Black Hole Kicks as New Gravitational Wave Observables

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(Received 21 April 2016; published 29 June 2016)

Generic black hole binaries radiate gravitational waves anisotropically, imparting a recoil, or kick, velocity to the merger remnant. If a component of the kick along the line of sight is present, gravitational waves emitted during the final orbits and merger will be gradually Doppler shifted as the kick builds up. We develop a simple prescription to capture this effect in existing waveform models, showing that future gravitational wave experiments will be able to perform direct measurements, not only of the black hole kick velocity, but also of its accumulation profile. In particular, the eLISA space mission will measure supermassive black hole kick velocities as low as $\sim 500\ \mathrm{km\,s^{-1}}$, which are expected to be a common outcome of black hole binary coalescence following galaxy mergers. Black hole kicks thus constitute a promising new observable in the growing field of gravitational wave astronomy.

DOI: 10.1103/PhysRevLett.117.011101

Introduction.—Merging black hole (BH) binaries have entered the realm of observational astronomy. On September 14, 2015, gravitational waves (GWs) emitted during the inspiral and merger of two stellar-mass BHs of $\sim 30\ M_\odot$ at $z \sim 0.1$ were detected by the two LIGO detectors [1]. GW150914 constitutes not only the first direct detection of GWs, but also the first observation of a stellar-mass BH binary. The identification of supermassive BH binary candidates has (so far) only been possible through electromagnetic observations [2,3]. The most promising candidates have been identified as double-core radio galaxies [4] and quasars with periodic behaviors [5,6]. Upcoming GW observations will revolutionize the field of BH binary astrophysics: stellar-mass BH binaries will be targeted by a worldwide network of ground-based interferometers [7–9] while in space the recent success of the LISA pathfinder mission [10] has helped paved the way for eLISA [11] which will observe hundreds (if not thousands) of supermassive BH binaries out to cosmological redshifts and open the era of multi-frequency GW astronomy.

In this Letter, we show that the enormous potential of future GW observations is further enriched by the direct observability of BH kicks. BH binaries radiate GWs anisotropically, which leads to a net emission of linear momentum and, by conservation of momentum, to a recoil of the final remnant. This effect has been studied extensively using post-Newtonian and numerical techniques; see, e.g., Ref. [12] and references therein. The key findings of these studies are that the merger of nonspinning BHs can only produce kicks of $\sim 170\ \mathrm{km\,s^{-1}}$ [13], but that recoil velocities as large as $\sim 5000\ \mathrm{km\,s^{-1}}$ are possible if rapidly rotating BHs with suitable spin orientations collide [14–16]. These exceptionally large recoils are commonly referred to as *superkicks* and their dynamics can be attributed to antiparallel spin components in the orbital plane [17].

BH kicks have striking astrophysical consequences, especially for supermassive BHs. Superkicks of $\mathcal{O}(1000)\ \mathrm{km\,s^{-1}}$ easily exceed the escape velocity of even the most massive galaxies [18], and may thus eject BHs from their hosts [19]. Such ejections would affect the fraction of galaxies hosting central BHs [20,21] and, consequently, the expected event rates for eLISA [22]. Even smaller recoil velocities $\lesssim 500\ \mathrm{km\,s^{-1}}$ affect the dynamics of galaxy cores by displacing the postmerger BHs for time scales as large as ~ 10 Myr [23,24]. BH kicks may lead to a variety of electromagnetic signatures [25], and observational strategies [26,27] have recently been proposed for their detection. Candidates are present, but their nature is debated (see Refs. [25,28] and references therein) and, overall, BH kicks remain elusive.

If GW observations of a BH binary provide accurate measurements of the component masses and spins, it is, in principle, possible to use numerical relativity (NR) results to infer the kick that the binary should have received around merger (this was not possibile for GW150914 [29]). Such an approach, however, would be of *indirect* nature and crucially relies on the validity of the assumptions in the numerical modeling process. For instance, it would *not* provide an additional consistency check of the predictions of general relativity (GR). As argued here, it is possible instead to *directly* measure BH kicks from the GW signal alone. If the kick is directed towards (away from) Earth, then the latter part of the waveform will be blueshifted (redshifted) relative to the early part. Roughly speaking, different, Doppler-shifted mass parameters would be inferred from the inspiral and ringdown parts of the signal if analyzed separately. More precisely, by observing the

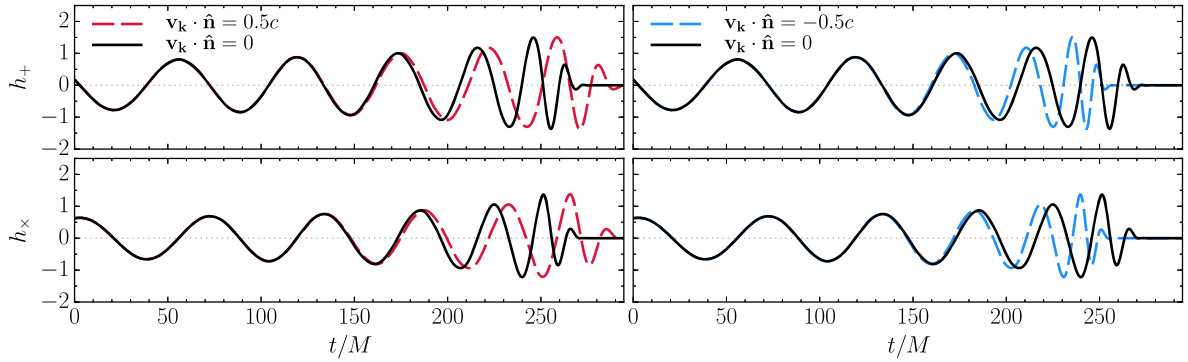

FIG. 1. GW shift due to BH kicks (artificially exaggerated to demonstrate the key features). As the kick velocity builds up during the last few orbits and merger, the emitted GWs are progressively redshifted (left) or blueshifted (right), depending on the sign of the projection of the kick velocity $\mathbf{v_k}$ onto the line of sight $\hat{\mathbf{n}}$. This is equivalent to differentially rescaling the binary's total mass in the phase evolution from M to $M(1+\mathbf{v_k}\cdot\hat{\mathbf{n}})$. These figures have been produced by artificially imparting kicks of $\mathbf{v_k}\cdot\hat{\mathbf{n}} = \pm 0.5c$ to nonspinning equal-mass binaries, assuming a Gaussian kick model with $\sigma = 60M$ [see Eqs. (4) and (5), with $\alpha_n = 0$ for $n \geq 1$].

differential Doppler shift throughout the signal, one can directly measure the change in speed of the system's center of mass as a function of time.

Doppler mass shift.—In the absence of a mass or length scale in vacuum GR, the GW frequency f enters the binary dynamics exclusively in the dimensionless form fM, where M is the total mass of the binary (hereafter, $G = c = 1$). This scale invariance implies a complete degeneracy between a frequency shift and a rescaling of the total mass of the system. For example, the cosmological redshift z of a BH binary merely enters in the predicted GW emission through a rescaling of the total mass by a factor $(1+z)$ and, hence, GW observation of the binary only measures the combination $M(1+z)$ [30]. BH kicks produce a similar effect: at linear order, the motion of the center of mass shifts the emitted GW frequency by a factor $1+\mathbf{v_k}\cdot\hat{\mathbf{n}}$ while leaving the amplitude unaffected ($\mathbf{v_k}$ is the kick velocity with magnitude v_k and the unit vector $\hat{\mathbf{n}}$ denotes the direction of the line of sight from observer to source). There is, however, one crucial difference: while cosmological redshift homogeneously affects the entire signal, a frequency shift due to BH kicks gradually accumulates during the last orbits and merger. This point is illustrated in Fig. 1: as a kick is imparted to the merging BHs, the emitted GWs are progressively blue- or redshifted. The frequency of the signal changes as if the mass of the system was varied from M in the early inspiral to $M(1+\mathbf{v_k}\cdot\hat{\mathbf{n}})$ by the end of the ringdown.

The detectability of this effect can be estimated using the following back-of-the-envelope argument. Imagine breaking a BH binary waveform into two parts: inspiral and ringdown, $h(t) = h_i(t) + h_r(t)$. For simplicity, assume that the kick is imparted instantaneously at merger so that only h_r is affected. Let M_i and M_r, respectively, denote the total binary mass as measured from h_i and h_r alone. Neglecting the energy radiated in GWs—this effect is not negligible in magnitude, resulting in a reduction of the mass by ~5%, but can be estimated accurately from the waveform and thus be accounted for—, the effect of a kick is to Doppler shift the final mass according to $M_r = M_i(1+\mathbf{v_k}\cdot\hat{\mathbf{n}})$. The inspiral part h_i of the GW signal generally contains a larger fraction of the signal-to-noise ratio (SNR) than the ringdown part h_r, so the detectability of the kick will be limited by the measurement of M_r: kicks of magnitude v_k can be detected if M_r is measured with a fractional accuracy of $\lesssim v_k/c$ (~1% for a superkick along the line of sight). The ringdown waveform can be modeled using the least damped quasinormal mode for a Schwarzschild BH [31], $h_r(t) \simeq A\exp(-0.089t/M_r)\sin(0.37t/M_r)$, which gives a squared SNR,

$$\rho_r^2 = \frac{1}{S_n}\int_0^\infty h_r(t)^2 dt \simeq \frac{2.66 M_r A^2}{S_n}, \tag{1}$$

assuming white noise in a detector with power spectral density (PSD) $S_n(f) = S_n = \text{const}$. The error on the measurement of M_r can be estimated using the linear signal approximation,

$$\left(\frac{1}{\Delta M_r}\right)^2 = \frac{1}{S_n}\int_0^\infty \left(\frac{\partial}{\partial M}h_r(t)\right)^2 dt \simeq \frac{25.6A^2}{M_r S_n}. \tag{2}$$

Therefore, the fractional error on M_r is given by

$$\frac{\Delta M_r}{M_r} \simeq \frac{0.322}{\rho_r}. \tag{3}$$

This back-of-the-envelope argument suggests that kicks along the line of sight with magnitude $v_k \sim 0.003c \simeq 900\ \mathrm{km\,s^{-1}}$ can be measured with GW observations if the SNR in the ringdown is $\rho_r \sim 100$. Direct detection of BH kicks will be very challenging, if not impossible, with current ground-based detectors. For instance, the rather loud event GW150914 has a ringdown SNR $\rho_r \sim 5$ [32], which would only allow us to measure unrealistically large

kicks, $v_k \sim 0.06c$. On the other hand, BH kicks are very promising observables for space-based detectors, where SNRs in the ringdown can reach $\rho_r \sim 10^3$ [33]. This will allow for measurements of supermassive BH kicks with magnitude as low as $v_k \sim 100\ \mathrm{km\,s^{-1}}$, which are expected to be ubiquitous [34,35]. The detectability of the kick is governed by the ringdown part of the SNR ρ_r, which has also been found to be important to detect the GW memory effect (see Ref. [36], where kicks are also mentioned) and test the Kerr hypothesis via BH spectroscopy [31].

Kicked waveforms.—In order to investigate the detectability of BH kicks more quantitatively, we need a waveform model that captures the cumulative frequency shift they introduce. Doppler shifts due to BH kicks can be straightforwardly incorporated into any preexisting waveform model (which does not include the kick) by substituting $M \to M \times [1 + v(t)]$ in the phase evolution, where $v(t)$ is the projection of the center-of-mass velocity due to the kick onto the line of sight. Here, we only consider the nonrelativistic Doppler shift; relativistic corrections enter at the order $\mathcal{O}(v_k)^2 \lesssim 10^{-4}$, well below the magnitude relevant for our analysis. The profile $v(t)$ is taken such that $v(t) \to 0$ as $t \to -\infty$ and $v(t) \to \mathbf{v_k} \cdot \hat{\mathbf{n}}$ as $t \to \infty$. A common observation in NR simulations is that the kick is imparted over a time $2\sigma \sim 20M$ centered on the merger, at a rate dv/dt, which is approximately of Gaussian shape [37,38], possibly with some deceleration after merger (*antikick*) [39,40]. In contrast to the kick speed, relatively little is known regarding the kick profile beyond these qualitative observations. We therefore adopt a flexible model for the kick profile. We expand dv/dt according to

$$\frac{d}{dt}v(t) = \mathbf{v_k} \cdot \mathbf{n}\,\frac{\sum_n \alpha_n \phi_n(t)}{\int_{-\infty}^{\infty} \sum_n \alpha_n \phi_n(t) dt}, \tag{4}$$

$$\phi_n(t) = \frac{1}{\sigma\sqrt{2^n n!\sqrt{\pi}}} \exp\left(-\frac{(t-t_c)^2}{2\sigma^2}\right) H_n\left(\frac{t-t_c}{\sigma}\right), \tag{5}$$

where H_n are the Hermite polynomials, t_c is the time of coalescence, σ controls the duration over which the kick is accumulated, and the α_n weigh the various components. The functions $\phi_n(t)$ constitute a complete basis (they are actually the familiar solutions for the quantum harmonic oscillator), and so they can model all possible kick profiles. This basis is particularly appealing, because the first two terms $n = 0, 1$ model Gaussian acceleration profiles and antikicks, respectively. The case $\sigma = 0$ and $\alpha_n = 0$ for $n \geq 1$ corresponds to a kick instantaneously imparted at t_c, as assumed in the back-of-the-envelope argument presented above. We have tested this prescription against 200 NR waveforms from the public Simulating eXtreme Spacetimes catalog [41], finding that the radiated-momentum profiles obtained from integrating the $l \leq 6$ modes of the Newman-Penrose scalar Ψ_4 are well approximated by the first two terms of the expansion of Eqs. (4) and (5). For systems with kicks above 500 $\mathrm{km\,s^{-1}}$, residuals in v_k are less than 17% in all cases, and typically less than 4% [42].

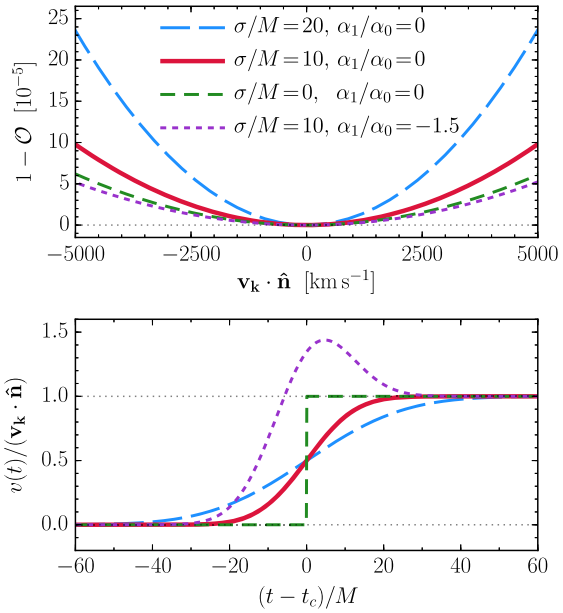

FIG. 2. Mismatches introduced by BH recoils. The top panel shows the mismatch $1 - \mathcal{O}$ between (i) a standard waveform of equal-mass nonspinning BH binaries of total mass M and (ii) a "kicked" waveform which includes the Doppler-shifting effects of a velocity profile $v(t)$. Each line corresponds to a different kick profile $v(t)$, as shown in the bottom panel. All models shown here assume $\alpha_n = 0$ for $n \geq 2$. The $\sigma = 10M$, $\alpha_1/\alpha_0 = 0$ model (solid line) is used in Fig. 3.

For a given waveform approximant, GW detector, and binary parameters, we generate two signals: a standard waveform $h_0(t)$ and a second "kicked" waveform $h_k(t)$. The two waveforms can be compared by calculating their overlap,

$$\mathcal{O} = \max_{t_c,\phi_c} \frac{(h_0|h_k)}{\sqrt{(h_0|h_0)(h_k|h_k)}}, \tag{6}$$

where $(h_0|h_k)$ is the noise-weighted inner product [43] and t_c (ϕ_c) is the time (phase) of coalescence. Approximately two waveforms are distinguishable (and the kick detectable) if $\mathcal{O} \lesssim 1 - \rho^{-2}$ [44], where $\rho = \sqrt{(h_0|h_0)}$ is the SNR (of the full waveform). This assumes the kick is not degenerate with other parameters, which is expected as the kick mostly affects the ringdown and not the entire signal.

This procedure is illustrated in Fig. 2 using a simple controlled experiment. We consider 6 inspiral cycles, merger, and ringdown of an equal-mass nonspinning BH binary (a similar setup to that used in Fig. 1). For simplicity, and to ensure that the results are not detector specific, the overlaps have been computed using a flat PSD. Artificially imposed recoils of $\sim$1000 $\mathrm{km\,s^{-1}}$ introduce mismatches $(1 - \mathcal{O}) \sim 10^{-5}$. Kicks are more likely to be detected if they are imparted over a longer period of time (i.e., larger σ)

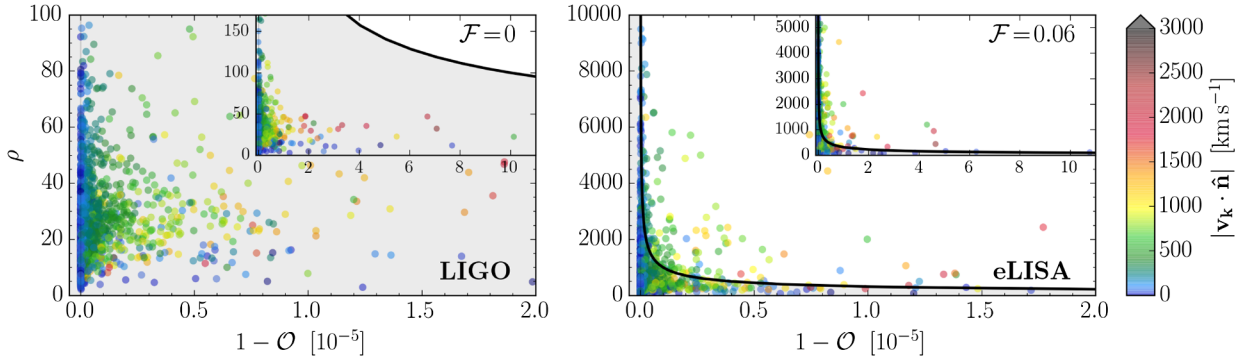

FIG. 3. Detectability of BH kicks with LIGO (left) and eLISA (right). For each simulated source we compute the overlap $\mathcal{O}$ between standard and kicked waveforms, and compare it with the SNR ρ. Kick velocities—here encoded in the color bar—are imparted using NR fitting formulas. BH kicks are detectable for the fraction $\mathcal{F}$ of the sources above the black line, $\mathcal{O} < 1-\rho^{-2}$. eLISA results have here been generated with the "N2A5L6" PSD of Ref. [47].

because dephasing starts to occur earlier in the inspiral (this effect can be seen in Fig. 1 where a larger value of $\sigma = 60M$ was used). Note that the overlaps are approximately symmetric with respect to the transformation $\mathbf{v_k} \to -\mathbf{v_k}$; i.e., blueshifts and redshifts are equally detectable. This property can be shown to hold exactly at linear order in v_k [42].

We next explore more realistic scenarios by using NR fitting formulas to predict the kick velocity. For this purpose, we generate two BH binary populations for the LIGO and eLISA detectors. LIGO (eLISA) sources were selected randomly from the following distributions: uniform total mass $M \in [10\ M_\odot, 100\ M_\odot]$ ($[10^5 M_\odot, 10^6 M_\odot]$) and mass ratio $q \in [0.05, 1]$; uniform dimensionless spin magnitudes $\chi_1, \chi_2 \in [0, 1]$; isotropic inclination and spin directions at a reference GW frequency $f_{\rm ref} = 20$ Hz (2 mHz); isotropic sky location; sources are distributed homogeneously in comoving volume with comoving distance $D_c \in [0.1\ \text{Gpc}, 1\ \text{Gpc}]$ ([1 Gpc, 10 Gpc]) assuming the Planck cosmology [45]. We use the LIGO "Zero-Det-High-P" PSD of Ref. [46] with lower cutoffs $f_{\rm low} = 10$ Hz, and the two possible eLISA PSDs "N2A5L6" and "N2A1L4" of Ref. [47] with $f_{\rm low} = 0.3$ mHz (the former being more optimistic; for simplicity, we neglect the spacecraft orbital motion which can be separately accounted for). For each binary, we estimate the kick velocity using the fitting formula summarized in Ref. [48]. In order to return accurate estimates, the kick formula requires as input the BH spin parameters at separations $r \sim 10M$, comparable to the initial separations of the NR simulations used in the formula's calibration. Otherwise, resonant effects [49] are not adequately accounted for and lead to erroneous kick magnitudes [50]. We bridge the separation range between $f_{\rm ref}$ and $r = 10M$ using the orbit-averaged post-Newtonian evolution code of Ref. [48]. The NR fitting formula then provides expressions for the kick components parallel and orthogonal to the binary orbital angular momentum $\mathbf{L}$: $v_\parallel$ and $v_\perp$. The projection of the kick velocity along the line of sight is given by

$$\mathbf{v_k}\cdot\hat{\mathbf{n}} = v_\parallel \cos\Theta \cos\iota - v_\perp \cos\Theta' \sin\iota, \tag{7}$$

where $\cos\iota = \hat{\mathbf{L}}\cdot\hat{\mathbf{n}}$ is the cosine of the inclination at $r = 10M$, Θ is related to the direction of the orbital-plane components of the spins at merger [37,51], and Θ' sets the direction of the orbital-plane component of the kick [42]. In practice, both Θ and Θ' depend on the initial separation of the binary in the NR simulations. While the Θ dependence has been studied extensively in the literature [37,51], the impact of Θ' and its relation with Θ have, to our knowledge, not yet been explored. In the following, both angles are drawn uniformly in $[0, \pi]$. For each system, we generate two waveforms, h_0 and h_k, using the inspiral-merger-ringdown approximant "IMRPhenomPv2" of Refs. [52–54], which accounts for spin precession. We have verified that our results for the overlaps are insensitive to the choice of the waveform approximant, even when nonprecessing models are used. In the following, we assume a "Gaussian" kick model, described by $\alpha_n = 0$ for $n \geq 1$ and $\sigma = 10M$ (solid curve in Fig. 2); cf. Ref. [37].

Our results are summarized in Fig. 3. As suggested by our previous argument, none of the LIGO sources have mismatches high enough to detect the kick. The eLISA case is different: ~1%–6% (depending on the PSD) of the simulated sources have $\mathcal{O} < 1-\rho^{-2}$ and therefore present detectable BH kicks. Kicks with a projected magnitude $\mathbf{v_k}\cdot\hat{\mathbf{n}} \gtrsim 500$ km s^{-1} at $\rho \gtrsim 1000$ will be generically observable, but even some of the lower kicks with $\mathbf{v_k}\cdot\hat{\mathbf{n}} \sim 100$ km s^{-1} may be accessible. In the fortunate case of a *superkick* directed along the line of sight ($|\mathbf{v_k}\cdot\hat{\mathbf{n}}| \sim 3000$ km s^{-1}), the effect may be so prominent to be distinguishable at SNRs as low as $\rho \sim 50$. As eLISA is expected to measure up to $\mathcal{O}(100)$ BH binaries per year [10,47], our study suggests that ~6 yr^{-1} (~30 in total for a 5-yr mission lifetime) sources may present detectable kicks. Although more realistic astrophysical modeling is needed to better quantify this fraction, our simple study shows that direct detection of BH recoils is well within the reach of eLISA. Third-generation ground-based detectors will also

present promising opportunities: repeating the calculations of the LIGO population of binaries but observed with ET (assuming the "ET-D-sum" PSD of Ref. [55], with $f_{\rm low} = 1$ Hz), we find ∼5% of binaries possess detectable kicks.

GW observations not only have the potential to measure the magnitude of the BH kick, but also the details of how the velocity accumulates with time. By expanding $v(t)$ according to Eqs. (4) and (5), one can take the kick model parameters $\mathbf{v_k} \cdot \hat{\mathbf{n}}$, σ, and α_n to be free parameters of the waveform model, and treat them on an equal footing with masses, spins, inclination angles, etc. Consider, for example, a *golden system* at $\rho = 10^4$ with component masses of $1.3 \times 10^6 M_\odot$ (chosen to maximize the mismatch caused by the kick), misaligned extremal spins and inclination such that $\mathbf{v_k} \cdot \hat{\mathbf{n}} \sim 5000\ \mathrm{km\,s^{-1}}$ km. A Fisher matrix calculation of the intrinsic parameters of this binary suggests that eLISA will be capable of measuring the kick velocity with precision $\Delta v_k \sim 200\ \mathrm{km\,s^{-1}}$, the kick duration with precision $\Delta\sigma \sim 1M$, and the presence of an antikick at the level of $\Delta(\alpha_1/\alpha_0) \sim 0.1$ (considering a two-component kick model, i.e., $\alpha_n = 0$ for $n \geq 2$) [42]. This Fisher matrix analysis revealed no strong degeneracies between the kick and other parameters, thus further justifying our previous use of the overlap as a detectability criterion for the kick. Finally, note that *superkicks* have $v_\parallel \gg v_\perp$, so that face-on or face-off binaries $|\hat{\mathbf{L}} \cdot \hat{\mathbf{n}}| \sim 1$ generate the largest velocity components along the line of sight and, hence, are most favorable for a direct kick measurement.

Conclusions.—BH kicks leave a clear imprint on the GW waveform emitted during the late stages of the inspiral, merger, and ringdown of BH binaries. eLISA and, likely, third-generation ground-based detectors will be able to *directly* detect the presence of a kick from the distortion of the waveform for a significant fraction of the binaries observed. By comparing the directly measured kicks (both magnitude and profile) to the NR kick predictions for a binary with measured masses and spins, it will be possible to verify whether linear momentum is radiated as predicted by GR. Much like the Hulse-Taylor pulsar provided the first evidence that GWs carry away energy in accordance with the expectation of GR, and GW150914 provided the first direct evidence of the GWs themselves [1], a direct measurement of a BH kick will provide the first direct evidence for the linear momentum carried by GWs.

We thank Ulrich Sperhake, Jonathan Gair, Enrico Barausse, Emanuele Berti, Christopher Berry, Vitor Cardoso, Alberto Sesana, Nathan Johnson-McDaniel, and the anonymous referees for fruitful discussions and suggestions. This document has been assigned LIGO document reference LIGO-P1600118. D. G. is supported by the UK STFC and the Isaac Newton Studentship of the University of Cambridge. C. J. M. is supported by the UK STFC. Partial support is also acknowledged from the H2020 ERC Consolidator Grant "Matter and strong-field gravity: New frontiers in Einstein's theory" Grant Agreement No. MaGRaTh–646597, the H2020-MSCA-RISE-2015 Grant No. StronGrHEP-690904, the STFC Consolidator Grant No. ST/L000636/1, the SDSC Comet and TACC Stampede clusters through NSF-XSEDE Award No. PHY-090003, the Cambridge High Performance Computing Service Supercomputer Darwin using Strategic Research Infrastructure Funding from the HEFCE and the STFC, and DiRAC's Cosmos Shared Memory system through BIS Grant No. ST/J005673/1 and STFC Grants No. ST/H008586/1 and No. ST/K00333X/1.

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