Three-dimensional Character of the Magnetization Dynamics in Magnetic Vortex Structures: Hybridization of Flexure Gyromodes with Spin Waves

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Three-dimensional linear spin-wave eigenmodes of a vortex-state Permalloy disk are studied by micromagnetic simulations based on the Landau-Lifshitz-Gilbert equation. The simulations confirm that the increase of the disk thickness leads to the appearance of additional exchange-dominated so-called gyrotropic flexure modes having nodes along the disk thickness, and eigenfrequencies that decrease when the thickness is increased. We observe the formation of a gap in the mode spectrum caused by the hybridization of the first flexure mode with one of the azimuthal spin-wave modes of the disk. A qualitative change of the transverse profile of this azimuthal mode is found, demonstrating that in a thick vortex-state disk the influence of the "transverse" and the "azimuthal" coordinates cannot be separated. The three-dimensional character of the eigenmodes is essential to explain the recently observed asymmetries in an experimentally obtained phase diagram of vortex-core reversal in relatively thick Permalloy disks.

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The study of spin-wave (SW) excitations in micro- and nanosized magnetic elements is one of the most important topics in modern magnetism. The dynamic spin-wave eigenmodes of finite-size magnetic elements not only determine the high-frequency properties of these elements, but also provide valuable information about remagnetization processes in nanomagnetic objects, as the change of the magnetic ground state of an element is, usually, happening through the softening of one of the SW eigenmodes of this element. When the ground state of magnetization in a magnetic element is spatially uniform the spatial distribution of the spin-wave excitations can, usually, be factorized, and represented as a product of three functions of three independent coordinates (separation ansatz).

The situation becomes much more complicated in the case when the magnetic element is thick, so that it has to be treated as *three-dimensional* (3D), and the magnetization ground state of the element is spatially *nonuniform*. In that case the possibilities of traditional analytic methods are limited; however, the essential information about SW excitations can be obtained from micromagnetic simulations. As it will be shown below, in a 3D spatially nonuniform case the dependence of the spin-wave mode profile on some of the spatial coordinates gets mixed, and the above discussed separation ansatz traditionally used in the analytic theory does not work anymore.

A relatively simple example of a magnetic element having spatially nonuniform ground state of static magnetization is a vortex-state magnetic disk [1–3]. In a cylindrical nanodisk with thickness h of a few tens of nanometers and diameter $2R$ of typically several hundred nanometers the magnetic ground state is a vortex. There the magnetization curls in the plane of the disk with a clockwise (CW) or counterclockwise (CCW) circulation. At the center of the disk in an area with a typical diameter of 10 to 20 nm the magnetization turns out of the plane [3] forming the vortex core, which points either up or down, corresponding to the two polarities $p = +1$ and $p = -1$.

The spectrum of magnetic excitations in relatively thin vortex-state magnetic disks, having a thickness in the range of the exchange length [$l_{ex} = \sqrt{2A/(\mu_0 M_s^2)}$ in Système International (SI) units [4]] in which the dynamical magnetization along the out-of-plane direction (i.e., transverse direction) can be considered uniform, has been intensively studied in [1,2,5–18]. The lowest frequency mode is the fundamental *gyrotropic mode* G_0 with the frequency in the range of 0.1–1.2 GHz [2,9,10,13]. It describes the translational gyrotropic motion of the vortex core and has an almost linear dependence of the mode frequency on the disk thickness. The higher frequency excitations are the *dipolar spin-wave modes* describing the dynamics of the in-plane part of the magnetic vortex [7,11]. The dipolar in-plane modes have a square-root-like thickness dependence of their eigenfrequencies in the range of several GHz [11,12], and are denoted by a pair of integers (n,m). The radial mode number $n \geq 1$ is the number of radial nodes in the out-of-plane component of the dynamical magnetization. The azimuthal mode number m can have negative and positive values and $|2m|$ is the number of the azimuthal nodes [2,7]. In the following, we consider primarily the azimuthal in-plane modes with $|m| = 1$.

In thicker vortex-state magnetic disks the problem of finding spin-wave eigenexcitations becomes a *three-dimensional* problem, as it becomes necessary to take into account the possible variation of the dynamic magnetization along the disk thickness and the formation of nodes in the mode magnetization profile along this direction. This is correct for all the dynamic modes of the magnetic vortex. In particular, for the lowest in frequency gyrotropic mode, this leads to the appearance in the disk spectrum of the so-called "higher order gyromodes" G_N (or "flexure" modes), having a flexed line of the gyrating vortex core with $N \geq 1$ nodes along the disk thickness, see Fig. 1 and Refs. [19–22]. These flexure modes are dominated by the exchange interaction, and their eigenfrequencies are inversely proportional to the square of the disk thickness h [19]. In principle all the SW eigenmodes of a vortex-state magnetic disk are 3D, but some of them, which have a spatially uniform profile of the dynamic magnetization along the disk thickness, are well described by the effectively two-dimensional models developed in [2,7,11,14,18].

It should be noted that in all of the above mentioned *analytical* approaches to the calculation of the frequencies of the SW modes of a vortex-state magnetic disk, where a linearized Landau-Lifshitz-Gilbert (LLG) equation [23,24] is used, the coordinate dependence of the mode's profiles is factorized in separate functions, and the "approximate eigenmodes" are obtained. Apart from the separation of coordinates, the main approximation in these analytical calculations is the neglect of the nondiagonal part of the dipole-dipole interaction appearing in the effective field of the LLG equation. This nondiagonal part describes the dipole-dipole hybridization of the different approximate eigenmodes taking place when their "approximate eigenfrequencies" are sufficiently close to each other and when their mode profiles have sufficient overlap [25].

Obviously, the presence of approximate eigenmodes in the spin-wave spectrum of a relatively thick magnetic disk with both *increasing* and *decreasing* frequency dependences on the disk thickness may lead to a crossing of the corresponding dispersion curves, and related hybridization of those approximate eigenmodes having similar symmetry of their spatial profiles [25].

A similar hybridization of the approximate eigenmodes is observed and calculated in relatively thin vortex-state magnetic disks for G_0 and the lowest dipolar SW modes having radial index $n = 1$ and opposite azimuthal indices $m = -1$ and $m = +1$ [6]. This hybridization leads to a frequency splitting of the azimuthal dipolar spin waves that agrees well with experimental results [15–17]. In addition, the hybridization of G_0, G_1 and dipolar spin waves leads to a reduction of the frequency of the gyrotropic mode for increasing disk thickness [26]. This reduced frequency is in better agreement to experimental results [19] compared to previous analytical theory [2,7].

In the present Letter, we investigate the numerically simulated "true" eigenmodes of relatively *thick* 3D magnetic disks (for details on the method see Supplemental Material [27]). Our initial goal is to interpret these true eigenmodes as hybridizations between the "approximate" (or diagonal) three-dimensional eigenmodes with separated spatial coordinates happening due to the nondiagonal dipolar interaction between them. In order to remain in the linear regime, only small amplitude excitations are used to analyze the eigenmodes. Below we especially focus on the spectral region where the frequencies of the approximate dipolar SW modes, having a uniform profile along the disk thickness in thin films, are close to the frequency of the gyrotropic flexure mode G_1 that has one node along the thickness of the disk. We demonstrate that the hybridization between these modes qualitatively changes not only the frequency behavior of these modes, opening a *spectral gap* near the point of degeneracy of the approximate eigenfrequencies of one of the azimuthal dipolar modes ($m = +1$) and G_1, but also the transverse dependence of this $m = +1$ SW mode. A node is created in its transverse profile at the region of the vortex core (see Figs. 3 and 4), leading to different dependences of the mode profile on the "transverse" coordinate close to the vortex center and outside of the core region. Thus, our numerical simulations show that in thick 3D vortex-state magnetic disks the dependence of the mode profile on the transverse and the "azimuthal" coordinates is mixed, and cannot be separated even in the linear regime.

The simulations are performed for circular Permalloy disks with a diameter of 500 nm. Figure 2 shows the frequencies of the true (hybridized) vortex eigenmodes as a function of the disk thickness h. The resolution in h is 0.7 nm. The respective dominant character of the eigenmode is given by the color of the line: Orange represents the fundamental gyrotropic mode G_0, red the first gyrotropic flexure mode G_1, and blue and green the azimuthal spin-wave modes with $n = 1$, $m = -1$ and $n = 1$, $m = +1$, respectively. For a vortex core pointing in the direction of

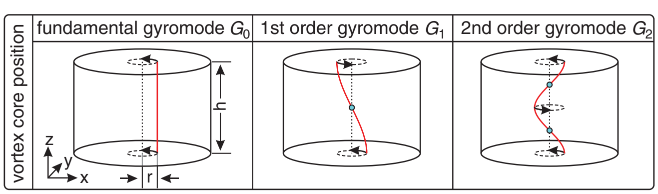

FIG. 1. Schematic representation of the gyrotropic modes G_0, G_1, and G_2. The blue dots indicate nodes of the vortex-core gyration radius.

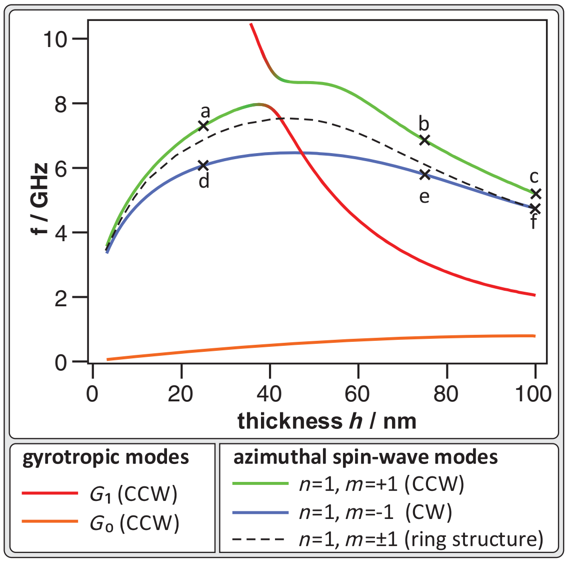

FIG. 2. Eigenfrequencies of the gyrotropic eigenmodes G_0 and G_1 and the azimuthal spin-wave modes $n = 1$, $m = \pm 1$ as a function of the disk thickness h obtained from micromagnetic simulations by Fourier analysis.

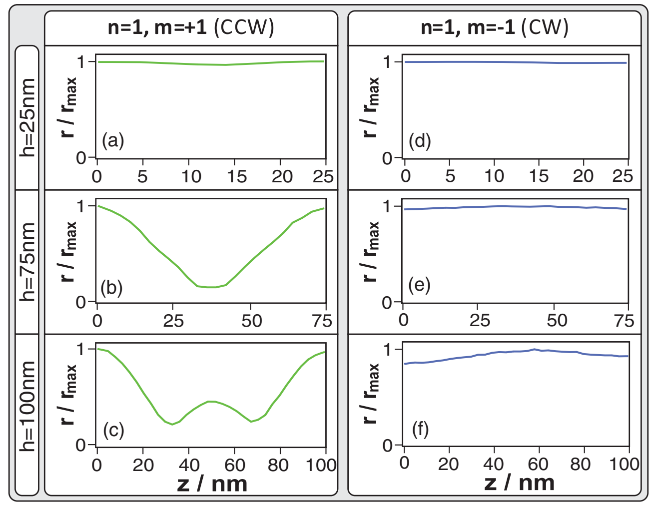

FIG. 3. The vortex-core gyration radius r obtained for the points (a)–(f) marked in Fig. 2 shows that the azimuthal spin-wave modes do not only hybridize with G_0 (a), (d)–(f), but also with G_1 (b) and G_2 (c) (see also Fig. 4).

the observer the senses of rotation of these modes are CCW for G_0, G_1 and the $n = 1$, $m = +1$ mode and CW for the $n = 1$, $m = -1$ mode.

In thin disks the dependence of the mode frequency of G_0 on the disk thickness is approximately linear, similar to the analytical prediction [2]. Furthermore, our numerical calculations show that, in agreement with the analytical calculations [6], G_0 is hybridized with both azimuthal dipolar SW modes with $n = 1$, $m = \pm 1$. This can be seen in Figs. 3(a) and 3(d) from the uniform vortex-core gyration along the disk thickness present for these SW modes. This hybridization leads to the frequency splitting of the dipolar modes with different values of the azimuthal index m [6,15], which was briefly discussed in the introduction. It also leads to the renormalization and a decrease of the fundamental gyromode frequency [1,19,26] compared to the "unhybridized" case [2] (see orange curve at large thickness in Figs. 2 and 3 in Ref. [19]).

Approximate analytical calculations [19] predict that for large thickness h the frequency of the nonhybridized approximate G_1 should scale as h^{-2} which is similar to the dependence we find in our numerical simulations (red curve in Fig. 2).

In Fig. 2 one can see a *strong hybridization* between this flexure mode G_1 (red curve) and one of the azimuthal dipolar modes (green curve, $m = +1$) in the vicinity of $h = 40$ nm. Obviously, this strong hybridization qualitatively changes the thickness dependence of the spin-wave spectrum of a vortex-state magnetic disk and opens a *spectral gap* near $h = 40$ nm.

The hybridization with G_1 is also evidenced by the gyration radius of the vortex core as a function of the transverse coordinate z numerically calculated at the frequencies corresponding to the dipolar mode $n = 1$, $m = +1$ at different values of the disk thickness [see Figs. 3(a)–3(c) and the corresponding dynamic mode profiles in Figs. 4(a)–4(c)]. For a disk thickness below $h = 40$ nm the mode $n = 1$, $m = +1$ is dominantly hybridized with G_0 [see nearly z-independent gyration radius for $h = 25$ nm in Fig. 3(a)], whereas above $h = 40$ nm the strong hybridization with G_1 can be seen by a deep minimum in the gyration radius in Fig. 3(b) ($h = 75$ nm). For even larger disk thickness, the mode $n = 1$, $m = +1$ is hybridized with the second flexure mode G_2 having two nodes in the transverse profile [note the two minima in the frame Fig. 3(c) for $h = 100$ nm]. In summary, these results imply that the dipolar mode $n = 1$, $m = +1$, which has the same CCW sense of the azimuthal rotation as all the gyrotropic modes, is hybridized with all the gyrotropic modes, in dependence of disk thickness. The hybridization is so strong that the profile of this mode along the transverse coordinate z qualitatively changes due to the hybridization. This property of the CCW azimuthal mode ($m = +1$) is further confirmed by the transverse mode profiles presented in Figs. 4(a)–4(c), showing that after the hybridization with the mode G_1 the transverse profile of the CCW mode acquired a node only at the core region which does not exist without the hybridization (for $h < 40$ nm), whereas the profile outside of the core remains unchanged. This means that for sufficiently thick magnetic disks the dependence of the mode profile on the azimuthal and transverse coordinates is mixed and cannot be separated. Therefore, the simple analytical formalism based this separation ansatz is not applicable for the quantitative description of the spin-wave eigenmodes in sufficiently thick 3D vortex-state magnetic disks.

In contrast, the dipolar mode $n = 1$, $m = -1$ having the opposite CW sense of the azimuthal rotation, is hybridized

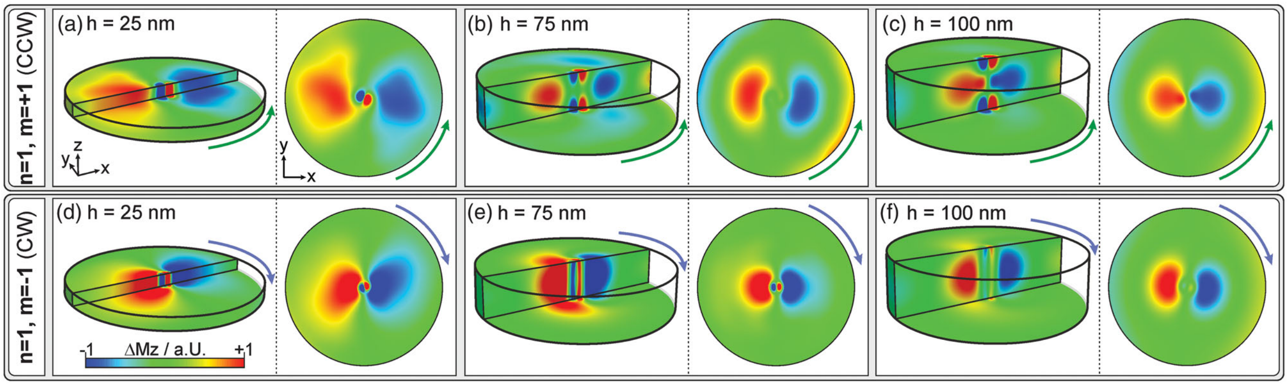

FIG. 4. Mode profiles for the points (a)–(f) marked in Fig. 2. All cuts at $z = 1/2h$ (respectively, right images) show the bipolar structure typical for a $n = 1$, $m = \pm 1$ azimuthal spin-wave mode. The 3D cuts (left images) show no minimum at the core region in the precession amplitude in (a) and (d)–(f), one minimum in (b), and two minima in (c), indicating a hybridization with G_0, G_1, and G_2, respectively.

with G_0 only, independent of disk thickness. This follows from the frames Figs. 3(d)–3(f) where the vortex gyration radius is virtually independent of the transverse coordinate z and from the corresponding mode profiles Figs. 4(d)–4(f). Since there is no significant hybridization present between this SW mode and the G_1 mode, no spectral gap forms at the crossing of these modes. The different hybridization behavior of the CW and CCW rotating spin-wave modes can be explained in the following way. The spatial profile of the CW ($n = 1$, $m = -1$) azimuthal dipolar mode is orthogonal to the profile of the mode G_1 along both azimuthal and transverse coordinates, and, therefore, no hybridization of these modes takes place. In contrast, for the CCW ($n = 1$, $m = +1$) azimuthal mode its azimuthal profile is similar to the azimuthal profile of the mode G_1 due to the CCW senses of rotation of both modes, and, in spite of the orthogonality of the transverse profiles of these two modes, due to the mixing of the azimuthal and transverse coordinates in a 3D case a strong hybridization of the modes takes place, and a spectral gap is formed.

A similar behavior is also found at the crossing of G_1 with higher order SW modes having mode numbers $n = 2$, $m = \pm 1$ and $n = 3$, $m = \pm 1$. Similar to the case of the $n = 1$ modes, the spectral gaps are only formed for the CCW rotating modes ($m = +1$) having an azimuthal profile similar to that of the mode G_1, but not for the CW rotating $m = -1$ modes (see Fig. II of the Supplemental Material [27]). No hybridization between G_1 and SW modes with $|m| \geq 2$ is found, which can be explained by a small or no overlap of the mode profiles caused by different symmetries (see Supplemental Material [27]).

We have also performed simulations for rings where the central part of the vortex structure is removed, so that there is no vortex core, and, therefore, no gyrotropic modes G_N. In this case the dipolar spin-wave modes with the opposite sense of the azimuthal rotation ($m = +1$ and $m = -1$) are degenerate in frequency. This is shown by the black dashed line in Fig. 2, which is always located between the eigenfrequencies of the $n = 1$, $m = \pm 1$ modes of the system with a vortex core. This means that the non-monotonic behavior of the frequency of the dipolar azimuthal modes in a magnetic disk, either with or without the vortex core, is not related to their hybridization with either of the gyrotropic modes, and is an inherent property of the dipole-dipole dispersion [see, e.g., dispersion equation (8) in [5]].

An analysis of the numerically calculated spatial profiles of the dynamic magnetization in the SW modes of the disk (Fig. 4) reveals a z dependence outside the core region, demonstrating that all true spin-wave eigenmodes have 3D character [Figs. 4(b), 4(c), 4(e), and 4(f) 3D cuts] even outside the core region. The transverse profiles of the true eigenmodes of the vortex-state magnetic disk have maxima at $z = h/2$, and are similar to the profiles of the exchange-dominated perpendicular standing spin waves (PSSWs) in planar films (see, e.g., [28]).

Besides the modification of the mode spectrum the hybridization between azimuthal spin waves and the first order flexure mode has also to be considered in experiments on vortex-core reversal. It was shown in various publications that the polarity of the vortex core can be reversed dynamically, for example, by exciting the fundamental gyrotropic mode [29,30] or spin waves [31]. Recently experiments on vortex-core reversal by pulsed excitation of spin waves have been performed [32]. The asymmetric experimental results of [32] could not be reproduced by two-dimensional micromagnetic simulations, and only three-dimensional simulations allow an adequate description of the experiment. It was suggested that while the energy transfer from the external field to the vortex structure is dominated by excitation of azimuthal spin waves, the z-dependent vortex-core trajectories found in the three-dimensional simulations are responsible for the asymmetry observed in the experiments. In the present Letter we have shown that the hybridization of the dipolar azimuthal spin-wave modes with the higher order gyromodes introduces a three-dimensional character to the vortex-core dynamics of dipolar spin waves and in this way explains the origin of the observed three-dimensional core trajectories in [32]. Therefore, we can conclude that this hybridization can have significant influence on the results of switching experiments.

To conclude, we have performed micromagnetic simulations of the linear dynamical behavior of a vortex-state cylindrical Permalloy disk based on the Landau-Lifshitz-Gilbert equation. The numerically calculated true spin-wave eigenmodes of the disk are interpreted in terms of hybridized three-dimensional approximate eigenmodes of a simplified version of the equation of motion, where the nondiagonal part of the dipole-dipole interaction has been neglected. Our three-dimensional numerical simulations confirm the well-known frequency splitting of the $n = 1$, $m = \pm 1$ azimuthal spin-wave modes [6] caused by their hybridization with the fundamental gyrotropic mode G_0. The simulations reveal a new effect appearing with increasing disk thickness: the formation of a spectral gap. This is caused by the mixed dependence of the spin-wave mode profiles on the azimuthal and transverse coordinates resulting in the strong hybridization of the first flexure gyrotropic mode G_1 and the dipolar azimuthal mode $n = 1$, $m = +1$ having the same CCW sense of the azimuthal rotation as the mode G_1. This also means that in sufficiently thick vortex-state magnetic disks the separation of the coordinate dependence (typically used in traditional two-dimensional analytical calculations) cannot be applied to model such systems and more involved theoretical approaches are required.

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