example $[28-31]$, and accuracies at the level of parts in $10³$ have been achieved [\[32\].](#page-4-9)

Here, we present the realization of a novel approach for the precise calibration of absolute external quantum efficiencies of p-i-n photodiodes based on a continuous-wave squeezed-light source. Our method does not require any calibrated standard for the incident light power, but only the measurements of squeezing and corresponding antisqueezing levels and a determination of optical loss of components. For this new application of quantum metrology we have realized a low-loss squeezed-light experiment, which allowed for the direct observation of up 15 dB squeezing. This is the strongest quantum noise reduction demonstrated to date and enabled the low error bar of our method.

We calibrated the quantum efficiency of an InGaAs $p-i-n$ photodiode to 99.5% with 0.5% $(k = 2)$ uncertainty. The device under test was a custom-made photodiode manufactured from the same wafer material as the photodiode used in GEO 600 since 2011 [\[20\].](#page-4-2)

The schematic of our experimental setup is illustrated in Fig. [1.](#page-1-0) The laser source was a monolithic nonplanar Nd: YAG ring laser with 2 W of continuous-wave single-mode output power at a wavelength of 1064 nm. About 350 mW were converted to approximately 180 mW at 532 nm in a second-harmonic generation unit. The second-harmonic light was used to pump a doubly resonant OPA in which squeezed vacuum states were generated by degenerate parametric down-conversion. A standing-wave cavity was formed by an external coupling mirror and the back surface of a 9.3 mm long periodically poled KTP (PPKTP) crystal. The crystal had a 10 mm radius of curvature (ROC) at the back and was dielectrically coated for a high reflectivity (HR) for the fundamental ($R = 99.96_{-0.01}^{+0.01}\%$ at 1064 nm) and the pump field $(R > 99.9\%$ at 532 nm). The plane crystal front face was antireflectively (AR) coated for both wavelengths. The coupling mirror (ROC = 25 mm) had a power reflectivity of $R =$ 97.5^{+0.5}% at 532 nm and $R = 87.5^{+0.5}_{-0.5}$ % at 1064 nm. The cavity round-trip length was approximately 74 mm, resulting in a free spectral range of about 4 GHz, a finesse of 243 at 532 nm (linewidth $= 16$ MHz), and a finesse of 47 at 1064 nm (linewidth $= 86$ MHz). The cavity length was held on resonance for the fundamental and pump laser fields via a high bandwidth Pound-Drever-Hall locking scheme relying on the detection of the green pump field with a high signal-to-noise ratio. To generate the control signal 10% of the pump field was detected with a photodetector (PD_{OPA}) via a beam splitter as shown in Fig. [1.](#page-1-0) The required phase modulation was generated with an EOM driven at 122 MHz. Both the quasiphase matching and the simultaneous resonance of fundamental and pump field were tuned via the stabilized crystal temperature. The squeezed light at a wavelength of 1064 nm was separated from the pump field at 532 nm using a DBS.

FIG. 1. Schematic of the experimental setup. Squeezed vacuum states of light (SQZ) at a wavelength of 1064 nm were generated in a doubly resonant, type I optical parametric amplifier (OPA) operated below threshold. SHG: second-harmonic generation; PBS: polarizing beam splitter; DBS: dichroic beam splitter; LO: local oscillator; PD: photodiode; MC1064: three-mirror ring cavity for spatiotemporal mode cleaning; EOM: electro-optical modulator; FI: Faraday isolator. The phase shifter for the relative phase θ between SQZ and LO was a piezo actuated mirror. Retroreflecting mirrors were used to recycle the residual reflection from the photodiodes used for homodyne detection. An auxiliary beam (not shown) was used for the alignment of the homodyne contrast.

The measurements of the vacuum noise, and the squeezed and antisqueezed quantum noise, were performed by means of balanced homodyne detection. As sketched in Fig. [1,](#page-1-0) for this detection scheme two input fields (LO and signal) need to interfere at the 50-50 beam splitter. Each of the resulting two output fields was detected with one $p-i-n$ photodiode (custom made by Laser Components). The diodes had an active area of 500 μ m diameter and were AR coated for a 20° angle of incidence. To recycle the residual reflection from each photodiode surface (which we determined to be 0.3%) two curved HR mirrors were used as retroreflectors. This recycling technique increased the quantum yield of the homodyne detector and hence the measurable squeezing level. A signal proportional to the difference of the photocurrents was generated, which was then fed into a spectrum analyzer. This yields the variance of a generic quadrature $\hat{X}(\theta) = \hat{X}_1 \cos \theta + \hat{X}_2 \sin \theta$, where θ is the phase angle between the local oscillator and the signal field. Consequently, the phase angle defines the quadrature under investigation. We used a piezo actuated mirror as the phase shifter to adjust θ . The potential power fluctuations of the local oscillator field due to beam pointing with respect to the eigenmode of MC1064 were found to be negligible over the necessary actuation range. We achieved a fringe visibility in the TEM_{00} mode of 99.6^{+0.05}% between the signal input and the local oscillator beam. For all measurements presented here a local oscillator intensity of 26.5 mW corresponding to a photon flux of 1.4×10^{17} s⁻¹ was used. This resulted in a clearance between the electronic detection dark noise and the vacuum

FIG. 2. Quantum noise and electronic dark noise at Fourier frequencies from 3 to 8 MHz. The vacuum noise reference level corresponds to local oscillator power of 26.5 mW. The measurement time for each individual trace was 211 ms. A nonclassical noise reduction of up to 15 dB below vacuum noise was directly observed. The degradation of the squeezing factor towards higher frequencies is a consequence of the OPA linewidth. The electronic detector dark noise was not subtracted from the data.

noise reference level of up to 28 dB at Fourier frequencies between 3 and 10 MHz.

The measurements in the frequency range from 3 to 8 MHz, as shown in Fig. [2](#page-2-0), were recorded with a Spectrum Analyzer with a resolution bandwidth (RBW) of 300 kHz and a video bandwidth (VBW) of 200 Hz. The electronic dark noise of the detection system was recorded with the signal and local oscillator input blocked. The vacuum noise was recorded with only the local oscillator port of the balanced beam splitter open. In this configuration no photons entered through the signal port such that the measured noise directly represents the vacuum noise reference level. The squeezed noise was recorded by additionally opening the signal input port and adjusting the phase between local oscillator and signal field such that the measured noise variance was minimized. A short optical path length was realized for both fields aiming at minimizing differential phase noise. Up to 15 dB squeezing was measured with merely 16 mW of second-harmonic pump power. In a separate measurement we confirmed the linearity of the detection system, including the spectrum analyzer, by measuring shot-noise levels versus local oscillator powers.

For a detailed analysis we compared the performance of our apparatus with a theoretical model. Neglecting phase noise, the output spectrum of the squeezed (−) and antisqueezed $(+)$ quadrature variances for an OPA below threshold can be computed as [\[33\]](#page-4-10)

$$
\Delta^2 \hat{X}_{+,-} = 1 \pm \eta_{\text{tot}} \frac{4\sqrt{P/P_{\text{thr}}}}{(1 \mp \sqrt{P/P_{\text{thr}}})^2 + 4(\frac{2\pi f}{\gamma})^2}, \quad (1)
$$

FIG. 3. Pump power dependence of the squeezing and antisqueezing spectra, experiment, and theory. The theoretical curves (dashed lines) were modeled with $\eta_{\text{tot}} = 0.975$ and $\theta_{\text{pn}} =$ 1.7 mrad for pump powers corresponding to P/P_{thr} of 8%, 33.9%, and 83.5% and a full-width-half-maximum linewidth $\gamma/2\pi=84$ MHz. The electronic dark noise was subtracted from the data, which were subsequently normalized to the vacuum noise level. The measurement time for each individual trace was 295 ms. Already with a pump power of 1.6 mW, a nonclassical noise reduction of up to 5 dB was obtained. With 7 mW our setup produced up to 10 dB squeezing with only 11 dB antisqueezing. The model is in good agreement with the 0.3 dB increase to 15.3 dB of maximum squeezing due to subtraction of the dark noise. This effect is negligible for all other traces in this representation.

where η_{tot} is the total detection efficiency $(\eta_{\text{tot}} = 1 - \text{total optical loss})$, P is the second-harmonic pump power, P_{thr} is the amount of pump power required to reach the threshold of OPA, and f is the sideband frequency of the measurement. The cavity decay rate $\gamma = c(T + L)l$, with the speed of light c, the cavity round-trip length *l*, the coupling mirror's power transmissivity T , and the round-trip loss L . In addition to optical loss, one needs to include the impact of phase fluctuations between the signal and LO field. While the effect of optical loss is the addition of contributions from the unsqueezed vacuum field to the squeezed quadrature, the effect of phase noise is to add extra noise through components proportional to the antisqueezed quadrature. Assuming that potential phase fluctuations follow a Gaussian distribution and that the standard deviation is small, a rms phase jitter θ_{pn} corresponds to the homodyne detector measuring at an offset phase angle θ_{pn} relative to the ideal quadrature that yields variances of the form [\[7\]](#page-3-5)

$$
V_{+,-} = \Delta^2 \hat{X}_{+,-} \cos^2(\theta_{pn}) + \Delta^2 \hat{X}_{-,+} \sin^2(\theta_{pn}). \quad (2)
$$

By varying η_{tot} , θ_{pn} , and P/P_{thr} this model was used to fit our measurements. The spectral distributions shown in Fig. [3](#page-2-1) are squeezing and antisqueezing measurements obtained with second-harmonic pump powers of 1.6, 7, and 16 mW, respectively. Here, the electronic dark noise was subtracted from the measured data (antisqueezing, vacuum, and squeezing) and subsequently all traces were renormalized to the (new) vacuum noise level. From the squeezing level of 15.3 dB shown here, we deduce that the directly measured squeezing as shown in Fig. [2](#page-2-0) was degraded by 0.3 dB due to contributions by electronic dark noise. The dashed lines in Fig. [3](#page-2-1) correspond to the above model with ratios P/P_{thr} of 8%, 33.9%, and 83.5% with a total detection efficiency $\eta_{\text{tot}} = 0.975$ and a phase noise $\theta_{pn} = 1.7$ mrad.

The detected squeezing level is not limited by phase noise but by $2.5^{+0.1}_{-0.1}\%$ total optical loss, of which we attribute $0.8^{+0.1}_{-0.1}$ % to the homodyne contrast of 99.6^{+0.05}% and $0.2^{+0.01}_{-0.01}\%$ to the transmission loss through lenses. The latter we measured in a separate experiment. We determined the OPA escape efficiency to $\eta_{\rm esc} = T/(T+L)$ = 99.05 $^{+0.4}_{-0.45}$ %. For this we took into account the loss of the PPKTP crystal comprising the residual transmission through the HR-coated backside 400^{+100}_{-100} ppm, which was measured in a separate experiment, the (negligible) absorption within the crystal $(12 ppm/cm [34,35])$ $(12 ppm/cm [34,35])$, and the residual reflection of the AR-coated frontside (two times 400^{+200}_{-200} ppm), and the coupling mirror transmissivity $T =$ $12.5^{+0.5}_{-0.5}\%$. The latter two correspond to the manufacturer specifications. This analysis accounts for $2.0^{+0.5}_{-0.6}$ % of the $2.5^{+0.1}_{-0.1}\%$ overall optical loss in our squeezing path. The remaining contribution to the loss budget is the quantum efficiency of the retroreflector enhanced photodiodes, which we therefore deduce to be 99.5% with 0.5% $(k = 2)$ uncertainty.

In conclusion, we have realized a low-loss and low phase noise squeezed-light experiment with a doubly resonant, nonmonolithic OPA cavity. From a comparison to a theoretical model we determined the total optical loss to be $2.5^{+0.1}_{-0.1}\%$ and the upper bound for phase noise to be 1.7 mrad. With this ultralow level of phase noise, our experiment meets the requirements of current and future gravitational-wave detectors [\[36,37\].](#page-4-12) With only 7 mW pump power, 10 dB squeezing with 11 dB antisqueezing were measured, which are the purest strongly squeezed states observed to date. The generation of the high squeezing factor (10 dB), together with a low antisqueezing factor (11 dB), which is close to the lower bound set by Heisenberg's uncertainty relation, is of high relevance for the application in GW detectors. Low antisqueezing is important to minimize backaction noise and to make backaction evading (quantum nondemolition) schemes, e.g., the one in [\[38\]](#page-4-13), more efficient. By the implementation of a scheme for the coherent control of audioband squeezed vacuum states [\[39\]](#page-4-14) our apparatus can be extended to be compatible with the application in gravitational-wave detectors. With an increased pump power up to 15 dB

squeezing was directly measurable for the first time. These states were used for an absolute calibration of the photoelectric external quantum efficiency of a custom-made p-i-n InGaAs photodiode to 99.5% with 0.5% $(k = 2)$ uncertainty at a wavelength of 1064 nm. Our method does not rely on the measurement of absolute optical power, as realized for example in [\[24\]](#page-4-5) by means of an electrically calibrated cryogenic radiometer (ECCR). Since no reference detector is required for comparison nor photon flux needed to be known, it is rather a direct measurement of the external quantum efficiency. The high precision documents the potential of this novel application of squeezed states in quantum metrology. Any additional reduction of uncertainties, especially of the OPA escape efficiency, will directly enhance the accuracy of this calibration method even further.

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