Composite Operators in the Twistor Formulation of $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory

Laura Koster,[1,*] Vladimir Mitev,[1,2,†] Matthias Staudacher,[1,‡] and Matthias Wilhelm[1,3,§]
[1]*Institut für Mathematik, Institut für Physik und IRIS Adlershof, Humboldt-Universität zu Berlin, Zum Großen Windkanal 6, 12489 Berlin, Germany*
[2]*PRISMA Cluster of Excellence, Institut für Physik, WA THEP, Johannes Gutenberg-Universität Mainz, Staudingerweg 7, 55128 Mainz, Germany*
[3]*Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark*
(Received 6 April 2016; published 29 June 2016)

We incorporate gauge-invariant local composite operators into the twistor-space formulation of $\mathcal{N}=4$ super Yang-Mills theory. In this formulation, the interactions of the elementary fields are reorganized into infinitely many interaction vertices and we argue that the same applies to composite operators. To test our definition of the local composite operators in twistor space, we compute several corresponding form factors, thereby also initiating the study of form factors using the position twistor-space framework. Throughout this Letter, we use the composite operator built from two identical complex scalars as a pedagogical example; we treat the general case in a follow-up paper.

DOI: 10.1103/PhysRevLett.117.011601

Introduction.—The study of the simplest interacting gauge theory in four dimensions, namely, $\mathcal{N}=4$ super Yang-Mills theory (SYM), has led to a plethora of important theoretical insights, such as holography (AdS/CFT), integrability in the planar limit, on-shell methods, and more. Moreover, the theory can be formulated in twistor space [1], which has been an efficient setting for computing on-shell quantities such as amplitudes [2–6], and relating them to lightlike Wilson loops [7–9]. The action of $\mathcal{N}=4$ SYM in twistor space is the sum of two parts $\mathcal{S}_1+\mathcal{S}_2$, with $\mathcal{S}_1$, introduced in Ref. [10], describing the self-dual part and $\mathcal{S}_2$, referred to as the interaction piece, immediately giving the maximally helicity-violating (MHV) tree-level scattering amplitudes [11]. However, computations involving off-shell quantities, such as form factors or correlation functions of gauge-invariant local composite operators (composite operators), are less straightforward in the twistor formalism, though some progress has been made [12,13]. To tackle off-shell objects, one needs a proper definition of composite operators in twistor space, which is the main subject of this Letter [14].

In the usual space-time formulation, the composite operator $\mathcal{O}$ consists of a single term that immediately determines its vertex. In contradistinction, we argue that the twistor-space representation of $\mathcal{O}$ (or rather its vertex) must contain infinitely many terms as it has to describe all interactions of $\mathcal{O}$ with elementary particles at minimal MHV degree. In particular, all MHV tree-level form factors of the operator $\mathcal{O}$ have to be given immediately by the operator vertex; an elementary counting of the MHV degree shows that they cannot contain any twistor-space propagators and hence also no interaction vertices.

In this Letter, we explicitly demonstrate this principle using the gauge-invariant local composite operator

$$\mathcal{O}'=\frac{1}{2}\mathrm{Tr}[\phi_{ab}^2], \quad (1)$$

built out of two identical complex scalars. In particular, we determine the correct twistor-space vertex for this operator. An algorithm for generating the vertices for all operators using Wilson loops will be given in a follow-up paper.

The definition of a composite operator $\mathcal{O}$ in twistor space can be probed by computing its tree-level MHV form factor with external on-shell states $A_1,\ldots,A_n$ and comparing this to data from the literature. Tree-level MHV form factors are the simplest quantities that contain a composite operator and hence provide an ideal testing ground for our definition of composite operators in twistor space [15]. Letting p_i be the momenta of the on-shell states, and q the momentum of $\mathcal{O}$, the form factor is defined as the expectation value

$$\mathcal{F}_{\mathcal{O}}(1^{A_1},\ldots,n^{A_n};\mathsf{q}) = \int\frac{d^4x}{(2\pi)^4}e^{-i\mathsf{q}x}\langle A_1(\mathsf{p}_1)\cdots A_n(\mathsf{p}_n)|\mathcal{O}(x)|0\rangle. \quad (2)$$

Form factors in $\mathcal{N}=4$ SYM theory are also interesting in their own right and have received increasing attention, both at weak coupling [16–37] and at strong coupling [38–40]. In comparison to amplitudes, however, where all tree-level expression [41] as well as the unregularized integrand of all loop-level expressions [42] have been found, much less is known for form factors; see Ref. [43] for the state of the art. In particular, the form factors of the operator $\mathcal{O}'$ are also phenomenologically interesting, as they are related to the Higgs-to-gluons amplitude in QCD; cf. Ref. [23].

A first attempt at composite operators…—We refer to Ref. [44] for an introduction to the supertwistor methods that we shall use here. We write supertwistors $\mathcal{Z}\in\mathbb{CP}^{3|4}$ as

$\mathcal{Z} = (\lambda_\alpha, \mu^{\dot\alpha}, \chi^a)$, where the χ^a are fermionic, $\alpha\, \dot\alpha \in \{1,2\}$, and $a \in \{1,2,3,4\}$. Supertwistor space is naturally related to chiral Minkowski superspace $\mathbb{M}^{4|8}$, which is obtained by appending eight Graßmann variables $\theta^{\alpha a}$ to each point $x^{\alpha\dot\alpha}$ in Minkowski space. Each point (x,θ) in $\mathbb{M}^{4|8}$ corresponds to a unique projective line in supertwistor space given by the set of supertwistors,

$$\mathcal{Z} = (\lambda_\alpha, ix^{\alpha\dot\alpha}\lambda_\alpha, i\theta^{\alpha a}\lambda_\alpha), \qquad \lambda \in \mathbb{CP}^1. \tag{3}$$

For brevity, we denote a line in supertwistor space by x instead of (x,θ) and we denote by $\mathcal{Z}_x(\lambda)$ the supertwistor (3) on the line x given by the spinor λ.

In order to obtain a field Φ in space-time, the standard prescription is to Penrose transform [45,46] a field in twistor space $\tilde\Phi$; i.e., integrating the twistor-space field $\tilde\Phi$ over the line in $\mathbb{CP}^{3|4}$ corresponding to (x,θ) as

$$\Phi(x) = \int_{\mathbb{CP}^1} D\lambda \tilde\Phi(\mathcal{Z}_x(\lambda)), \tag{4}$$

where $D\lambda = \langle\lambda d\lambda\rangle/2\pi i$. The angular brackets are defined as $\langle\lambda\lambda'\rangle = \lambda^\alpha\lambda'_\alpha$ with $\epsilon^{\alpha\beta}\lambda_\alpha = \lambda^\beta$ and $\epsilon^{12} = 1$. For future reference, we note that integrals over the spinors λ are always taken over the projective line $\mathbb{CP}^1$, so that we can omit this from the integral sign.

The twistor action [1] is written using a single connection superfield $\mathcal{A}$ introduced in Ref. [47]. This superfield combines the on-shell degrees of freedom of $\mathcal{N}=4$ SYM theory—the two helicity ±1 gluons $g^\pm$, the four helicity $\frac{1}{2}$ fermions $\bar\psi_a$, and their antiparticles ψ^a and the six scalars ϕ_{ab}—as

$$\mathcal{A}(\mathcal{Z}) = g^+ + \chi^a\bar\psi_a + \frac{1}{2}\chi^a\chi^b\phi_{ab} + \frac{1}{3!}\chi^a\chi^b\chi^c\psi^d\epsilon_{abcd} + \chi^1\chi^2\chi^3\chi^4 g^-, \tag{5}$$

where the components fields $g^\pm, \ldots$ do not depend on the Graßmann variables χ. According to Eq. (4), a natural first attempt [48,49] for the (gauge-covariant) scalar field $\phi_{ab}(x)$ is given by a Penrose transform

$$\phi_{ab}(x) \overset{?}{=} \int D\lambda h_x^{-1}(\lambda) \frac{\partial^2\mathcal{A}(\lambda)}{\partial\chi^a\partial\chi^b} h_x(\lambda)_{|\theta=0}, \tag{6}$$

where $\mathcal{A}(\lambda) \equiv \mathcal{A}(\mathcal{Z}_x(\lambda))$. In Eq. (6), we set $\theta = 0$, implying $\chi^a = i\theta^{\alpha a}\lambda_\alpha = 0$, after taking the derivatives, because we are only interested in the ϕ_{ab} component of Eq. (5). In addition, we have introduced $h_x(\lambda)$—the frame on x that trivializes the connection $\mathcal{A}$ along the line x and thus ensures gauge invariance when taking traces of products of these fields [49].

Therefore, using the ansatz (6), the operator $\mathcal{O}'$ built out of two scalars would read

$$\mathcal{O}'(x) = \frac{1}{2}\mathrm{Tr}[\phi_{ab}^2](x) \overset{?}{=} \frac{1}{2}\int D\lambda D\lambda' \mathrm{Tr}\left[\frac{\partial^2\mathcal{A}(\lambda)}{\partial\chi^a\partial\chi^b} \times U_x(\lambda,\lambda')\frac{\partial^2\mathcal{A}(\lambda')}{\partial\chi'^a\partial\chi'^b}U_x(\lambda',\lambda)\right]_{|\theta=0}, \tag{7}$$

where $U_x(\lambda,\lambda') = h_x(\lambda)h_x(\lambda')^{-1}$ is the parallel propagator for the connection $\mathcal{A}$. It can be expanded as

$$U_x(\lambda,\lambda') \equiv U_x(\mathcal{Z}_x(\lambda),\mathcal{Z}_x(\lambda')) = 1 + \sum_{m=1}^\infty \int \frac{\langle\lambda\lambda'\rangle D\tilde\lambda_1\cdots D\tilde\lambda_m \mathcal{A}(\tilde\lambda_1)\cdots\mathcal{A}(\tilde\lambda_m)}{\langle\lambda\tilde\lambda_1\rangle\langle\tilde\lambda_1\tilde\lambda_2\rangle\cdots\langle\tilde\lambda_m\lambda'\rangle}. \tag{8}$$

In order to obtain form factors (2), which are naturally expressed in momentum space, we insert external on-shell momentum states [5] of (super-)momentum $\mathsf{P} = (\mathsf{p}_{\alpha\dot\alpha}, \eta_a) = (p_\alpha, \bar p_{\dot\alpha}, \eta_a)$:

$$\mathcal{A}_{\mathsf{P}}(\mathcal{Z}) = 2\pi i \int_{\mathbb{C}} \frac{ds}{s} e^{s(\mu^{\dot\alpha}\bar p_{\dot\alpha} + \chi^a\eta_a)} \bar\delta^2(s\lambda - p), \tag{9}$$

where $\bar\delta^2(\lambda) = \bar\delta^1(\lambda_1)\bar\delta^1(\lambda_2)$ with the $\bar\delta^1(z)$ denoting the δ function on the complex plane. Let us compute $\mathcal{F}_{\mathcal{O}'}(1^+,\ldots,i^{\phi_{ab}},\ldots,j^{\phi_{ab}},\ldots,n^+;\mathsf{q})$: the (color-ordered) form factor of $\mathcal{O}'$ with n external particles, two of which are scalars ϕ_{ab} at positions i and j, while the remaining ones are positive-helicity gluons. Here, only the terms in Eq. (8) with the appropriate number of $\mathcal{A}$'s contribute, namely, those with $j-i-1$ from one U_x and $n+i-j-1$ from the other. Inserting the on-shell states Eqs. (9) into Eq. (8) as well as directly into Eq. (7), and then integrating over s and the corresponding λ, effectively cancels s and replaces $\lambda_\alpha \to p_\alpha$, $\mu^{\dot\alpha} \to ix^{\alpha\dot\alpha}p_\alpha$, and $\chi^a \to i\theta^{\alpha a}p_\alpha$ due to the $\bar\delta^2$ function and the parametrization (3). Selecting the coefficient of the corresponding η's and Fourier transforming in x as $\int[d^4x/(2\pi)^4]e^{-i\mathsf{q}x}$ yields the desired form factor

$$\mathcal{F}_{\mathcal{O}'}(1^+,\ldots,i^{\phi_{ab}},\ldots,j^{\phi_{ab}},\ldots,n^+;\mathsf{q}) = -\frac{\langle ij\rangle^2\delta^4(\mathsf{q}-\sum_{k=1}^n \mathsf{p}_k)}{\langle 12\rangle\cdots\langle n1\rangle}, \tag{10}$$

which perfectly agrees with the result of Ref. [17]. We would like to comment that the parallel propagators $U_x(\lambda,\lambda')$ in Eq. (7), introduced in order to ensure gauge invariance, are responsible for inserting infinitely many vertices in the description of the composite operators related to their MHV coupling to positive-helicity gluons.

…and where it fails.—While the successful computation of the form factor (10) is encouraging, the ansatz (7) fails once one considers external states involving fermions. Specifically, let us take the MHV form factor

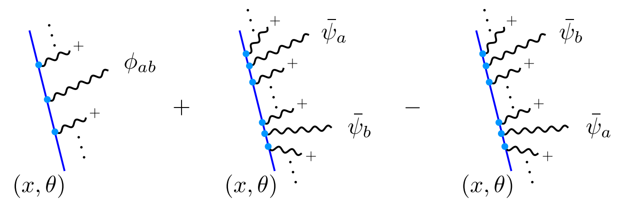

FIG. 1. The vertex of an operator containing a scalar ϕ_{ab} includes all its MHV-preserving splitting terms.

$$\mathcal{F}_{\mathcal{O}'}(1^{\overline{\psi}_a}, 2^{\overline{\psi}_b}, 3^{\phi_{ab}}; \mathsf{q}) = \frac{\delta^4(\mathsf{q} - \sum_{k=1}^{3} \mathsf{p}_k)}{\langle 12 \rangle}, \qquad (11)$$

which was first calculated in Ref. [19]. However, using the twistor-space machinery and Eq. (7), we would obtain zero. This can be seen as follows. On the one hand, the contribution to the form factor (11) has to come solely from the operator $\mathcal{O}'$ itself, since the inclusion of an interaction vertex from $\mathcal{S}_2$ connected by a propagator to $\mathcal{O}'$ would increase the MHV degree. On the other hand, every form factor obtained from Eq. (7) will necessarily contain two on-shell scalars due to the $\partial^2\mathcal{A}/(\partial\chi^a\partial\chi^b)$ terms, while Eq. (11) contains only one.

We conclude that it is necessary to add extra terms encoding the contribution to the form factor (11) directly into the twistor-space expression of the operator $\mathcal{O}'$.

Our proposal.—As previously argued, the twistor-space avatar of any operator $\mathcal{O}$ has to contain the terms that allow the elementary fields to split into different ones while preserving the MHV degree. Hence, we propose to complete Eq. (6) to

$$\begin{aligned}\phi_{ab}(x) = &\int D\lambda h_x^{-1}(\lambda) \frac{\partial^2 \mathcal{A}(\lambda)}{\partial\chi^a \partial\chi^b} h_x(\lambda)_{|\theta=0} \\ &+ \int \frac{D\lambda D\lambda'}{\langle\lambda\lambda'\rangle} h_x^{-1}(\lambda) \frac{\partial\mathcal{A}(\lambda)}{\partial\chi^a} U_x(\lambda,\lambda') \frac{\partial\mathcal{A}(\lambda')}{\partial\chi'^b} h_x(\lambda')_{|\theta=0} \\ &- (a \leftrightarrow b). \end{aligned} \qquad (12)$$

It is depicted in Fig. 1. An immediate observation is that in Eq. (12) the χ derivatives are now distributed supersymmetrically, unlike in Eq. (6).

Our proposal for the correction of the expression (7) is now obtained by squaring Eq. (12) and taking the trace. Since it has nine terms, we refrain from writing it out. Using Eq. (12) and the methods previously employed leads to the correct result (11). We can do even better and straightforwardly derive the MHV super form factor of $\mathcal{O}'$:

$$\mathcal{F}_{\mathcal{O}'}(1, \ldots, n; \mathsf{q}) = \frac{\delta^4(\mathsf{q} - \sum_{k=1}^{n} \mathsf{p}_k) \prod_{c=a,b} (\sum_{i<j} \langle ij \rangle \eta_{ic} \eta_{jc})}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}. \qquad (13)$$

This agrees with the results of Ref. [19] and with Eqs. (10) and (11) when the respective components are specified.

Looking at Eq. (12) and recalling the product rule, one is tempted to try to generate Eq. (12) by some kind of double derivative. For this, we must replace the operator at point x by a polygonal lightlike Wilson loop [50] with an appropriate number of edges $x_1, \ldots, x_n$:

$$\mathcal{W} = \mathrm{Tr}[U_{x_1}(\mathcal{Z}_1, \mathcal{Z}_2) \cdots U_{x_n}(\mathcal{Z}_n, \mathcal{Z}_1)], \qquad (14)$$

where the $\mathcal{Z}_i$ are the twistors at the intersection of the line x_{i-1} and x_i. We can then act on $\mathcal{W}$ with four θ derivatives and finally shrink the Wilson loop back to a point, recovering our expression for $\mathcal{O}'$. We describe this procedure in full detail in a forthcoming publication [51], where we also derive the analogs of Eq. (12) for the rest of the field content of $\mathcal{N} = 4$ SYM theory.

Summary and outlook.—In this Letter, we described how to incorporate composite operators into the twistor-space formulation of $\mathcal{N} = 4$ SYM theory. Just as translating the action to twistor space shuffles the interaction terms into infinitely many vertices, so does the translation of the composite operators require the repackaging of infinitely many operator vertices. Form factors provide the ideal testing ground for our construction as they are the simplest quantities that contain composite operators. Thus, we simultaneously initiated the study of form factors in $\mathcal{N} = 4$ SYM theory using the position twistor-space framework.

In a forthcoming publication [51], we use the Wilson loop that we hinted at above to derive the tree-level MHV super form factors of all composite operators. We extend the framework to N^{k}MHV form factors and correlation functions in a further publication [52].

We are thankful to Tim Adamo, Simon Caron-Huot, and especially Lionel Mason for insightful comments and discussions. This research is supported in part by the SFB 647 *"Raum-Zeit-Materie. Analytische und Geometrische Strukturen"*. L. K. would like to thank the CRST at Queen Mary University of London and especially Gabriele Travaglini for hospitality during a crucial stage of preparation of this work. V. M. would like to thank the Simons Summer Workshop, where part of this work was performed. M. W. was supported in part by DFF-FNU through Grant No. DFF-4002-00037.

*laurakoster@physik.hu-berlin.de
†vmitev@uni-mainz.de
‡matthias@math.hu-berlin.de
§matthias.wilhelm@nbi.ku.dk

[1] R. Boels, L. Mason, and D. Skinner, Supersymmetric gauge theories in twistor Space, J. High Energy Phys. 02 (2007) 014.

[2] N. Arkani-Hamed, F. Cachazo, C. Cheung, and J. Kaplan, A duality for the S matrix, J. High Energy Phys. 03 (2010) 020.

[3] L. J. Mason and D. Skinner, Dual superconformal invariance, momentum twistors and grassmannians, J. High Energy Phys. 11 (2009) 045.

[4] M. Bullimore, L. J. Mason, and D. Skinner, MHV diagrams in momentum twistor space, J. High Energy Phys. 12 (2010) 032.

[5] T. Adamo and L. Mason, MHV diagrams in twistor space and the twistor action, Phys. Rev. D **86**, 065019 (2012).

[6] T. Adamo, M. Bullimore, L. Mason, and D. Skinner, Scattering amplitudes and wilson loops in twistor space, J. Phys. A **44**, 454008 (2011).

[7] S. Caron-Huot, Notes on the scattering amplitude—Wilson loop duality, J. High Energy Phys. 07 (2011) 058.

[8] L. J. Mason and D. Skinner, The complete planar S-matrix of $\mathcal{N}=4$ SYM as a Wilson loop in twistor space, J. High Energy Phys. 12 (2010) 018.

[9] A. V. Belitsky, G. P. Korchemsky, and E. Sokatchev, Are scattering amplitudes dual to super Wilson loops?, Nucl. Phys. **B855**, 333 (2012).

[10] E. Witten, Perturbative gauge theory as a string theory in twistor space, Commun. Math. Phys. **252**, 189 (2004).

[11] R. Boels, L. J. Mason, and D. Skinner, From twistor actions to MHV diagrams, Phys. Lett. B **648**, 90 (2007).

[12] L. Koster, V. Mitev, and M. Staudacher, A twistorial approach to integrability in $N=4$ SYM, Fortschr. Phys. **63**, 142 (2015).

[13] D. Chicherin, R. Doobary, B. Eden, P. Heslop, G. P. Korchemsky, L. Mason, and E. Sokatchev, Correlation functions of the chiral stress-tensor multiplet in $\mathcal{N}=4$ SYM, J. High Energy Phys. 06 (2015) 198.

[14] The problem addressed in this Letter does not occur in Ref. [13] due to the use of the Lagrangian-insertion technique and it was explicitly deferred to this Letter in Ref. [12].

[15] In Ref. [19], dual MHV rules, which can be rewritten in terms of twistor diagrams, were used to study form factors of $\mathcal{O}'$. Whereas the twistors there correspond to dual momentum space, we are studying form factors via twistors in position space.

[16] W. L. van Neerven, Infrared behavior of on-shell form-factors in a $\mathcal{N}=4$ supersymmetric Yang-Mills field Theory, Z. Phys. C **30**, 595 (1986).

[17] A. Brandhuber, B. Spence, G. Travaglini, and G. Yang, Form factors in $\mathcal{N}=4$ super Yang-Mills and periodic Wilson loops, J. High Energy Phys. 01 (2011) 134.

[18] L. V. Bork, D. I. Kazakov, and G. S. Vartanov, On form factors in $\mathcal{N}=4$ sym, J. High Energy Phys. 02 (2011) 063.

[19] A. Brandhuber, O. Gurdogan, R. Mooney, G. Travaglini, and G. Yang, Harmony of super form factors, J. High Energy Phys. 10 (2011) 046.

[20] L. V. Bork, D. I. Kazakov, and G. S. Vartanov, On MHV form factors in superspace for $\mathcal{N}=4$ SYM theory, J. High Energy Phys. 10 (2011) 133.

[21] J. M. Henn, S. Moch, and S. G. Naculich, Form factors and scattering amplitudes in $\mathcal{N}=4$ SYM in dimensional and massive regularizations, J. High Energy Phys. 12 (2011) 024.

[22] T. Gehrmann, J. M. Henn, and T. Huber, The three-loop form factor in $\mathcal{N}=4$ super Yang-Mills, J. High Energy Phys. 03 (2012) 101.

[23] A. Brandhuber, G. Travaglini, and G. Yang, Analytic two-loop form factors in $\mathcal{N}=4$ SYM, J. High Energy Phys. 05 (2012) 082.

[24] L. V. Bork, On NMHV form factors in $\mathcal{N}=4$ SYM theory from generalized unitarity, J. High Energy Phys. 01 (2013) 049.

[25] O. T. Engelund and R. Roiban, Correlation functions of local composite operators from generalized unitarity, J. High Energy Phys. 03 (2013) 172.

[26] H. Johansson, D. A. Kosower, and K. J. Larsen, Two-loop maximal unitarity with external masses, Phys. Rev. D **87**, 025030 (2013).

[27] R. H. Boels, B. A. Kniehl, O. V. Tarasov, and G. Yang, Color-kinematic duality for form factors, J. High Energy Phys. 02 (2013) 063.

[28] B. Penante, B. Spence, G. Travaglini, and C. Wen, On super form factors of half-BPS operators in $\mathcal{N}=4$ super Yang-Mills, J. High Energy Phys. 04 (2014) 083.

[29] A. Brandhuber, B. Penante, G. Travaglini, and C. Wen, The last of the simple remainders, J. High Energy Phys. 08 (2014) 100.

[30] L. V. Bork, On form factors in $\mathcal{N}=4$ SYM theory and polytopes, J. High Energy Phys. 12 (2014) 111.

[31] M. Wilhelm, Amplitudes, form factors and the dilatation operator in $\mathcal{N}=4$ SYM theory, J. High Energy Phys. 02 (2015) 149.

[32] D. Nandan, C. Sieg, M. Wilhelm, and G. Yang, Cutting through form factors and cross sections of non-protected operators in $\mathcal{N}=4$ SYM, J. High Energy Phys. 06 (2015) 156.

[33] F. Loebbert, D. Nandan, C. Sieg, M. Wilhelm, and G. Yang, On-shell methods for the two-loop dilatation operator and finite remainders, J. High Energy Phys. 10 (2015) 012.

[34] L. V. Bork and A. I. Onishchenko, On soft theorems and form factors in $\mathcal{N}=4$ SYM theory, J. High Energy Phys. 12 (2015) 030.

[35] R. Frassek, D. Meidinger, D. Nandan, and M. Wilhelm, On-shell diagrams, Graßmannians and integrability for form factors, J. High Energy Phys. 01 (2016) 182.

[36] R. Boels, B. A. Kniehl, and G. Yang, Master integrals for the four-loop Sudakov form factor, Nucl. Phys. **B902**, 387 (2016).

[37] R. Huang, Q. Jin, and B. Feng, Form factor and boundary contribution of amplitude, arXiv:1601.06612.

[38] L. F. Alday and J. Maldacena, Comments on gluon scattering amplitudes via AdS/CFT, J. High Energy Phys. 11 (2007) 068.

[39] J. Maldacena and A. Zhiboedov, Form factors at strong coupling via a Y-system, J. High Energy Phys. 11 (2010) 104.

[40] Z. Gao and G. Yang, Y-system for form factors at strong coupling in AdS_5 and with multi-operator insertions in AdS_3, J. High Energy Phys. 06 (2013) 105.

[41] J. M. Drummond and J. M. Henn, All tree-level amplitudes in $\mathcal{N}=4$ SYM, J. High Energy Phys. 04 (2009) 018.

[42] N. Arkani-Hamed, J. L. Bourjaily, F. Cachazo, S. Caron-Huot, and J. Trnka, The all-loop integrand for scattering amplitudes in planar $\mathcal{N}=4$ SYM, J. High Energy Phys. 01 (2011) 041.

[43] M. Wilhelm, Form factors and the dilatation operator in $\mathcal{N}=4$ super Yang-Mills theory and its deformations, arXiv:1603.01145.
[44] T. Adamo, Twistor actions for gauge theory and gravity, arXiv:1308.2820.
[45] R. Penrose, Solutions of the zero-rest-mass equations, J. Math. Phys. **10**, 38 (1969).
[46] R. Penrose, On the twistor descriptions of massless fields, Conference: C78-07-10.4, (1978), p. 55–91.
[47] V. Nair, A current algebra for some gauge theory amplitudes, Phys. Lett. B **214**, 215 (1988).
[48] T. Adamo, Correlation functions, null polygonal Wilson loops, and local operators, J. High Energy Phys. 12 (2011) 006.
[49] T. Adamo, M. Bullimore, L. Mason, and D. Skinner, A proof of the supersymmetric correlation function / Wilson loop correspondence, J. High Energy Phys. 08 (2011) 076.
[50] M. Bullimore and D. Skinner, Holomorphic linking, loop equations and scattering amplitudes in twistor space, arXiv:1101.1329.
[51] L. Koster, V. Mitev, M. Staudacher, and M. Wilhelm, All tree-level MHV form factors in $\mathcal{N}=4$ SYM from twistor space, arXiv:1604.00012.
[52] L. Koster, V. Mitev, M. Staudacher, and M. Wilhelm (to be published).