New Ambiguity in Probing *CP* Violation in Neutrino Oscillations

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If neutrinos get mass via the seesaw mechanism the mixing matrix describing neutrino oscillations can be effectively nonunitary. We show that in this case the neutrino appearance probabilities involve a new *CP* phase ϕ associated with nonunitarity. This leads to an ambiguity in extracting the "standard" three-neutrino phase δ_{CP}, which can survive even after neutrino and antineutrino channels are combined. Its existence should be taken into account in the planning of any oscillation experiment aiming at a robust measurement of δ_{CP}.

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Introduction.—The celebrated discovery of neutrino oscillations and the precision measurements of the corresponding parameters have opened a new era in particle physics. So far experiments have measured two neutrino mass differences and three mixing angles [1]. Four out of these measurements are very precise [2–4], while the octant of the atmospheric mixing angle θ_{23} still remains uncertain. In order to complete such simple three-neutrino paradigm, the hunt for leptonic *CP* violation stands out as the next challenge, taken up by experiments such as T2K and NOνA aimed at determining the Dirac *CP* phase δ_{CP}. (The so–called Majorana phases [5] do not affect the oscillation probabilities, only lepton number violating processes [6–8].) It has long been noted, however, [5] that such a simple closed picture holds true only for the simplest benchmark, in which there are just the three families of conventional orthonormal neutrinos.

One of the most popular ways to induce neutrino mass is the (type-I) seesaw mechanism [5,9–13]. The latter invokes the tree-level exchange of heavy, so far undetected, "right-handed" neutrinos. Such messenger particles may be accessible at the Large Hadron Collider [14–17]. In this case, they are expected to couple in the charged current with appreciable strength, leading to a rectangular form of the mixing matrix characterizing the leptonic weak interaction [5]. The outcome is that the effective mixing matrix describing neutrino oscillations will not in general be unitary. As a result more parameters are required in order to fully describe neutrino oscillations, posing an important challenge for future neutrino experiments [18,19].

In this Letter we focus on the description of neutrino oscillations with a nonunitary neutrino mixing matrix, particularly on the role of the extra *CP* phase required to describe oscillations under this hypothesis. In order to carry out this study, we find it most convenient to make use of the original symmetric parametrization [5] of the neutrino mixing matrix [20], in which the possible "confusion" between the "standard" and "new" *CP* violating phase combinations in the neutrino oscillation probability can be clearly seen. We illustrate this new ambiguity in extracting the Dirac *CP* phase for different L/E choices and different values of the new parameters characterizing nonunitarity. The ambiguities we find are genuinely new, without a counterpart within the standard three-neutrino oscillation paradigm. (They add to the well-known ambiguities associated with the mass hierarchy and θ_{23} octant [21–24].)

We would like to stress that the extra *CP* phase leading to the one-parameter degeneracy in the neutrino conversion rates constitutes a natural feature of neutrino oscillations within a broad class of seesaw theories [19]. The effects of these new degeneracies will have to be taken into account in the planning of current and upcoming experiments aiming at a robust determination of the leptonic Dirac *CP* violation phase δ_{CP}, such as T2K, NOνA, DUNE, MOMENT, etc.

New degeneracies in oscillations.—In the presence of heavy neutral leptons, the mixing matrix describing the leptonic weak interactions will be a rectangular $3\times(3+m)$ matrix [5], K, with m denoting the number of heavy states. As a result, the effective 3×3 mixing submatrix describing neutrino oscillations will be nonunitary. Using the original symmetric form in Ref. [5] one can write the latter, in full generality, as [18]

$$N=\begin{pmatrix}\alpha_{11} & 0 & 0\\ \alpha_{21} & \alpha_{22} & 0\\ \alpha_{31} & \alpha_{32} & \alpha_{33}\end{pmatrix}U^{3\times 3}, \qquad (1)$$

where $U^{3\times 3}$ is the usual three-neutrino unitary mixing matrix. The description of unitarity violation involves three real parameters, α_{ii}, that should be close to one, and three small complex off-diagonal parameters, α_{ij}. Within such a nonunitary framework the neutrino appearance probability *in vacuo*, $P_{\mu e}$, will be similar in form to that found in the unitary case, but with U replaced by the matrix N. (Expressions for P_{ee} and $P_{\mu\mu}$ were given in Ref. [18].

For such CP conserving channels nonunitarity hardly affects the determination of oscillation parameters, which are rather robust.)

This probability can be simplified by neglecting the cubic products of the small parameters α_{21}, $\sin\theta_{13}$ and $\sin(\Delta m^2_{21}L/4E)$. In this case the previous expression reduces to the very simple and compact master formula [18]

$$P_{\mu e} = \alpha_{11}^2\alpha_{22}^2 P_{\mu e}^{3\times 3} + \alpha_{11}^2\alpha_{22}|\alpha_{21}|P_{\mu e}^{I} + \alpha_{11}^2|\alpha_{21}|^2, \quad (2)$$

where the new physics information related to the seesaw mechanism is encoded in the α parameters describing nonunitarity, coming from Eq (1). Here we have used the original symmetric parametrization of the lepton mixing matrix [5] and denoted the standard three-neutrino conversion probability by $P_{\mu e}^{3\times 3}$. The latter is given explicitly in Refs. [25–27].

Notice that Eq. (2) represents, in closed form, the neutrino transition probability *in vacuo* in the presence of nonunitarity. This expression bears some formal similarity to the Kuo-Pantaleone formula [28]. The last term in Eq. (2) is a small "zero-distance" effect characterizing the effective nonorthonormality of the flavor neutrino states [29]. The corrections to the standard three-neutrino form are expected to be small; however, they involve a new CP phase, contained in the interference term $P_{\mu e}^{I}$, so far unrestricted. Its explicit form *in vacuo* is given by

$$\begin{aligned} P_{\mu e}^{I} = &-2\sin 2\theta_{13}\sin\theta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{CP}+\phi) \\ &-\cos\theta_{13}\cos\theta_{23}\sin 2\theta_{12}\sin 2\Delta_{21}\sin\phi, \end{aligned} \quad (3)$$

where we have set $\Delta_{ij} \equiv (\Delta m^2_{ij}L/4E_\nu)$. The CP violation phase-invariant parameter $\delta_{CP} = -(\phi_{12} - \phi_{13} + \phi_{23})$ denotes the standard CP phase, while the CP violation phase associated with "new physics" is given as $\phi = \phi_{12} - \mathrm{Arg}(\alpha_{21})$. (The ϕ_{ij} are the phases associated with each complex rotation in the symmetric parametrization [5].) The presence of this extra phase will lead to a degeneracy in the conversion probability.

Notice that, for values of L/E relevant for current and future long baseline neutrino experiments, the dependence of the appearance probability on the CP phases will be mainly determined by the interplay between two terms, one coming from the standard $P_{\mu e}^{3\times 3}$, and the other one from the interference term $P_{\mu e}^{I}$, namely:

$$\begin{aligned} &2\alpha_{11}^2\alpha_{22}^2\sin\theta_{13}\sin\theta_{23}\sin\Delta_{31}\sin 2\Delta_{21} \\ &\times\left[\sin 2\theta_{12}\cos\theta_{23}\cos(\Delta_{31}+\delta_{CP}) \right. \\ &\left. -2\frac{\cos\theta_{13}}{\alpha_{22}}\frac{|\alpha_{21}|}{\sin 2\Delta_{21}}\sin(\Delta_{31}+\delta_{CP}+\phi)\right]. \end{aligned} \quad (4)$$

By examining the brackets, one sees that, as expected, for vanishing α_{21}, we recover just the standard appearance probability, while a relatively large α_{21} value clearly leads to a degeneracy between δ_{CP} and ϕ.

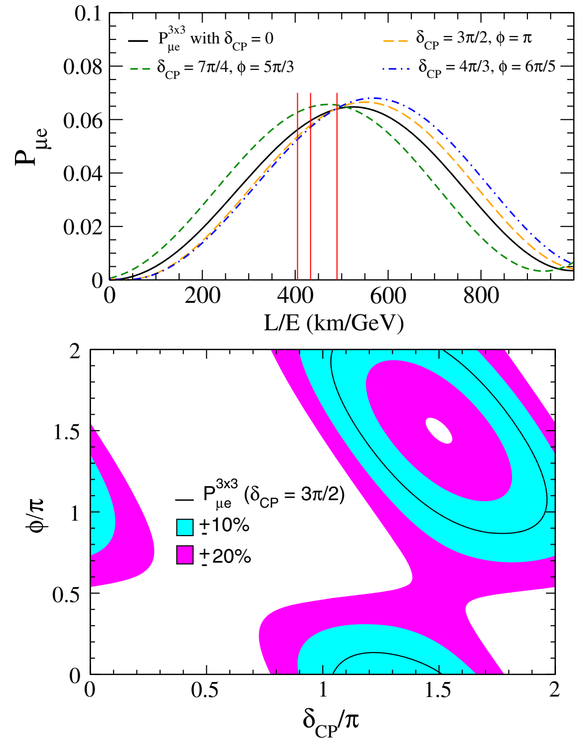

FIG. 1. Top: Vacuum appearance probability $P_{\mu e}$ versus L/E for different phase combinations, illustrating a degeneracy for $L/E = 500$ km/GeV. Vertical lines indicate the mean value of L/E for NOνA (405), DUNE (433), and T2K (490 km/GeV). Bottom: Isocontours of $P_{\mu e}$ as a function of the two CP phases. The solid line corresponds to the standard value $P_{\mu e}^{3\times 3}$ with $\delta_{CP} = 3\pi/2$ while colored regions denote the corresponding 10% and 20% deviations, as indicated.

This fact is illustrated in Fig. 1, where we show the conversion probability as a function of L/E for different values of the CP phases (top panel). One finds that, for a given L/E, the same conversion probability can be obtained for several CP phase combinations. Values of L/E for T2K, NOνA, and DUNE are indicated with vertical lines for illustration. For the nonunitarity parameters we have considered $\alpha_{11}^2 = \alpha_{22}^2 = 0.999$, and $|\alpha_{21}| = 2.5\times 10^{-2}$, consistent with the current bounds obtained in Ref. [18]. All over the paper, the neutrino oscillation parameters have been taken to their best fit value obtained in Ref. [2], with θ_{23} in the second octant. Normal mass hierarchy has been assumed. These degeneracies are further illustrated at the bottom panel of Fig. 1 which shows the CP isocontours that lead to the same probability to within 10% and 20%, given a true value of the standard three–neutrino probability with $\delta_{CP} = 3\pi/2$, as indicated by the current best fit point [2]. In this figure we have fixed $L/E = 500$ km/GeV which lies very close to the value characterizing the T2K experiment.

Coping with the new ambiguity.—In Fig. 1 we saw how the new degeneracy associated with nonunitarity leads to ambiguities in $P_{\mu e}$. The comparison between the neutrino

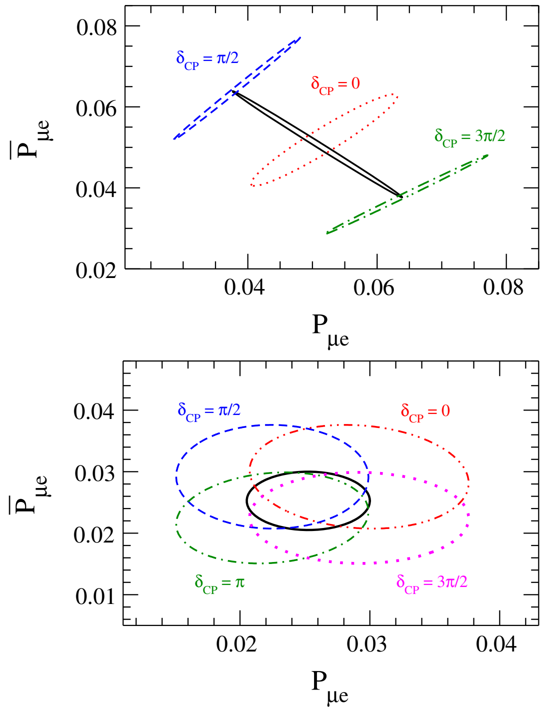

FIG. 2. Bi-probability plots for two different choices of L/E. The standard CP phase δ_{CP} is fixed for each ellipse (except for the standard one denoted in black, where it varies freely), while the new phase ϕ is allowed to vary from 0 to 2π. The upper panel, with $L/E = 490$ km, corresponds to T2K while the bottom panel, with $L/E = 250$ km, has been chosen for comparison.

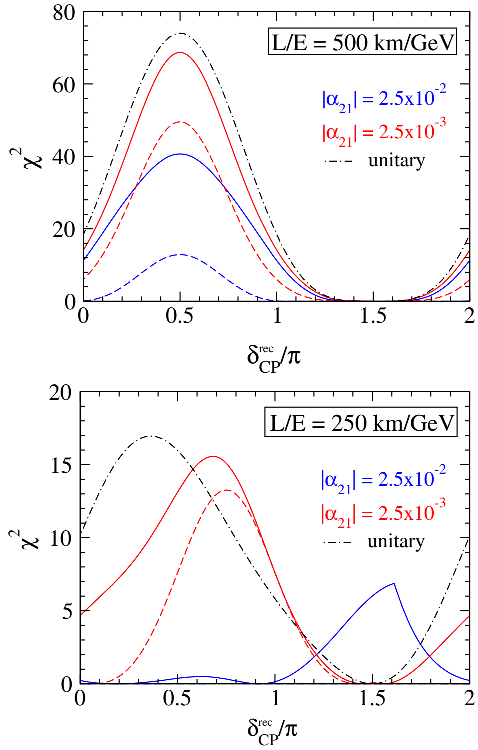

FIG. 3. δ_{CP} determination in the presence of nonunitarity for two different values of L/E and $|\alpha_{21}|$. The true value of δ_{CP} has been taken equal to $3\pi/2$. Dashed (solid) lines correspond to the reconstructed value from the neutrino (combined neutrino + antineutrino) appearance probability. The dot–dashed line shows the reconstruction of the standard CP phase in the unitary case.

and the antineutrino channels could provide a way to disentangle the CP phase δ_{CP} from the new "seesaw" phase ϕ coming from nonunitarity. Indeed, in the unitary case, the knowledge of a point $(P_{\mu e}, \bar{P}_{\mu e})$ in the bi-probability plot will determine the standard CP phase up to the trigonometric $\delta_{CP} \to \pi - \delta_{CP}$ ambiguity.

In order to check whether this also holds true in the presence of nonunitarity we consider the bi-probability plots in Fig. 2. The upper panel shows that, for values of L/E close to 500 km/GeV, the combination of neutrino and antineutrino measurements removes the degeneracies between the CP phases present in each channel separately. In fact, this can be understood from a detailed analysis of the CP-dependent terms in $P_{\mu e}$ as given by Eq. (2). One finds that, for $L/E = 500$ km/GeV, some of these terms cancel exactly. The degeneracies in the phases δ_{CP} and ϕ due to the remaining terms, present in both the neutrino and antineutrino channels separately, disappear once the two channels are combined. Fortunately, neutrino long-baseline experiments are usually tuned to the ratio $L/E = 500$ km/GeV, where the oscillation maximum is located.

However, for L/E values far from 500 km/GeV, the interplay between the different CP-dependent terms in $P_{\mu e}$ is rather involved. As a result, the phase degeneracies present in the neutrino channel may persist even after the combined two-channel analysis including antineutrino observations. Indeed, as can be seen in the bottom panel of Fig. 2, the ambiguities in general remain even with the combined measurements of the appearance probabilities in neutrino ($P_{\mu e}$) and antineutrino channel ($\bar{P}_{\mu e}$). Therefore, the conventional strategy will not in general be enough to ensure an unambiguous determination of the standard CP phase in the present case.

Likewise, one can obtain a quantitative measure of the reconstruction sensitivity of the standard phase δ_{CP} in the presence of nonunitarity, as shown in Fig. 3. (These results have been obtained by fitting the neutrino oscillation probability, assumed to be measured with a 10% uncertainty.) One finds that by combining the two channels the reconstruction is very much improved and is close to that obtained in the standard unitary case, just a bit worse due to the presence of the extra degree of freedom ϕ. This holds for $L/E = 500$ km/GeV or close. In contrast, for L/E values far from the above, say 250 km/GeV, the reconstruction sensitivity is lost completely. Indeed, with the neutrino channel alone one has no sensitivity at all, with the corresponding dashed blue line being hardly visible, overlapping the horizontal axis. Combining neutrino and antineutrino channels does not solve the situation, as a local χ^2 maximum appears at $\delta_{CP} = 3\pi/2$, the true simulated value. One also finds how for $\alpha_{21} \to 0$ the standard case is recovered, lifting the new degeneracy relatively well for $L/E\,500$ km/GeV and $\alpha_{21} < 2.5 \times 10^{-3}$ (top panel). Unfortunately, however, stringent direct limits on α_{21} are inexistent. There are only indirect restrictions from charged lepton flavor violation processes, difficult to quantify in a model-independent way. For a recent discussion in the context of seesaw models see Ref. [30]. (In that paper it was shown that values of α_{21} up to 3×10^{-3} are in agreement with constraints from LFV searches at 90% C.L. However, those bounds hold within a restrictive "minimal ansatz." Here we prefer to be conservative and apply only the truly model-independent bounds on α_{21} derived in Ref. [18].) We must stress that, for simplicity, we have restricted our study to neutrino oscillations *in vacuo*. This is reasonable because we are focussing on degeneracies associated with intrinsic CP violation. In this sense it is relevant to investigate whether the vacuum probabilities provide a robust signature of CP violation. Although the inclusion of matter effects will be necessary for realistic predictions for very long baseline experiments such as DUNE [31], it is expected to modify but not destroy the existence of the new degeneracies noted here.

Conclusions.—We have argued, on the basis of the seesaw mechanism, that the lepton mixing matrix describing neutrino oscillations is likely to be nonunitary. We have focussed on the description of neutrino oscillations in the nonunitary case, in particular on the effects of the extra CP phase present in the oscillation probabilities. We have identified degeneracies in the appearance probability $P_{\mu e}$ for different combinations of the standard three-neutrino phase δ_{CP} and the "new" CP phase ϕ associated with the new parameters describing nonunitarity. These ambiguities are beyond the conventional ones, having no analogue within the standard unitary three-neutrino oscillation benchmark. We have discussed the resulting ambiguities in oscillation probabilities for various L/E and nonunitarity parameter choices. We have outlined the simplest strategies to help in coping with the presence of these new degeneracies. The standard strategy of determining δ_{CP} from the combination of neutrino and antineutrino observations, that holds in the unitary case, turns out to be insufficient in removing the degeneracies between two CP phases δ_{CP} and ϕ for values of L/E far from the "magic" value of 500 km/GeV. In short, we showed how "generic" neutrino oscillation measurements are not individually robust with respect to the unitarity violation effects expected within a class of seesaw schemes. New strategies and/or combined studies using data from different experiments may be necessary in order to ensure unambiguous CP measurements. Such efforts offer a valuable window for complementary tests of lepton flavor conservation and weak universality. Before closing, let us also mention that CP ambiguities will also arise within generic nonstandard interaction schemes not directly related to a seesaw mechanism as the origin of neutrino mass. Likewise, dedicated studies, analogous to those in Refs. [32–35] will be required here in order to cover each experimental setup.

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