Constraints on Cosmology and Gravity from the Dynamics of Voids

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The Universe is mostly composed of large and relatively empty domains known as cosmic voids, whereas its matter content is predominantly distributed along their boundaries. The remaining material inside them, either dark or luminous matter, is attracted to these boundaries and causes voids to expand faster and to grow emptier over time. Using the distribution of galaxies centered on voids identified in the Sloan Digital Sky Survey and adopting minimal assumptions on the statistical motion of these galaxies, we constrain the average matter content $\Omega_m = 0.281 \pm 0.031$ in the Universe today, as well as the linear growth rate of structure $f/b = 0.417 \pm 0.089$ at median redshift $\bar{z} = 0.57$, where b is the galaxy bias (68% C.L.). These values originate from a percent-level measurement of the anisotropic distortion in the void-galaxy cross-correlation function, $\varepsilon = 1.003 \pm 0.012$, and are robust to consistency tests with bootstraps of the data and simulated mock catalogs within an additional systematic uncertainty of half that size. They surpass (and are complementary to) existing constraints by unlocking cosmological information on smaller scales through an accurate model of nonlinear clustering and dynamics in void environments. As such, our analysis furnishes a powerful probe of deviations from Einstein's general relativity in the low-density regime which has largely remained untested so far. We find no evidence for such deviations in the data at hand.

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Introduction.—After the epoch of recombination, the initially tiny Gaussian density perturbations in the early Universe have grown increasingly nonlinear under the influence of gravity, generating what is known as the cosmic web. Because the gravitational force is attractive, structures with densities above the mean always contract in comoving coordinates, while underdense ones expand. The latter are referred to as cosmic voids and have progressively occupied most of the available space in the Universe. Traditionally the formation of structure is viewed as hierarchical buildup of smaller dense clumps of matter into ever-larger objects. We take the dual perspective where structure formation is seen as the emptying out of void regions onto the walls, filaments, and clusters that surround them.

This void-centric point of view offers distinct advantages when probing the observed accelerated expansion of the Universe for two reasons: first, void dynamics are less nonlinear and, hence, more amenable to modeling than the high-density regime; second, the accelerated expansion began at a density below the cosmic average. For this reason theories that attempt to explain the acceleration without introducing dark energy explicitly modify general relativity (GR) in the low-density regime. The effects of such modifications would therefore be most prominent in voids rather than in dense environments such as the solar system, galaxies, or clusters of galaxies.

While the dominant matter content of the Universe is invisible (dark), luminous tracers such as galaxies allow for the observation of the process of structure formation directly via their peculiar motions that follow the dynamics of voids. Although the individual velocity of galaxies cannot be determined in most cases, its line-of-sight component causes a Doppler shift in their spectrum, in addition to the Hubble redshift of each galaxy. This leads to a unique pattern of redshift-space distortions (RSDs) in the distribution of galaxies around void centers, which allows for the inferring of their velocity flow statistically [1–3]. The relation between galaxy density and velocity in voids can then be used to test the predictions of GR on cosmological scales [4]. So far most studies have focused on correlations between galaxies in this context, but in the dynamics of voids nonlinearities are less severe [4,5]. As a consequence a large amount of smaller-scale information is unlocked for cosmological inference, resulting in a substantial decrease of statistical errors.

Another type of distortion in the distribution of galaxies can be generated by the so-called Alcock-Paczyński (AP) effect [6]. Galaxy surveys measure the redshifts δz and angles $\delta\vartheta$ between any two galaxies on the sky, but these can only be converted to the correct comoving distances parallel ($r_\parallel$) and perpendicular ($r_\perp$) to the line of sight, if the expansion history and the geometry of the universe is known,

$$r_\parallel = \frac{c}{H(z)}\delta z, \qquad r_\perp = D_A(z)\delta\vartheta. \tag{1}$$

The expansion history is described by the Hubble rate

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}, \tag{2}$$

and the geometry by the angular diameter distance

$$D_A(z) = \frac{c}{H_0\sqrt{-\Omega_k}}\sin\left(H_0\sqrt{-\Omega_k}\int_0^z \frac{1}{H(z')}dz'\right). \tag{3}$$

These, in turn, depend on the Hubble constant H_0, the matter and energy content Ω_m and Ω_Λ, and the curvature Ω_k of the Universe today. Therefore, a spherically symmetric structure may appear as an ellipsoid when incorrect cosmological parameters are assumed. The correct parameters can be obtained by demanding that the average shape of cosmic voids be spherically symmetric [7–11], i.e., the ellipticity

$$\varepsilon \coloneqq \frac{r_\parallel}{r_\perp} = \frac{D_A^{\mathrm{true}}(z)H^{\mathrm{true}}(z)}{D_A^{\mathrm{fid}}(z)H^{\mathrm{fid}}(z)} \tag{4}$$

should be unity for the average distribution of galaxies around voids. In this case, $r_\parallel$ and $r_\perp$ refer to distances between galaxies and void centers with a total separation of $r = (r_\parallel^2 + r_\perp^2)^{1/2}$, and we distinguish between the unknown true and the assumed fiducial values of D_A and H.

Model.—In this Letter we apply these two concepts to voids identified in the distribution of galaxies observed with a redshift survey. Thereby, we closely follow the methodology presented in Ref. [4], which has been extensively tested on simulated mock-galaxy catalogs. The starting point is the Gaussian streaming model [12], providing the average distribution of galaxies around voids (in short: void stack) in redshift space via their cross-correlation function

$$1 + \xi_{\mathrm{v}g}(\mathbf{r}) = \int \frac{1 + b\delta_{\mathrm{v}}(r)}{\sqrt{2\pi}\sigma_v}\exp\left(-\frac{(v_\parallel - v_{\mathrm{v}}(r)\frac{r_\parallel}{r})^2}{2\sigma_v^2}\right)dv_\parallel. \tag{5}$$

Here, r and v denote void-centric distances and velocities of galaxies in real space. Because distances are observed in redshift space, one has to take into account the contribution from peculiar motions,

$$r_\parallel = \tilde{r}_\parallel - \frac{v_\parallel}{H(z)}(1+z), \tag{6}$$

where the tilde symbol indicates redshift space. Moreover, b describes the linear bias parameter for galaxies and σ_v their velocity dispersion. In simulations we have verified that the linear galaxy-bias assumption applies as long as the density fluctuations are moderate, i.e., $|\delta_{\mathrm{v}}(r)| \lesssim 1$. The radial density profile of voids in real space can be parametrized with an empirical fitting function obtained from simulations, such as that given in Ref. [5],

$$\delta_{\mathrm{v}}(r) = \delta_c\frac{1-(r/r_s)^\alpha}{1+(r/r_{\mathrm{v}})^\beta}, \tag{7}$$

with a central underdensity δ_c, scale radius r_s, slopes α and β, and the effective void radius r_{v}. The latter is not a free parameter, but determined via $r_{\mathrm{v}} = (3V_{\mathrm{v}}/4\pi)^{1/3}$, where V_{v} is the total volume of a void. The velocity profile can be obtained via mass conservation [13]. Up to linear order in density, it is given by

$$v_{\mathrm{v}}(r) = -\frac{f(z)H(z)}{(1+z)r^2}\int_0^r \delta_{\mathrm{v}}(q)q^2dq, \tag{8}$$

where $f(z)$ is the linear growth rate of density perturbations. Assuming GR and a flat ΛCDM cosmology it can be expressed as [14]

$$f(z) \simeq \left(\frac{\Omega_m(1+z)^3}{\Omega_m(1+z)^3 + \Omega_\Lambda}\right)^{0.55}. \tag{9}$$

Theories of modified gravity predict deviations from GR—and thus Eq. (9)—to be most pronounced in unscreened low-density environments [15], potentially making voids a smoking gun for the detection of a fifth force. We have explicitly checked the range of validity for Eq. (8) in the void environments we analyze using simulations [4,5]. Note that the parameters (f, b, δ_c) are mutually degenerate in this model, but the combinations f/b and $b\delta_c$ can be constrained independently.

Data.—Our results are shown in Fig. 1 for cosmic voids identified in the Sloan Digital Sky Survey (SDSS) DR11 at a median redshift $\bar{z} = 0.57$ [16]. The different panels show void stacks of increasing effective void radius from left to right and top to bottom. Deviations from spherical

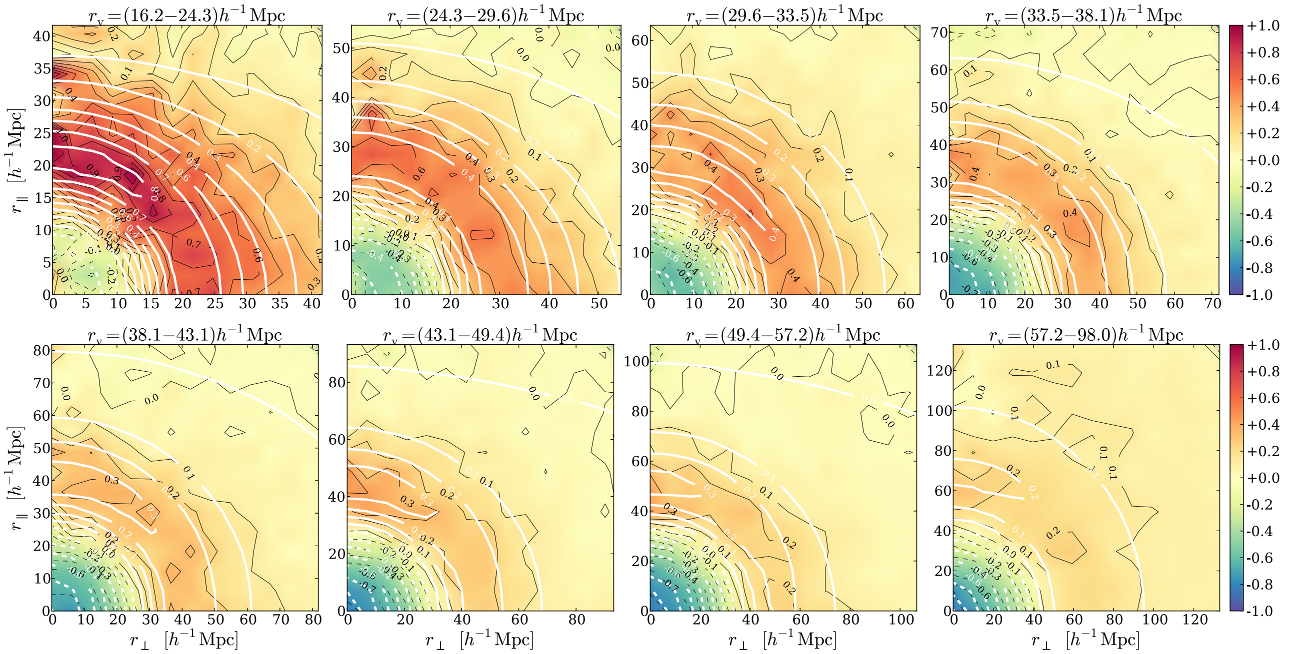

FIG. 1. Void stacks from the SDSS-III DR11 CMASS galaxies at median redshift $\bar{z} = 0.57$ in bins of increasing effective void radius r_v. Void centers are at the origin and the statistical distribution of galaxies in void-centric distances along and perpendicular to the line of sight ($r_\parallel$, $r_\perp$) is color coded: red indicates more and blue, fewer galaxies than average. By construction the average is set to zero (yellow). Black solid (dashed) lines show positive (negative) contours of the data; white lines show the maximum-likelihood fit of the model. Because of the symmetry of the stacks, only one quadrant is shown. The enhanced ridge feature along $r_\parallel$ is caused by the coherent outflow of galaxies from the interior of voids. This allows us to infer the strength of gravity (growth rate f/b) when compared to directions perpendicular to the line of sight $r_\perp$.

symmetry are significant and clearly visible even by eye. These are due to RSDs caused by peculiar velocities in the statistical distribution of galaxies around voids. On large-enough scales most galaxies are attracted coherently by overdensities of the matter distribution and do not change directions, which leads to the characteristic compression of the ridge feature around the void centers along the line of sight. This squashing of overdensities in redshift space is known as the Kaiser effect [17]. On smaller scales the velocity dispersion of galaxies becomes dominant over their coherent flow, causing an elongation of overdense structures along the line of sight that opposes the latter; this is commonly referred to as the finger-of-God (FOG) effect. However, the scales considered in this analysis are still large, and the density fluctuations are small enough for the Kaiser effect to be the dominant one, as evident in Fig. 1. It is also worth noticing the increase of central underdensities towards smaller voids, which is caused by finite-sampling effects when approaching the average galaxy separation of the sample. This effect does not, however, influence the anisotropic component of the void stacks, so it can be marginalized over via the free parameters in Eq. (7).

Analysis.—In order to compare our model from Eq. (5) with the observational data, we employ a Markov chain Monte Carlo (MCMC) technique [16]. The best-fit solutions are shown as white contour levels in Fig. 1 and the posterior distributions in the $\Omega_m - f/b$ plane for the individual void stacks are shown in Fig. 2. In general a very reasonable agreement with our assumed fiducial cosmology is achieved, especially for intermediate-size voids within the range 30 h^{-1} Mpc $\lesssim r_v \lesssim 60$ h^{-1} Mpc. On smaller scales the effects of nonlinear RSDs (FOG) may cause systematic deviations that are not accounted for in our model [4]. On the other hand, our largest void stack necessarily exhibits the widest range of void sizes, as the void abundance drops exponentially in this regime. Therefore, both the RSD signal and the void profile get smeared over a wider range of scales, which can result in a biased fit. Nevertheless, the posteriors on Ω_m and f/b are all consistent with each other across a wide range of scales, providing largely independent and competitive constraints to the existing literature.

This is particularly the case when we choose to combine all the void stacks and infer the posterior parameter distribution jointly in a single MCMC chain that takes into account all the data at once. The resulting posterior distribution is presented in Fig. 3, including the marginal distributions for both Ω_m and f/b individually. Our fiducial cosmology consistently falls inside the innermost confidence level of their joint posterior, and the standard deviation from the marginal distributions amounts to ~11% for Ω_m and ~21% for f/b, relative to their mean values. This implies $\varepsilon = 1.003 \pm 0.012$, a ~1% precision on the AP parameter from Eq. (4), which is nearly a factor

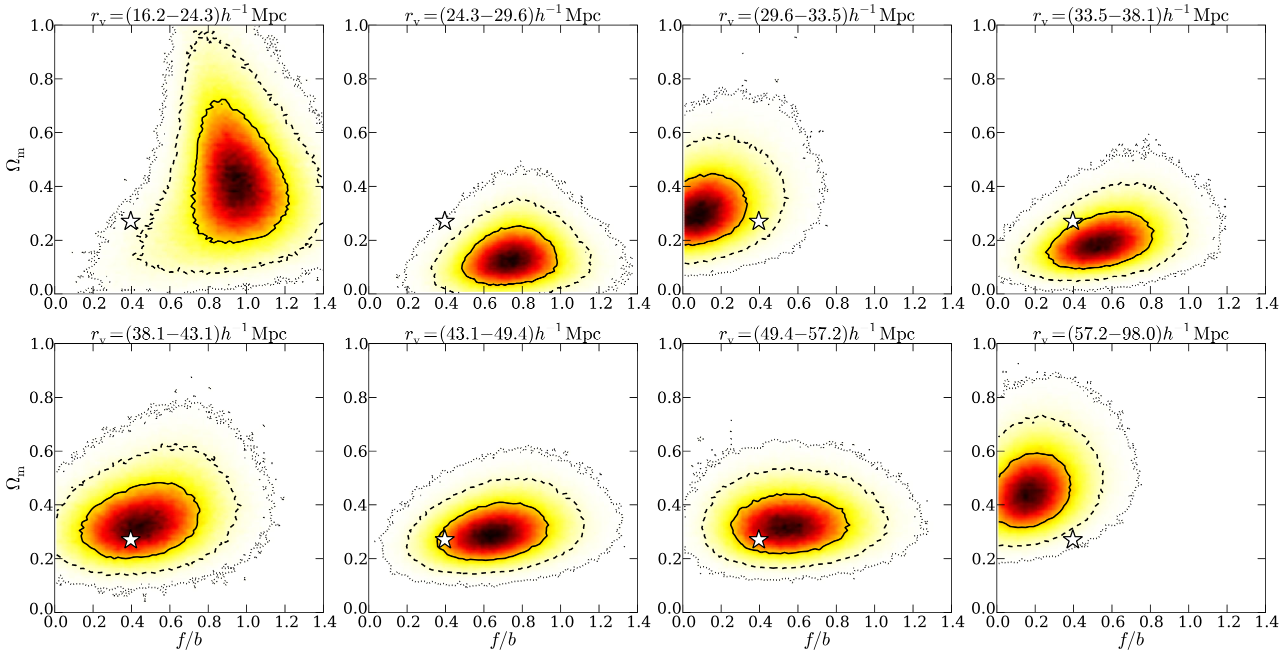

FIG. 2. Constraints on matter density Ω_m and growth rate f/b from each individual void stack of Fig. 1. Solid, dashed, and dotted contour lines represent 68.3%, 95.5%, and 99.7% credible regions, respectively. Stars indicate fiducial values of $\Omega_m = 0.27$ and $f/b = 0.40$.

of 4 smaller than current state-of-the-art galaxy clustering constraints from RSDs (e.g., Ref. [18]), but obtained from a different regime of large-scale structure. We tested the robustness of our constraints using bootstraps of the data and mock catalogs and identify an additional systematic uncertainty of approximately 0.5σ caused by a residual dependence on the choice of our fiducial cosmology (see [16]). Moreover, so far we have neglected the large-scale regime of the void-galaxy cross-correlation function. It exhibits the baryon acoustic oscillation (BAO) feature, a relic clustering excess from the very early Universe. The latter provides a standard ruler and allows for the breaking of the degeneracy between $D_A(z)$ and $H(z)$ in Eq. (4), resulting in even tighter cosmological constraints. The BAO feature in the clustering statistics of cosmic voids has recently been detected in the same data [19] (using a different void definition); it provides complementary information to the RSD analysis conducted in this Letter.

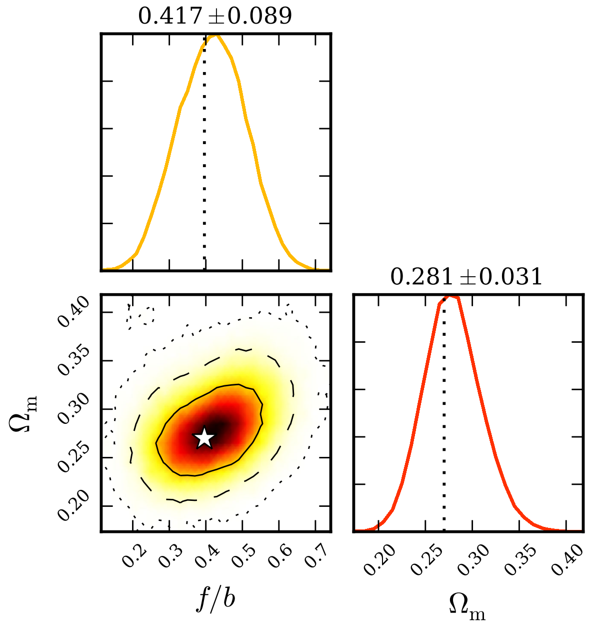

FIG. 3. Joint constraints on matter density Ω_m and growth rate f/b from all void stacks at median redshift $\bar{z} = 0.57$ combined. Their mean and standard deviation is shown above the marginal distributions. The star and dotted lines indicate fiducial values of $\Omega_m = 0.27$ and $f/b = 0.40$.

The consequences of modifications to GR are expected to be most striking in the low-density regime of the cosmic web [15]. For example, voids extracted from simulations in $f(R)$ gravity exhibit significantly higher radial velocity flows compared to standard GR [20]. If present, this effect would be absorbed into our constraint on f/b by biasing it high via Eq. (8). We find no significant evidence for such a bias at the current level of precision.

Conclusions.—Our analysis demonstrates that a substantial amount of unexplored cosmological information can be made available through the analysis of cosmic voids. Besides their dynamics studied in this Letter, voids also act as gravitational lenses [21–23], exhibit rich clustering statistics [24–26] including the BAO feature [19], and constrain cosmology through their abundance and shapes [27,28]. These complementary cosmological observables break parameter degeneracies [29] and are promising probes of dark energy, GR [20,30,31], or the impact of massive neutrinos [32] on cosmological scales. Different void finders most likely yield various trade-offs between the strength of the sought-after signal and the ability to model it, so the optimal void definition will depend on the specific application. We leave further investigations along these lines to future work.

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