Passive CPHASE Gate via Cross-Kerr Nonlinearities

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A fundamental and open question is whether cross-Kerr nonlinearities can be used to construct a controlled-PHASE (CPHASE) gate. Here we propose a gate constructed from a discrete set of atom-mediated cross-Kerr interaction sites with counterpropagating photons. We show that the average gate fidelity F between a CPHASE and our proposed gate increases as the number of interaction sites increases and the spectral width of the photon decreases; e.g., with 12 sites we find $F > 99\%$.

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Photons are attractive in quantum information processing as flying qubits and as a quantum computing platform. To realize the full benefits of quantum photonic applications, a nonlinearity or photon-photon interaction is usually required. However, photons only interact in contrived situations [1]; thus, most interactions between photons are effective, i.e., mediated by matter. For optical quantum computing, in a dual rail encoding, a natural entangling gate is the controlled-PHASE (CPHASE) gate [2,3]. Unfortunately, the photon-photon interactions required for a CPHASE gate are hard to engineer. Thus, much of the progress in the field of optical quantum computing has focused on the Knill-Laflamme-Milburn scheme [4] or measurement-based quantum computing [5–7], which circumvent these issues by use of nondeterministic measurement-induced nonlinearities.

Cross-Kerr interactions have been suggested as a route to a deterministic Fredkin gate by Milburn [8] and a CPHASE gate by Chuang and Yamamoto [9]. These proposals have received less attention than linear-optical schemes due to two obstacles. First, bulk cross-Kerr nonlinearities have historically been very small [10]. However, experiments in cavity-QED [11], circuit-QED [12], and ensemble systems [13], have already demonstrated large cross phase shifts of order one radian per photon.

Second, single-mode analyses fail to account for multimode effects that preclude a high-fidelity CPHASE gate, as pointed out by Shapiro [14] and Gea-Banacloche [15]. In principle, a CPHASE gate could be implemented by a frequency-local interaction, i.e., with a Hamiltonian proportional to $a^\dagger(\omega)a(\omega)b^\dagger(\omega)b(\omega)$. However, physically realistic cross-Kerr effects are spatially localized, e.g., $a^\dagger(x)a(x)b^\dagger(x)b(x)$, because they must be mediated by atoms. This creates a tension between the spectral width of the quanta and the response time of the Kerr medium. If two temporally broad (spectrally narrow) photons impinge on the medium, they are likely to both be absorbed by the atoms, but not at the same interaction site, so no interaction occurs. When temporally narrow (spectrally broad) photons impinge on the medium, the atoms cannot absorb the photons before they leave the interaction site, and again no interaction occurs. Shapiro [14] arrives at similar conclusions via a phenomenological model of the cross-Kerr interaction, which includes a fidelity-degrading phase-noise [16] term. In an intermediate regime, a more fundamental problem with spatially local interactions is that they generate spectral entanglement [15], e.g., when different frequencies gather different cross-phase shifts, or there is frequency mixing. As a consequence of these arguments, it has become folklore that the multimode nature of photons is a fundamental obstacle for constructing a CPHASE gate from Kerr nonlinearities, even in the absence of other imperfections.

Here we provide a counterexample to this claim, by constructing a high-fidelity CPHASE gate using photons that counterpropagate through N atom-mediated cross-Kerr interaction sites. In particular, as N increases and the spectral width of the photons decreases, our proposal tends to a perfect CPHASE gate. Furthermore, because we do not rely on any phenomenology, our results unambiguously show that the multimode nature of the field is not a fundamental obstacle to quantum computation.

There are other proposals for CPHASE gates based on atom-mediated interactions, see Refs. [17–21]. Our proposal was motivated by Ref. [22], where a CPHASE gate was built by a random walk of counterpropagating qubit waves. Counterpropagating photonic wave packets, with interactions mediated by Rydberg atoms or atomic vapors, were investigated in Refs. [23–26]. Our Letter improves on previous proposals in two ways. First, our construction requires no active elements, such as error correction, control pulses, switches, or memories. Second, high fidelities ($F > 99\%$) are obtainable with relatively few interaction sites ($N = 12$).

Our main goal is to construct a gate that entangles two qubits encoded in dual-rail states (see, e.g., [2]) or, equivalently, enact the two-mode transformation,

$$|0\rangle_a \otimes |0\rangle_b \to |0\rangle_a \otimes |0\rangle_b, \tag{1a}$$

$$|0\rangle_a \otimes |1_\xi\rangle_b \to |0\rangle_a \otimes |1_\xi\rangle_b, \tag{1b}$$

$$|1_\xi\rangle_a \otimes |0\rangle_b \to |1_\xi\rangle_a \otimes |0\rangle_b, \tag{1c}$$

$$|1_\xi\rangle_a \otimes |1_\xi\rangle_b \to e^{i\phi}|1_\xi\rangle_a \otimes |1_\xi\rangle_b, \tag{1d}$$

where a and b are photonic modes, $|0\rangle$ indicates a multimode vacuum, $|1_\xi\rangle = \int d\omega\xi(\omega)a^\dagger(\omega)|0\rangle$ is a single photon in the wave packet $\xi(\omega)$, and $[a(\omega), a^\dagger(\omega')] = \delta(\omega - \omega')$. Any nontrivial phase ($0 < \phi < 2\pi$) in Eq. (1d) enables quantum computation, but we are interested in the case $\phi = \pi$, which corresponds to the CPHASE gate.

To characterize the action of a medium on multimode light, we use the S matrix from scattering theory. The S matrix is a unitary matrix connecting asymptotic input and output field states, i.e., $|\omega_{\text{out}}\rangle = S|\nu_{\text{in}}\rangle$, while capturing the relevant effects of the medium. The ideal S matrices corresponding to Eqs. (1a)–(1d) would be $S_{\text{id},1}(\omega_k; \nu_k) = \delta(\omega_k - \nu_k)$ for single-photon states and $S_{\text{id},2}(\omega_a, \omega_b; \nu_a, \nu_b) = e^{i\phi}S_{\text{id},1}(\omega_a; \nu_a)S_{\text{id},1}(\omega_b; \nu_b)$ for two-photon states, where input (output) frequencies are denoted by ν_k (ω_k), for $k = \{a, b\}$. Typically, however, the actual S matrices for matter-mediated interactions are of the form $S_{\text{act},1}(\omega_k;\nu_k) = e^{i\phi_k}\delta(\omega_k - \nu_k)$ and $S_{\text{act},2}(\omega_a, \omega_b; \nu_a, \nu_b) = S_{\text{act},1}(\omega_a; \nu_a)S_{\text{act},1}(\omega_b; \nu_b) + C\delta(\omega_a + \omega_b - \nu_a - \nu_b)$, where the coefficient C depends on all frequencies and the parameters of the interaction mediators [27,28]. The phase $e^{i\phi_k}$ in $S_{\text{act},1}$ leads to a deformation of the single-photon wave packets, while the function $\delta(\omega_a + \omega_b - \nu_a - \nu_b)$ in $S_{\text{act},2}$, which arises from energy conservation [27], is usually identified as the source of spectral entanglement.

One important choice we make is to ignore single-photon deformation, which is enforced by mapping all S matrices as $S \to S^\dagger_{\text{act},1}(\omega_a; \nu_a)S^\dagger_{\text{act},1}(\omega_b; \nu_b)S$. Most previous proposals do not do this (e.g., [15]), which accounts for part of the discrepancy in the maximum fidelities obtained. Single-photon deformation could have two negative effects for our proposal. First, it might disrupt linear-optical steps of the computation. This can be avoided by ensuring all photons are deformed equally at each computational time step [29]. Second, our results are obtained for specific input wave packet shapes, so single-photon effects could significantly degrade the fidelity of subsequent gates; later, we show that this is not the case for a few rounds of deformation. It is then possible to use measurement-based quantum computing, where each photon experiences at most two CPHASE gates [30], or teleportation-based error correction [31]. Finally, it is also possible to physically undo this deformation if necessary, as proposed in, e.g., Ref. [19].

Ideally, we would like to show that the S matrix for our proposal approaches $S_{\text{id},2}$ in some limit. However, it is sufficient for this to hold only for the particular states that we are considering. Thus, to gauge the quality of our operation, we use the average gate fidelity [32]

$$F(\phi) := \int d\psi \langle\psi|S_{\text{id}}(\phi)^\dagger S_{\text{act}}|\psi\rangle\langle\psi|S^\dagger_{\text{act}}S_{\text{id}}(\phi)|\psi\rangle \tag{2}$$

where the integration is taken over the Haar measure of the joint Hilbert space (for further details, see the Supplemental Material [33]). For our gate to be useful for quantum computation, it suffices that $F = 1 - \epsilon$, where ϵ is some constant threshold [35].

Single- and two-site gate fidelities.—We begin by examining F for wave packets scattering from a single site and from two sites in a co- and counterpropagating arrangement. The discrete Kerr interaction we consider is depicted in Fig. 1(a). The unit cell we repeat in Fig. 1(b)—call it $G = (L, H)$—can be described using LH [36–38] parameters from input-output theory, where L is a vector of operators that couple the field to the system and H is the system Hamiltonian. The LH parameters for our unit cell are

$$G = \left[\begin{pmatrix}\sqrt{\gamma}A_- \\ \sqrt{\gamma}B_-\end{pmatrix}, \quad \frac{\Delta}{2}(\mathbb{1} - A_z) + \frac{\Delta}{2}(\mathbb{1} - B_z) + \chi(\mathbb{1} - A_z)(\mathbb{1} - B_z)\right], \tag{3}$$

where A_- and A_z are the atomic lowering and Pauli Z operators for atom A, and likewise for B_- and B_z. We

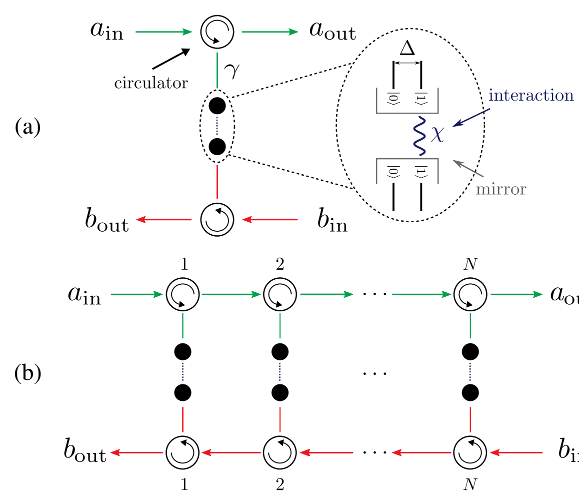

FIG. 1. (a) The physical system inside our unit cell. It consists of two coupled two-level atoms, with internal energies Δ, and which interact via $H = \chi(\mathbb{1} - A_z)(\mathbb{1} - B_z) = \chi|1,1\rangle\langle 1,1|$. The input-output fields couple to the atoms via the relation $a_{\text{out}} = \sqrt{\gamma}A_- + a_{\text{in}}$, and similarly for mode b. It was shown in Ref. [28] that this system gives rise to the same S matrices for single- and two-photon scattering as a pair of crossed cavities with cross-Kerr interaction between them. In the limit $\chi \to \infty$, this reduces to a three-level atom in a "V" configuration, such as considered in Ref. [19]. (b) Our main proposal using N discrete interaction sites with counterpropagating photons.

cascaded N unit cells with co- and counterpropagating fields and computed the corresponding S matrices, as detailed in Ref. [28] and which we reproduce in the Supplemental Material [33]. The final ingredient needed to calculate F is the wave packet shape, which we choose to be Gaussian with detuning ω_0 (i.e., carrier frequency $\omega_c = \Delta + \omega_0$) and bandwidth σ, i.e.,

$$\xi(\omega) = \frac{1}{(2\pi\sigma^2)^{1/4}} \exp\left[-\frac{(\omega-\omega_c)^2}{4\sigma^2}\right]. \tag{4}$$

In Fig. 2, left column, we display the plots for parameters (ω_0, γ, χ) which maximize the fidelity of counterpropagating wave packets. Relative to the single site, we observe a clear increase of the maximum obtainable fidelity when the photons are counterpropagating, and a decrease when they are copropagating, as illustrated in the top row. In the limit of large χ and $\omega_0 = 0$, the phase shift is always either 0 or π, corresponding to the identity or CPHASE gate respectively (see the bottom row). We observe that counterpropagating wave packets tend to perform better than copropagating for a large region of the parameter space, but there are exceptions.

In Fig. 2, middle column, we display a parameter regime where co-propagating photons obtain their maximum fidelity and outperform the other two cases. The explanation for this is the following. In this regime, the copropagating case seems to suffer more spectral entanglement, but also acquire a larger phase shift, than the other two (see the dashed lines), and the tradeoff between these effects leads to a higher fidelity with the CPHASE gate. However, these effects are linked in such a way that this peak fidelity and the maximum phase are still much inferior to the best obtained by the single-site and counterpropagating cases in other parameter regimes. Nonetheless, this suggests it is possible to use a perturbative approach to construct long weakly coupled atom chains where the rate at which phases and spectral entanglement accumulate are more benign (e.g., [19]). It is also interesting that the peak of the fidelity in the copropagating case happens for larger σ than for the counterpropagating case in all three columns, which could lead to a CPHASE gate for spectrally broader photons.

In Fig. 2, right column, we display parameters that maximize the single-site fidelity. As we generically expect, the counterpropagating wave packets outperform the single-site and copropagating ones both in fidelities and phase shifts. This happens even when $\omega_0 \neq 0$, indicating that our conclusions are somewhat robust with respect to being off-resonance.

N-site gate fidelities.—We now investigate the average fidelity of our proposal to the CPHASE gate as we increase the number of interaction sites. Based on observations from the two-site case, we restrict our analysis to counter-propagating photons, working on-resonance ($\omega_0 = 0$), and take a $\chi \to \infty$ limit because this yields the most promising results. We also take $\gamma = 1$, because choosing other values effectively rescales σ when working on-resonance. Thus, the average gate fidelity F is a function of the number of interaction sites N and the photon bandwidth σ: $F(\sigma, N)$.

In Fig. 3(a) we plot the average gate fidelity, as a function of σ, for increasing N. Notice that, as the number of interaction sites increases, the maximum average fidelity

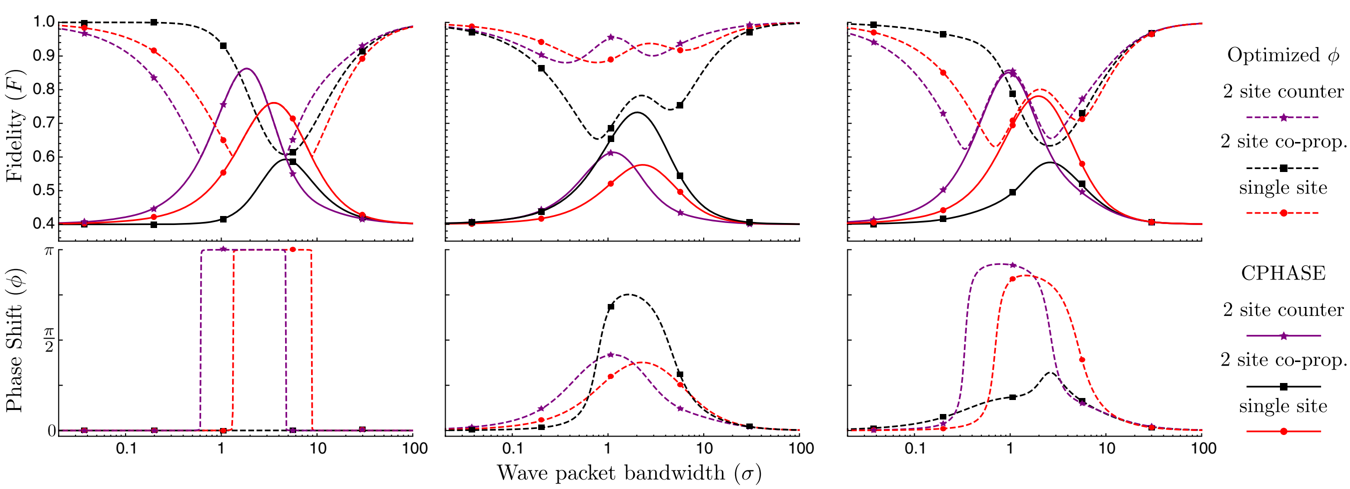

FIG. 2. In the top row, solid lines represent the average gate fidelity with respect to the CPHASE gate, i.e., $F(\phi = \pi)$, while dashed lines denote the fidelity $F(\phi)$ maximized with respect to some ϕ. The bottom row plots the corresponding phase shift $\phi_{\mathrm{opt}} = \mathrm{argmax}_{\phi} F(\phi)$. We compare three cases of interest: (i) a single-site Kerr interaction (circles), (ii) a two-site interaction with copropagating photons (squares), (iii) the two-site interaction with counterpropagation (stars). In the left column we have chosen $(\omega_0, \gamma, \chi) = (0, 10, 10000)$, which maximizes F for the counterpropagating case, resulting in $F_{\mathrm{counter}} = 0.8628$ (only for this case, whenever $F \gtrsim 0.6$, the dashed and solid lines coincide). In the middle column we chose $(\omega_0, \gamma, \chi) = (0, 6, 2.67)$ to maximize F for the copropagating case, obtaining $F_{\mathrm{co\text{-}prop.}} = 0.7326$, and in the right column we chose $(\omega_0, \gamma, \chi) = (1.1, 4.5, 5)$ to optimize the single-site F, obtaining $F_{\mathrm{single}} = 0.7810$.

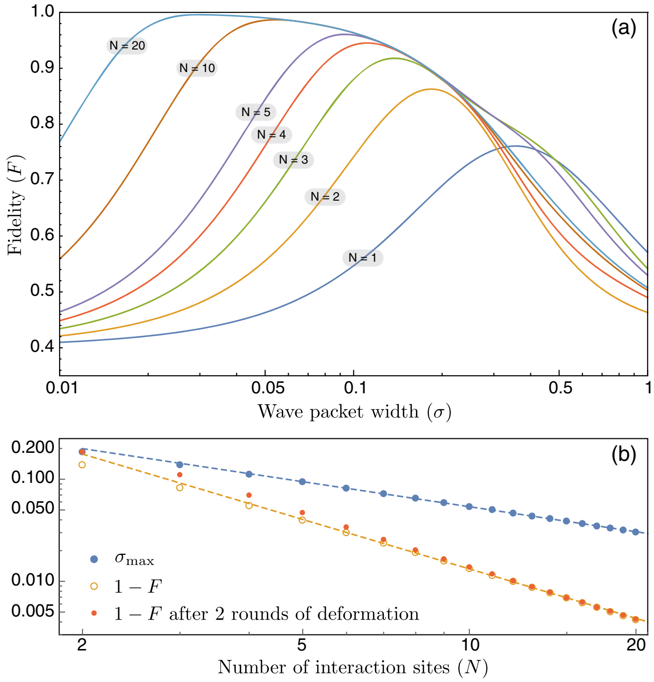

FIG. 3. (a) Average gate fidelity between our proposal and the CPHASE as a function of frequency bandwidth σ for increasing number of interaction sites N. (b) We take the maximum of the gate fidelity in (a), and plot the infidelity $(1-F)$ and the corresponding maximizing $\sigma_{\max}$ as functions of the number of interaction sites N. Small red dots correspond to $1-F$ for photons that have undergone two rounds of single-photon deformation. The dashed lines correspond to the fits $1-F(\sigma_{\max}, N) = 0.537N^{-1.61}$ and $\sigma_{\max} = 0.350N^{-0.81}$, where we fit to $N \in [4, 20]$.

increases, indicating that the resulting operation is sequentially closer to a CPHASE gate. Also notice that, in addition to attaining higher maximum values, the fidelity curve is also becoming broader (albeit in logarithmic scale). This means that as the number of sites is increased, the proposed CPHASE gate becomes more broadband, or robust with respect to the spectral bandwidth of the photon. The highest value for the fidelity in Fig. 3(a) is 0.996, when $N = 20$.

In Fig. 3(b), we investigate the maximum of the average fidelity $F_{\max}$ and its corresponding value of $\sigma_{\max}$ as a function of N. We see that $1 - F_{\max}$ is monotonically decreasing and $\sigma_{\max}$ slowly tends towards the plane-wave limit. For the observed behavior of Figs. 3(a)–3(b) we predict that, in order to obtain $F_{\max} = 0.999$, we would need $N \simeq 50$ and $\sigma \simeq 0.014 s^{-1}$. Figure 3(b) also shows that, for $N > 5$, the fidelity is not significantly affected by using single-photon wave packets that have suffered one or two rounds of deformation.

Another feature apparent in Fig. 3(a) is that, for fixed σ, the advantage gained from increasing N eventually saturates. This is explored further in Fig. 4. An intuitive explanation is as follows: Interpret $1/\gamma$ as the typical time scale before an excited atom reemits a photon; then,

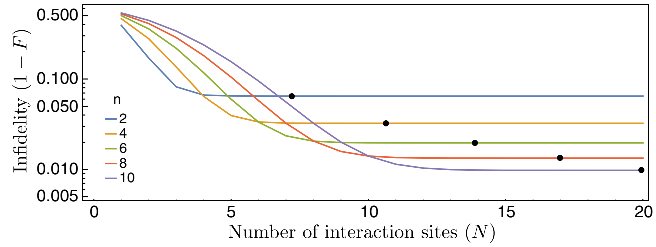

FIG. 4. Saturation of the average gate infidelity for fixed σ as the number of interaction sites N increases. For illustration, we choose the σ s that maximize the fidelity for specific numbers of interaction sites n, i.e., $\sigma^n_{\max} = \mathrm{argmax}_\sigma F(\sigma, n)$ where $n = \{2, 4, 6, 8, 10\}$. The lines then correspond to the fidelities at $\sigma^n_{\max}$ for increasing N, i.e., $F(\sigma^n_{\max}, N)$. Finally, the black dots in each curve correspond to $N = 1/\sigma^n_{\max}$, where we predict the fidelity to saturate.

$t_m \approx N/\gamma$ is the time that each wave packet remains inside the medium. Thus, if the wave packets have temporal width of $t_w \approx 1/\sigma$, when N is roughly γ/σ the chain becomes "long enough" to contain the entire wave packets, and the interaction saturates.

Our results show that, to obtain higher-fidelity gates, one has to move to smaller values of σ together with longer atomic chains. In fact, in Ref. [28], we and Gea-Banacloche have shown that, in the limit where $\sigma \to 0$ and $N \to \infty$, the S matrix for the N-site case tends to the ideal one (modulo single-photon deformation) $S_2(\omega_a, \omega_b; \nu_a, \nu_b) = -S_{\mathrm{act},1}(\omega_a; \nu_a) S_{\mathrm{act},1}(\omega_b; \nu_b)$. This is independent of the specific wave packet shape, further motivating our choice to ignore single-photon deformation. The results presented here are more relevant for implementations, as one only needs to increase N and decrease σ until the fidelity surpasses the threshold necessary for fault-tolerant computation.

Discussion.—Our goal was to determine if it is possible to build a passive CPHASE gate using cross-Kerr interactions, and we have shown that it is. Importantly, our results do not contradict those of Ref. [14], which uses a phenomenological model of a cross-Kerr medium. Using that model, a CPHASE gate might indeed be unachievable. However, our results are based on a fully multimode treatment of the field and a fully microscopic treatment of the interaction mediators. Thus, we believe they provide a counterexample against the stronger claim, frequently propagated the literature, that the multimode nature of the field is a fundamental physical obstacle to implementing a CPHASE gate. Furthermore, our proposal enjoys two advantages over prior proposals. First, our gate is passive; i.e., it does not require active error correction, such as the principal mode projection technique used in Ref. [19]. Also, our proposal requires fewer interaction sites to achieve a fixed fidelity; e.g., in Ref. [19] the authors estimate they need 10^6 interaction sites for a 95% fidelity with a CPHASE gate, whereas our proposal achieves that value with 5 sites.

Admittedly, our proposal is a proof-of-principle result that will be challenging to construct in practice. Further, we hope our construction will inspire others to devise simpler and less resource-intensive proposals. There are plenty of avenues to explore, e.g., placing the atomic interaction sites inside cavities to gain a cavity enhancement [39], or varying the atom parameters along the chain [40] while simultaneously varying the input photon wave packet shape. Our analysis did not include any additional imperfections, and we leave as future work the adaptation of our model to include other effects such as losses, emission into nonguided modes, coupling to a thermal bath, etc. In the Supplemental Material [33] we propose an update to a set of rules initially laid out by Gea-Banacloche [15] that must be satisfied by any theoretical proposal for a realistic CPHASE gate, based on conclusions drawn from this Letter and [28].

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