Universal Charge Diffusion and the Butterfly Effect in Holographic Theories

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We study charge diffusion in holographic scaling theories with a particle-hole symmetry. We show that these theories have a universal regime in which the diffusion constant is given by $D_c = C v_B^2/(2\pi T)$, where v_B is the velocity of the butterfly effect. The constant of proportionality C depends only on the scaling exponents of the infrared theory. Our results suggest an unexpected connection between transport at strong coupling and quantum chaos.

Introduction.—The transport properties of strongly correlated materials display a remarkable degree of universality. In particular, many materials with fundamentally different microscopic physics exhibit a linear resistivity over a broad temperature regime. A long-standing idea to explain such universality has been that transport is governed by a fundamental dissipative time scale, $\tau \sim \hbar/(k_B T)$ [1–3]. Recently, theoretical attention has been refocused on this claim following the direct observation of this "Planckian" time scale in a wide range of materials [4,5].

Perhaps the most famous example of how τ could lead to universal behavior is found in the proposed viscosity bound of Kovtun, Son, and Starinets [6]. Assuming that the viscosity η of a relativistic theory is controlled by this time scale leads to a conjecture that η/s should be given by

$$\frac{\eta}{s} \sim \frac{1}{4\pi}\frac{\hbar}{k_B}, \qquad (1)$$

where s is the entropy density. The original evidence for this claim stemmed from the observation that such a value is generic to many holographic theories with a gravity dual [7–9]. However, more recent developments in holography have shown that this relationship can receive large corrections, for instance, in anisotropic systems or those without translational symmetry [10–16].

Nevertheless, the idea that τ underpins the transport coefficients of strongly coupled matter has survived. Indeed, noticing that the viscosity controls momentum diffusion, it was proposed in Ref. [5] that one can reformulate the Kovtun, Son and Starinets bound in terms of the diffusion constants

$$D \sim \frac{\hbar v^2}{k_B T}, \qquad (2)$$

where v is a characteristic velocity of the theory.

A simple place in which to test Eq. (2) is in the context of a particle-hole symmetric theory. In this case the electrical current decouples from momentum and one has a finite conductivity even in a translationally invariant theory. In particular, for a holographic Conformal Field Theory

with an Einstein gravity dual then the charge diffusion constant indeed takes a universal form [17]

$$D_c = \frac{\hbar c^2}{4\pi k_B T}\frac{d+1}{d-1}, \qquad (3)$$

where d is the number of spatial dimensions and the characteristic velocity of a relativistic theory is the speed of light, $v = c$. Outside the framework of a relativistic theory, however, it has proved challenging to identify a velocity to appear in Eq. (2).

The purpose of this Letter is to point out that a natural candidate for such a velocity in a strongly coupled theory is provided by the butterfly effect [18–23]. In particular, the chaotic properties of strongly interacting large N gauge theories have recently been intensely studied using the holographic correspondence [18–27]. In such theories the butterfly effect refers to the exponential growth in the commutators of generic Hermitian operators which occurs after the thermal time scale β [19–21],

$$\langle [\hat{W}_x(t_w), \hat{V}_y(0)]^2 \rangle_\beta \sim f_1 e^{\lambda_L(t_w - t_* - |x-y|/v_B)} + \dots, \qquad (4)$$

where t_* is the scrambling time, λ_L is the Lyapunov exponent, and v_B is known as the butterfly velocity.

For any holographic theory with a classical gravity dual, both the Lyapunov exponent λ_L and the butterfly velocity v_B can be extracted from properties of a black hole horizon [19–21]. Similarly, it has long been established that the dc transport coefficients of conserved quantities can also be related to the horizon via the membrane paradigm [9]. As such, the holographic correspondence hints at an intimate connection between transport and the butterfly effect.

Moreover, the butterfly velocity v_B provides a natural analogue of the speed light in Eq. (3), which can be defined even in nonrelativistic theories. Specifically, the commutator in Eq. (4) determines how a perturbation to the system by $\hat{V}_y$ propagates to affect a later, distant measurement by $\hat{W}_x$. The butterfly velocity therefore describes the finite speed at which information spreads and, hence, has been

argued in Ref. [27] to act as a state-dependent Lieb-Robinson velocity [28]. These general considerations therefore motivate us to propose v_B as the characteristic velocity through which to formulate the diffusion bound of Hartnoll [5]. That is, the diffusion constants should be bounded by

$$D \sim \frac{\hbar v_B^2}{k_B T}, \qquad (5)$$

where the saturation of such a bound would correspond to a Planckian dissipation time $\tau \sim \hbar/k_B T$.

In the remainder of this Letter we wish to provide initial evidence in support of Eq. (5) by studying the charge diffusion constant of simple holographic scaling geometries. In particular, we will consider theories whose infrared physics is described by a dynamical critical exponent z, a hyperscaling violation exponent θ, and an anomalous dimension Φ for the charge density. Since previous studies of the butterfly effect in holography have focused on conformal field theories [19–21]; our first task is to calculate the velocity v_B for these more general geometries.

Armed with this velocity, we can then proceed to compare with the diffusion constant. Our central result is that in these theories there is a universal regime in which D_c indeed satisfies a relationship of the form (5)

$$D_c = \frac{d_\theta}{\Delta_\chi} \frac{v_B^2}{2\pi T}, \qquad (6)$$

where d_θ is the effective spatial dimensionality of the fixed point, Δ_χ is the scaling dimension of the susceptibility, and we have reverted to high energy units $\hbar = k_B = 1$. Finally, we close this Letter with a brief discussion of the implications of Eq. (6) for our general proposal and the extension to other diffusion constants.

Butterfly effect in scaling geometries.—Like in classical physics, the butterfly effect in a quantum system is associated with whether the effects of a small perturbation can eventually become large at late times. In holographic theories, the butterfly effect corresponds to the fact that the energy of an in-falling particle near a black hole horizon is exponentially boosted at late times [19]. The backreaction of this particle on the geometry creates a shock wave along the horizon that causes the growth of commutators Eq. (4).

From the form of this shock-wave geometry, it is possible to read off both the Lyapunov exponent and the velocity v_B. For the most part, previous holographic studies of chaos have focused on conformal field theories in which the velocity v_B is just a constant [19–21]. We will therefore begin by adapting the shock-wave techniques of Refs. [19–21] to calculate v_B for a more general family of metrics. These are described by an infrared geometry,

$$ds_{d+2}^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r)d\vec{x}_d^2, \qquad (7)$$

where r is the additional radial coordinate of the gravitational theory. At zero temperature we will assume we have power law solutions,

$$U(r) = L_t^{-2} r^{u_1}, \qquad V(r) = L_x^{-2} r^{2v_1}, \qquad (8)$$

where we take $u_1 > 1$, $v_1 > 0$ so that $r \to 0$ corresponds to the infrared of our theory. To turn on a finite temperature, we can introduce a horizon at $r = r_0$:

$$U(r) = L_t^{-2} r^{u_1}\left(1 - \frac{r_0^\delta}{r^\delta}\right), \qquad V(r) = L_x^{-2} r^{2v_1}, \qquad (9)$$

with $\delta = dv_1 + u_1 - 1$. The temperature of the boundary quantum field theory is then related to the horizon radius r_0 by the usual formula $4\pi T = U'(r_0)$.

Although written in unusual coordinates, these metrics simply correspond to a family of hyperscaling violating geometries [29–35] where the critical exponents z, θ are related to the power laws in the metric via

$$u_1 = \frac{2z - 2\theta/d}{z - 2\theta/d}, \qquad 2v_1 = \frac{2 - 2\theta/d}{z - 2\theta/d}. \qquad (10)$$

As usual, the dynamical critical exponent z characterizes the different scaling of space and time at the fixed point $[x] = -1$, $[T] = -[t] = z$. The hyperscaling violation exponent θ corresponds to the fact that the metric transforms nontrivially under scaling and is responsible for an effective shift in the dimensionality of the free energy, $[f] = z + d - \theta = z + d_\theta$.

Additionally, we have retained various parameters in our solution, L_t, L_x, that might normally be set to unity. These parameters, which set various scales in our infrared theory, are nonuniversal and will depend on the embedding of our metric into an asymptotically anti–de Sitter (AdS) spacetime. We will retain them in order to emphasize that the relationship between the diffusion constant and the butterfly effect [Eq. (6)] is independent of this UV data.

Shock-wave geometries.—We now wish to calculate the velocity of the butterfly effect dual to the metrics Eq. (9) by constructing the relevant shock-wave geometries [19,20]. In order to do this, we first need to pass to Kruskal coordinates (u, v). We therefore define

$$uv = -e^{U'(r_0) r_*(r)}, \qquad u/v = -e^{-U'(r_0)t}, \qquad (11)$$

where the tortoise coordinate is given as usual by $dr_* = dr/U(r)$. In terms of these coordinates our black hole metric now reads

$$ds^2 = A(uv)dudv + V(uv)d\vec{x}_d^2,$$

$$A(uv) = \frac{4}{uv}\frac{U(r)}{U'(r_0)^2}, \qquad V(uv) = V(r), \qquad (12)$$

with horizons now located at $u = 0$ and $v = 0$.

To study the butterfly effect, we consider releasing a particle from $x = 0$ on the boundary of AdS at a time t_w in the past. Then for late times (i.e., $t_w > \beta$) the energy density of this particle in Kruskal coordinates is exponentially boosted and localized on the $u = 0$ horizon,

$$\delta T_{uu} \sim E e^{(2\pi/\beta)t_w} \delta(u)\delta(\vec{x}), \tag{13}$$

where E is the initial asymptotic energy of the particle. As a result, even the effects of an initially small perturbation cannot be neglected and after the scrambling time $t_* \sim \beta \log N^2$ the backreaction of the stress tensor Eq. (13) on the metric becomes significant.

Within generic theories of Einstein gravity coupled to matter, the resulting geometry takes a universal form—it is a shock wave that is localized on the horizon [19–21, 36–38]. In particular, such a solution corresponds to a shift in the v coordinate $v \to v + h(x)$ as one crosses the $u = 0$ horizon. The resulting metric can then be written as

$$ds^2 = A(uv)dudv + V(uv)d\vec{x}^2 - A(uv)\delta(u)h(x)du^2, \tag{14}$$

where one finds a solution to the Einstein equations provided the shift obeys [38]

$$(\partial_i\partial_i - m^2)h(x) \sim \frac{16\pi G_N V(0)}{A(0)} E e^{(2\pi/\beta)t_w}\delta(\vec{x}) \tag{15}$$

and the screening length m is given by

$$m^2 = \frac{d}{A(0)}\frac{\partial V(uv)}{\partial(uv)}\bigg|_{u=0}. \tag{16}$$

Remarkably, after using the background equations of motion, one finds that the equations (15) and (16) for the shift still hold even when there is a nontrivial stress tensor supporting the background geometry [27,38]. The only way the matter content of the theory effects the shock wave is indirectly through the determination of the metric functions $A(uv)$ and $V(uv)$.

The net result is that all we need to do to study the butterfly effect is therefore to solve Eq. (15). At long distances $x \gg m^{-1}$ the metric is simply given by

$$h(x) \sim \frac{E e^{(2\pi/\beta)(t_w - t_*) - m|x|}}{|x|^{(d-1)/2}}. \tag{17}$$

Since it is the formation of this shock-wave geometry that is responsible for the growth of commutators [19,20], one can immediately read off the Lyapunov exponent λ_L and velocity v_B of these holographic theories as

$$\lambda_L = \frac{2\pi}{\beta}, \qquad v_B = \frac{2\pi}{\beta m}. \tag{18}$$

While the Lyapunov exponent is universal in all these theories, and saturates the proposed bound on chaos [23],

the velocity is model dependent. In order to extract v_B for our metrics [Eq. (9)], we observe that we can rewrite the screening length Eq. (16) in terms of more familiar coordinates (r, t) as

$$m^2 = d\pi T V'(r_0), \tag{19}$$

where the prime $(')$ indicates a radial derivative. The butterfly velocity therefore takes a remarkably simple form:

$$v_B^2 = \frac{4\pi T}{dV'(r_0)}. \tag{20}$$

For the case of the AdS-Schwarzchild solution, dual to a CFT, we have $V'(r_0) \sim r_0 \sim T$, giving a constant velocity as expected. In our more general scaling geometries, however, this velocity has a nontrivial temperature dependence:

$$v_B^2 \sim T^{2-2/z}. \tag{21}$$

Charge diffusion.—Now that we have the butterfly velocity v_B, we can proceed to study the diffusion of a $U(1)$ charge in the background Eq. (9). We therefore consider coupling a gauge field A_μ to our theories using an action

$$S = \int d^{d+2}x\sqrt{-g}\left[-\frac{1}{4}Z(r)F^{\mu\nu}F_{\mu\nu}\right], \tag{22}$$

where $Z(r)$ is a position-dependent Maxwell coupling. In the context of our scaling geometries, this position dependence can arise from a coupling between the gauge field and a logarithmically running dilaton that supports our background geometry Eq. (9). We will therefore assume that it takes a power law form $Z(r) = Z_0 r^\gamma$.

In terms of the boundary field theory, this running of the Maxwell coupling corresponds to the possibility that the $U(1)$ charge density can have an anomalous dimension in the IR [39–42]. That is, the chemical potential μ and charge density ρ dual to the gauge field A_μ have scaling dimensions

$$[\mu] = z - \Phi, \qquad [\rho] = d - \theta + \Phi, \tag{23}$$

where the anomalous dimension Φ is related to the running of the Maxwell coupling by

$$\gamma = \frac{2\Phi - 2\theta/d}{z - 2\theta/d}. \tag{24}$$

Extracting the diffusion constant for a current with particle-hole symmetry is then a standard calculation in holography. The Einstein relation $D_c = \sigma/\chi$ relates the diffusion constant to the conductivity σ and the susceptibility $\chi = (\partial\rho/\partial\mu)_T$. Both of these quantities can then be calculated using the membrane paradigm [9]. The electrical conductivity takes a particularly simple form—it is just

related to the effective Maxwell coupling on the horizon:

$$\sigma = V^{d/2-1}Z(r)|_{r_0}. \qquad (25)$$

Obtaining the susceptibility is only slightly more complicated. If we consider turning on a small chemical potential μ, the Maxwell equation implies that the electric flux is a constant

$$-\sqrt{-g}Z(r)g^{rr}g^{tt}\partial_r A_t = \rho, \qquad (26)$$

where ρ is interpreted as the charge density of the boundary theory. Since the metric functions are independent of μ at leading order, we can read off the susceptibility

$$\chi^{-1} = \frac{\partial\mu}{\partial\rho}\bigg|_{\rho=0} = \int_\infty^{r_0} dr \frac{1}{\sqrt{-g}Z(r)g^{rr}g^{tt}} \qquad (27)$$

and, hence, arrive at the diffusion constant

$$D_c = [V^{d/2-1}Z(r)]_{r_0} \int_\infty^{r_0} dr \frac{1}{\sqrt{-g}Z(r)g^{rr}g^{tt}}. \qquad (28)$$

At this point it is important to emphasize that, in general, the susceptibility, and hence the diffusion constant, depends via Eq. (27) on the details of the full bulk geometry. Since the butterfly velocity depends only on the local properties of the horizon, these effects will not always be related in a simple manner.

In particular, the behavior of the susceptibility will depend on whichever region of the geometry dominates the integral in Eq. (27). For our scaling geometries Eq. (9), this results in two qualitatively different regimes, depending on the scaling dimension,

$$\Delta_\chi = [\rho] - [\mu] = d_\theta + 2\Phi - z, \qquad (29)$$

of the susceptibility.

Nonuniversal regime.—When the scaling dimension is negative, $\Delta_\chi/z < 0$, then it is the UV region of the geometry that controls the susceptibility. Since in this case the diffusion constant is sensitive to the full geometry, we should not expect to find universal behavior in general. In particular, after performing the integral Eq. (27), we find that at low temperatures the susceptibility takes a constant value set by the cutoff $\chi \sim \Lambda_{\mathrm{UV}}^{\Delta_\chi/z}$. The resulting diffusion constant therefore scales as

$$D_c \sim \left(\frac{\Lambda_{\mathrm{UV}}}{T}\right)^{-\Delta_\chi/z} T^{1-2/z}, \qquad (30)$$

and, hence, in this regime it is parametrically larger than v_B^2/T [Eq. (20)] by powers of the cutoff.

Universal regime.—In contrast, when the scaling dimension $\Delta_\chi/z > 0$, it is the infrared region of the geometry that dominates the integral and gives a susceptibility $\chi \sim T^{\Delta_\chi/z}$. As a result, the charge fluctuations are now controlled by the near-horizon geometry, and so we are able to relate D_c to the butterfly effect. Indeed, if we explicitly evaluate our expression Eq. (28), we arrive at a formula for the diffusion constant:

$$D_c = \frac{z - 2\theta/d}{\Delta_\chi} L_x^2 r_0^{1-2v_1}, \qquad (31)$$

where the explicit dependence on the dilaton profile has canceled between the conductivity and susceptibility. If we now note that our formula (20) for the butterfly velocity is equivalent to

$$\frac{v_B^2}{2\pi T} = \frac{z - 2\theta/d}{d_\theta} L_x^2 r_0^{1-2v_1}, \qquad (32)$$

we see that the diffusion constant always takes the form

$$D_c = \frac{d_\theta}{\Delta_\chi} \frac{v_B^2}{2\pi T}, \qquad (33)$$

which is the central result we presented in the introduction Eq. (6).

It is worth emphasising that both the diffusion constant D_c and the butterfly velocity v_B depend, through L_x and r_0, on the details of our holographic theory. In particular, they are sensitive to how our metric Eq. (9) is embedded into an asymptotically AdS space-time. The relationship between them, on the other hand, is completely universal—it depends only on the scaling exponents of the infrared fixed point.

Discussion.—In summary, we have proposed that a natural way to define the diffusion bound of Hartnoll [5] in a strongly coupled system is to use the velocity v_B of the butterfly effect. Our main piece of evidence in support of this proposal was a calculation of the charge diffusion constant of simple holographic scaling geometries. We found that these theories had a universal regime in which the diffusion constant could be tied to properties of the black hole horizon. Since the butterfly velocity is also determined by the horizon, we were able to establish our central result [Eq. (33)]. The net result is that we have provided an intuitive picture of charge diffusion in these models. That is, in this regime we can think of charge transport as being governed by a diffusive process with a velocity v_B and a Planckian time scale $\tau \sim 1/T$.

More generally, we saw that the diffusion constant will not take a universal value but rather depends on the details of the full geometry. Since these effects increased the diffusion constant, this is consistent with the proposal that Eq. (5) corresponds to a "lowest possible value" for D_c. However, it is worth noting that it does not seem possible

to formulate a strict bound, at least in terms of v_B, since the coefficients in Eq. (33) depend on the universality class. Nevertheless, in generic theories these are order 1 numbers, and so the essential point is that as $T \to 0$, the diffusion constant will be at least as big as v_B^2/T.

While our focus in this Letter has been on studying charge diffusion in hyperscaling violating geometries, it is clearly of interest to understand to what extent our proposal Eq. (5) holds more generally. One obvious direction to pursue would be to investigate how higher derivative corrections to the bulk action affect our result [Eq. (33)]. Additionally, it is important to extend our analysis to include other diffusion constants. In the Supplemental Material [43] we, therefore, calculate the momentum diffusion constant (i.e., viscosity) of these holographic scaling geometries and demonstrate that it is also consistent with Eq. (5).

Finally, in Ref. [44] we will provide a further test of our proposal by studying more general holographic models with momentum relaxation [42,45–51]. This will also allow us to study the energy diffusion constant, which diverges in the translationally invariant theories considered here. Remarkably, we find that when momentum relaxation is strong, the diffusion constants do not become arbitrarily small, but rather are universally given by Eq. (5). As such, these models constitute strong further evidence that the diffusion constants of holographic theories are indeed bounded in terms of v_B.

Note added.—Recently, we became aware that Roberts and Swingle have simultaneously computed the velocity of the butterfly effect in hyperscaling violating geometries [27]. Their formula agrees with our Eq. (20).

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