Towards a Theory of Metastability in Open Quantum Dynamics

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By generalizing concepts from classical stochastic dynamics, we establish the basis for a theory of metastability in Markovian open quantum systems. Partial relaxation into long-lived metastable states—distinct from the asymptotic stationary state—is a manifestation of a separation of time scales due to a splitting in the spectrum of the generator of the dynamics. We show here how to exploit this spectral structure to obtain a low dimensional approximation to the dynamics in terms of motion in a manifold of metastable states constructed from the low-lying eigenmatrices of the generator. We argue that the metastable manifold is in general composed of disjoint states, noiseless subsystems, and decoherence-free subspaces.

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Introduction.—Stochastic many-body systems often display a complex and slow relaxation towards a stationary state. A common phenomenon is that of metastability, where the initial relaxation is into long-lived states, with subsequent decay to true stationarity occurring at much longer times. This separation of times in the dynamics has evident experimental manifestations, for example, in two-step decay of time correlation functions. Metastability is a common occurrence in classical soft matter [1], glasses being the paradigmatic example [2,3].

There is much current interest in the nonequilibrium dynamics of quantum many-body systems, both closed (i.e., isolated) and open (i.e., interacting with an environment). This includes issues such as thermalization [4–7], many-body localization [8–10], and aging and glassy behavior, where questions about time scales and partial versus full relaxation play central roles [11–16]. From the quantum information perspective, decoherence-free subspaces [17–20] and noiseless subsystems [21–23], where parts of the Hilbert space are protected against external noise, are ideal scenarios for implementing quantum information processing [24]. Since experiments are performed in finite time, it is sufficient (and practical) to consider manifolds of coherent states that are only stable over experimental time scales, i.e., metastable, with respect to noise.

Given this broad range of problems, it would be highly desirable to have a unified theory of quantum metastability. In this Letter we lay the ground for such a theory for the case of open quantum systems evolving with Markovian dynamics. Our starting point is a well-established approach for metastability in classical stochastic systems [25–29]. We develop an analogous method for quantum Markovian systems based on the spectral properties of the generator of the dynamics. The separation of time scales implies a splitting in the spectrum, and this spectral division allows us to construct metastable states from the low-lying eigenmatrices of the generator. Based on perturbative calculations for finite systems, we argue that the manifold of metastable states is in general composed of disjoint states, noiseless subsystems, and decoherence-free subspaces. We illustrate these possibilities with simple examples. We further discuss how to reduce the overall dynamics to a low-dimensional effective motion in the metastable manifold, and consider the associated behavior of time correlations.

Quantum metastability and spectral properties.—We consider an open quantum system evolving under Markovian dynamics, with the Linbladian master equation $(d/dt)\rho(t) = \mathcal{L}\rho(t)$ [30–33], where the generator of the dynamics $\mathcal{L}$ is

$$\mathcal{L}(\cdot) \coloneqq -i[H,(\cdot)] + \sum_j \left(J_j(\cdot)J_j^\dagger - \frac{1}{2}\{J_j^\dagger J_j,(\cdot)\}\right). \quad (1)$$

The state of the system at time t is $\rho(t)$, the system Hamiltonian is H, and $\{J_j\}$ are quantum jump operators [34]. While in general the linear operator $\mathcal{L}$ is not diagonalizable, one can find its eigenvalues $\{\lambda_k, k = 1,2,\ldots\}$ [which we order by decreasing real part, $\mathrm{Re}(\lambda_k) \ge \mathrm{Re}(\lambda_{k+1})$] each corresponding to an eigenspace or a Jordan block. Since $\mathcal{L}$ generates a proper quantum stochastic (completely positive trace-preserving) dynamics of $\rho(t)$, its largest eigenvalue vanishes, $\lambda_1 = 0$, and its associated right eigenmatrix R_1 is the stationary state, $R_1 = \rho_{\mathrm{SS}}$ (the corresponding left eigenmatrix being the identity $L_1 = I$) [35]. The real parts of the eigenvalues $\{\lambda_{k>1}\}$ give the relaxation rates of all the modes of the system dynamics. In particular, the second eigenvalue λ_2 determines the spectral gap, whose inverse is related to the longest time scale τ of the relaxation of the system to the stationary state, i.e., $\|\rho(t)-\rho_{SS}\| \sim e^{-t/\tau}$ with $\tau \sim 1/|\mathrm{Re}(\lambda_2)|$ (where $\|A\| \coloneqq \mathrm{Tr}\sqrt{A^\dagger A}$).

Metastability manifests as a long time regime when the system appears stationary, before eventually relaxing to ρ_{SS}. This occurs when low lying eigenvalues become separated from the rest of the spectrum. Let us assume that this separation occurs between the mth mode and the rest, that is, $|\mathrm{Re}(\lambda_m)| \ll |\mathrm{Re}(\lambda_{m+1})|$. We can then write for the time evolution from an initial state ρ_{in}

$$\rho(t) = e^{t\mathcal{L}}\rho_{\mathrm{in}} = \rho_{SS} + \sum_{k=2}^{m} e^{t\lambda_k} c_k R_k + [e^{t\mathcal{L}}]_{\mathcal{I}-\mathcal{P}}\rho_{\mathrm{in}}, \quad (2)$$

where $c_k = \mathrm{Tr}(L_k\rho_{\mathrm{in}})$ are coefficients of the initial state decomposition into the eigenbasis of $\mathcal{L}$ [35]. In Eq. (2) we have introduced the projection $\mathcal{P}$ on the subspace of the first m eigenmatrices, $\mathcal{P}\rho \coloneqq \rho_{SS}\mathrm{Tr}(\rho) + \sum_{k=2}^{m} R_k \mathrm{Tr}(L_k\rho)$, and $[e^{t\mathcal{L}}]_{\mathcal{P}} \coloneqq \mathcal{P}e^{t\mathcal{L}}\mathcal{P}$. Expanding the exponentials in the sum, and assuming $\lambda_1, \ldots, \lambda_m$ are real, Eq. (2) can be rewritten as [36]

$$\rho(t) = \rho_{SS} + \sum_{k=2}^{m} c_k R_k + O(\|[t\mathcal{L}]_{\mathcal{P}}\|) + O(\|[e^{t\mathcal{L}}]_{\mathcal{I}-\mathcal{P}}\|). \quad (3)$$

The dynamics will appear stationary for any initial condition when the last two terms are small. This defines a range $\tau'' \ll t \ll \tau'$ where metastability occurs. Intuitively, the last term can be discarded if $\tau'' \sim 1/|\mathrm{Re}(\lambda_{m+1})|$ and the overlap of the initial state with the suppressed modes is not too large, so that the sum over many modes of small amplitude can be neglected. Thus, for times $\tau'' \ll t$ the system relaxes into a state in the metastable manifold (MM). Apparent stationarity requires $\|[t\mathcal{L}]_{\mathcal{P}}\| \ll 1$, which defines the upper limit of the metastable interval: $\tau' \sim 1/|\mathrm{Re}(\lambda_m)|$ (for m not too large).

More generally, the eigenvalues could be complex, appearing in conjugate pairs, $\lambda_{k,1} = \lambda_{k,2}^*$, with imaginary parts that cannot be discarded. Taking this into account, a metastable state ρ_{MS} in the MM would read in general [37]

$$\rho_{MS} = \rho_{SS} + \sum_{k}^{m} c_k{}'(t) R_k{}'. \quad (4)$$

When λ_k is real, we have that $c_k{}'(t) \coloneqq c_k$ and $R_k{}' \coloneqq R_k$. For conjugate pairs, $\lambda_{k,1} = \lambda_{k,2}^*$, we have that $c_{k,1} = c_{k,2}^*$ and $c_{k,1}{}' \coloneqq |c_{k,1}|\cos(\omega_k t + \delta_k)$ and $c_{k,2}{}' \coloneqq |c_{k,1}|\sin(\omega_k t + \delta_k)$, where $R_{k,1}{}' \coloneqq R_{k,1} + R_{k,2}$ and $R_{k,2}{}' \coloneqq i(R_{k,1} - R_{k,2})$ with $\delta_k \coloneqq \arg(c_k)$, $\omega_k \coloneqq \mathrm{Im}(\lambda_k)$. In Eq. (4) we have discarded the second line of Eq. (3), which leads ρ_{MS} to be approximately positive with its negative part bounded by the corrections to the invariance of the MM in Eq. (3). The remaining time dependence in Eq. (4) constitutes rotations within the MM that leave the MM invariant, which necessarily correspond to nondissipative evolution for $\tau'' \ll t \ll \tau'$, which we also discuss below.

Beyond the metastable regime, $t \gtrsim \tau'$, the dynamics will correspond to motion in the MM towards the true stationary state, which is reached at times $t \gg \tau$. This effective dimensional reduction due to a separation of time scales is a key result of this Letter.

Geometrical description of quantum metastability.—The MM can be described geometrically by generalizing the classical method of Refs. [25–29]. In the metastable regime the system state is well approximated by a linear combination of the m low-lying modes, see Eqs. (4). A metastable state is determined by a vector $(c_2{}', \ldots, c_m{}')$ in $\mathbb{R}^{m-1}$. We thus refer to the MM as being $(m-1)$ dimensional, but note that each point on this manifold represents a D^2 density matrix ρ_{MS}, where $D = \dim(\mathcal{H})$ is the dimension of the Hilbert space $\mathcal{H}$ of the system. Furthermore, the MM is a convex set as it is a linearly transformed convex set of initial states ρ_{in}.

Let us first consider the case of $m = 2$. Because of the convexity of the MM, any metastable state is a mixture of extreme metastable states (EMSs). In this case they are just two, $\tilde{\rho}_1$ and $\tilde{\rho}_2$, obtained from

$$\tilde{\rho}_1 = \rho_{SS} + c_2^{\max} R_2, \qquad \tilde{\rho}_2 = \rho_{SS} + c_2^{\min} R_2, \quad (5)$$

where $c_2^{\max}$ and $c_2^{\min}$ are the maximal and minimal eigenvalues of L_2, respectively [35]. Note that $\tilde{\rho}_{1,2}$ are (approximately) positive despite R_2 being nonpositive. From Eq. (3) it follows, up to corrections, that $\rho(t) = p_1\tilde{\rho}_1 + p_2\tilde{\rho}_2$ with probabilities $p_{1,2} = \mathrm{Tr}(\tilde{P}_{1,2}\rho_{\mathrm{in}})$, where

$$\tilde{P}_1 = (L_2 - c_2^{\min} I)/\Delta c_2, \qquad \tilde{P}_2 = (-L_2 + c_2^{\max} I)/\Delta c_2,$$

and $\Delta c_2 \coloneqq c_2^{\max} - c_2^{\min}$. Note that the observables $\tilde{P}_{1,2}$ satisfy $\tilde{P}_{1,2} \geq 0$ and $\tilde{P}_1 + \tilde{P}_2 = I$. This leads to $\tilde{\rho}_1$ and $\tilde{\rho}_2$ being (approximately) disjoint [38].

Example I: three-level system. Consider the three-level system of Fig. 1(a), with the Hamiltonian $H = \Omega_1(|1\rangle\langle 0| + |0\rangle\langle 1|) + \Omega_2(|2\rangle\langle 0| + |0\rangle\langle 2|)$ and a jump operator $J = \sqrt{\kappa}|0\rangle\langle 1|$. When $\Omega_2 \ll \Omega_1$, the dynamics can be "shelved" for long times in $|2\rangle$, giving rise to intermittency in quantum jumps [32], which can be seen as the coexistence of "active" and "inactive" dynamical phases [44]. Figure 1(b) shows the spectrum of $\mathcal{L}$: the gap is small for $\Omega_2 \ll \Omega_1$, the two leading eigenvalues detach from the rest (i.e., $m = 2$), and the dynamics is metastable. Figure 1(c) illustrates the trace distance of the state $\rho(t)$ to the MM starting from $\rho_{\mathrm{in}} \neq \rho_{SS}$: an initial decay on times of order of τ'' to the nearest point on the MM (in this case to an EMS) is followed by decay to ρ_{SS} on times of order $\tau' = \tau$ (since $m = 2$). The MM for this $m = 2$ case is a one-dimensional simplex (i.e., a convex set whose interior points uniquely represent probability distributions on the vertices), see Fig. 1(d).

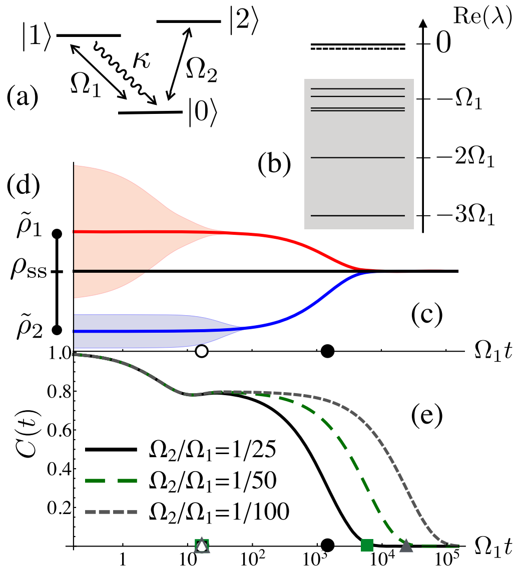

FIG. 1. Example of metastability in a three-level system. (a) Level scheme and transitions. (b) Spectrum of $\mathcal{L}$ showing the separation of time scales between (λ_1, λ_2) (full and dashed) and $\{\lambda_{k>2}\}$ (shaded), for the case $\kappa = 4\Omega_1$, $\Omega_2 = \Omega_1/10$. (c) Illustration of the distance of the state $\rho(t)$ to the MM. We consider $\rho(t)$ starting from pure states corresponding to the eigenvectors of L_2 with maximal [top (red)] and minimal eigenvalues [bottom (blue)] $c_2^{\max}$ and $c_2^{\min}$. The full curves indicate the nearest state on the MM, $\rho_{\rm MS}(t)$, to the full state $\rho(t)$. The shaded region indicates the scale of the "error" $\|\delta\rho(t)\|$ with $\delta\rho(t) \coloneqq \rho(t) - \rho_{\rm MS}(t)$. On times of order τ'' (open circle) the state $\rho(t)$ relaxes to the MM (in this case to either of the EMSs $\tilde{\rho}_{1,2}$), as seen by the shaded region decreasing to zero. On times of order τ (filled circle) there is an eventual relaxation to the stationary state $\rho_{\rm SS}$ (central, black line). Since $m = 2$, in this case $\tau' = \tau$. (d) The MM is a one-dimensional simplex. (e) Normalized autocorrelation $C(t)$ of the observable $|1\rangle\langle 1| - |2\rangle\langle 2|$, in the stationary state. For decreasing Ω_2/Ω_1 (i.e., decreasing gap), metastability in the regime τ'' (open symbols) to τ (filled symbols) is increasingly pronounced.

For $m > 2$ the convex set MM of possible coefficients can have more than m extreme points. For classical dynamics it has been proven that this set is well approximated by a simplex [27], whose vertices correspond to m disjoint EMSs and its barycentric coordinates to the probabilities of a metastable state decomposed as a mixture of the EMSs, cf. Fig. 1(d). For quantum dynamics and $m > 3$, we expect the structure of the MM to be richer than just a simplex. As we describe below, the MM can in general also include decoherence-free subspaces (DFSs) [17–19] and noiseless subsystems (NSSs) [21,22] which are protected from dissipation in the metastable regime, as the next example shows.

Example II: collective dissipation and a metastable DFS. Consider a two-qubit system with the Hamiltonian $H = \Omega_1\sigma_1^x + \Omega_2\sigma_2^x$, and a collective jump operator $J = \sqrt{\gamma_1}n_1\sigma_2^- + \sqrt{\gamma_2}(1-n_1)\sigma_2^+$. When $\Omega_{1,2} \ll \gamma_{1,2}$ there is a small gap and the four leading eigenvalues of $\mathcal{L}$ detach from the rest, Fig. 2(a). This is related to the fact that any superposition of $|01\rangle$ and $|10\rangle$ is annihilated by J. Figure 2(b) maps out the MM by randomly sampling all (pure) initial states $\rho_{\rm in}$ from $\mathcal{H}$ and obtaining their corresponding metastable state via Eq. (4): the MM is an affinely transformed Bloch ball corresponding to a DFS of a qubit within the metastable regime $\tau'' \ll t \ll \tau'$. It is important to note that (i) this coherent structure is not the consequence of a symmetry, as for $\gamma_1 \neq \gamma_2$ the system dynamics neither has a U(2) nor an up-down nor a permutation symmetry, cf. Ref. [45]; (ii) the smallest m for which we can obtain a DFS is $m = 4$, as in this case.

Structure of metastable manifold.—We aim to find the general structure of the MM for two classes of systems for which $\mathcal{L}$ has a small gap: (A) finite systems where the gap closes at some limiting values of the parameters in $\mathcal{L}$ (such as $\Omega_2 \to 0$ in example I, and $\Omega_{1,2} \to 0$ in example II), and (B) scalable systems of size N where the gap closes only in the thermodynamic limit $N \to \infty$ (such as the dissipative Ising model of Ref. [48]).

For class A we prove via non-Hermitian degenerate perturbation theory [38] that the structure of a metastable state $\rho_{\rm MS} \in$ MM is given by the following block structure

$$\rho_{\rm MS} = \sum_{l=1}^{m'} p_l \tilde{\rho}_l \otimes \omega_l + \text{corrections}, \tag{6}$$

with $\mathcal{H}$ being the orthogonal sum $\mathcal{H} = \bigoplus_l \mathcal{H}_l \otimes \mathcal{K}_l$, where $\tilde{\rho}_l$ are fixed states on $\mathcal{H}_l$ (cf. the EMSs above), ω_l are arbitrary states on $\mathcal{K}_l$, and p_l are probabilities. Up to the corrections, this is a general structure of a manifold of stationary states of open quantum Markovian dynamics [49]. The metastable regime is given by $\tau_0 \ll t \ll s^{-2}\tau_0$, where τ_0 is the relaxation time for the unperturbed dynamics and s is the scale of the perturbation [38]. The corrections in Eq. (6) are of the order of the corrections to the invariance of the MM during the metastable time regime, cf. Eq. (3). The $(m-1)$ coefficients $(c_2{}', \ldots, c_m{}')$ that determine $\rho_{\rm MS}$, see Eq. (4), correspond approximately to an affine transformation of the m entries of $p_l\omega_l$ $(l = 1, \ldots, m')$ in Eq. (6) with $\sum_{l=1}^{m'} p_l = 1$ [38]. Therefore, the MM approximately represents the degrees of freedom of the classical-quantum space in Eq. (6).

For class B we conjecture that the coefficients representing the MM converge to the degrees of freedom of a classical-quantum space as in Eq. (6), when the separation in the spectrum becomes more and more pronounced as $N \to \infty$. Note that the dimensionality of the MM does not change with N and thus the convergence is well defined. This general conjecture is based on the necessary condition that the low-lying spectrum of $\mathcal{L}$ features only trivial Jordan blocks [50]. Note that a conjecture of the $\rho_{\rm MS}$ structure being approximately that of stationary states, cf. Eq. (6), is a stronger claim. A proof of the former conjecture for class

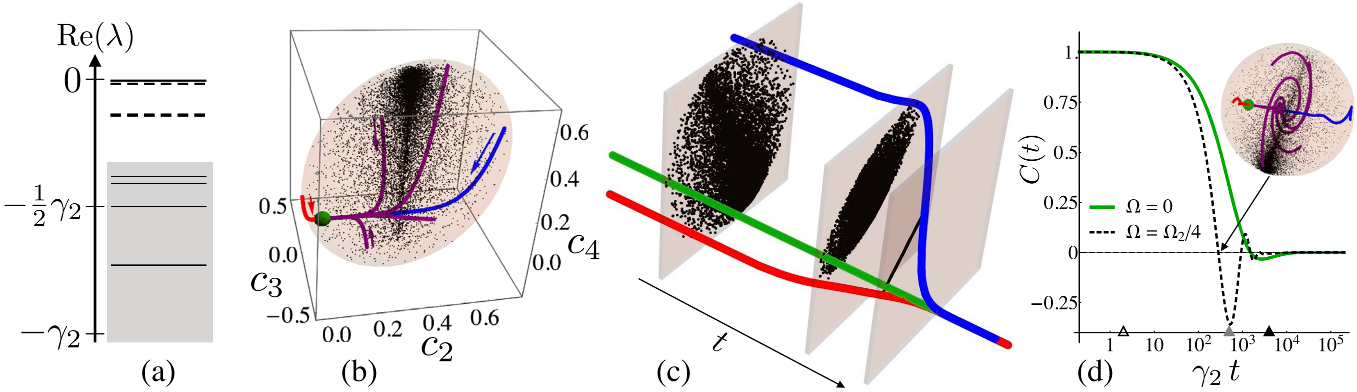

FIG. 2. Example of a coherent metastable manifold. (a) Spectrum of $\mathcal{L}$ for example II (at $\gamma_1 = 4\gamma_2$, $\Omega_1 = 2\Omega_2 = \gamma_2/50$). The first four eigenvalues ($m = 4$) split from the rest (shaded) and define the MM. Note the further splitting between (λ_1, λ_2) and (λ_3, λ_4) (which are almost degenerate). (b) The MM is a qubit. The dots represent the metastable states reached from random initial pure states. They map out under Eq. (4) an affinely transformed Bloch ball (shaded). The large dot (green) is ρ_{SS}; the curves indicate paths in the MM taken by the states evolving from the extreme eigenvectors of L_2 (red and blue), and L_3 and L_4 (purple) towards ρ_{SS}. (c) Time evolution in the MM (affinely transformed to a Bloch ball—the planes are projections in the direction of the eigenbasis of ρ_{SS} and another orthogonal direction): the MM contracts towards a one-dimensional simplex before relaxing eventually to ρ_{SS}, due to the splitting between the first two eigenvalues and the next two, see panel (a). (d) Normalized autocorrelation $C(t)$ for the observable $\sigma_1^z - \sigma_2^z$ (green, solid). Same for the case where there is an extra perturbing Hamiltonian $\Delta H = \Omega\sigma_1^x \otimes \sigma_2^x$, which induces a rotation in the MM, manifesting in oscillations in $C(t)$ in the metastable regime (black, dashed). This realizes in a metastable system the proposal of Refs. [46,47] for implementing operations in a DFS.

B appears challenging at this moment; see the comment in Ref. [38].

The blocks in Eq. (6) can be of three kinds. (i) When $\dim(\mathcal{K}_l) = 1$, the lth block is a disjoint EMS. This is the case in example I, where there are two EMSs $\tilde{\rho}_{1,2}$, with metastable states being mixtures of them. For classical systems the MM is always approximately a simplex of m disjoint EMSs [27] with probabilities representing the classical degrees of freedom. (ii) When $\dim(\mathcal{K}_l) > 1$ and $\dim(\mathcal{H}_l) = 1$, $\mathcal{K}_l$ is a DFS protected from the noise. This is the case in example II where the MM is a qubit. (iii) When $\dim(\mathcal{K}_l) > 1$ and $\dim(\mathcal{H}_l) > 1$, $\mathcal{K}_l$ is also protected from noise and termed a NSS. The structures (ii) and (iii) correspond to quantum degrees of freedom (ω_l) and do not appear in the case of classical dynamics [27]. In general the number of blocks in Eq. (6) is $m' \le m$, with equality occurring only when there are no DFSs or NSSs.

Effective motion in the metastable manifold.—In the metastable regime, $\tau'' \ll t \ll \tau'$, the metastable states appear stationary, or perhaps rotate within the MM. This latter case corresponds to either (i) coherent motion in the DFSs or NSSs where the matrices ω_l of Eq. (6) evolve unitarily in time, or (ii) classical rotations with a frequency that is limited by the dimensionality of the MM [51]. For class A systems only case (i) is possible [38,46,47].

For longer times, $t \gtrsim \tau'$, the MM contracts exponentially towards ρ_{SS}. This is illustrated in Fig. 2(c) for example II. This low dimensional evolution in the MM is well described by an effective generator $\mathcal{L}_{\text{eff}} \coloneqq [\mathcal{L}]_{\mathcal{P}}$, which can be considered as the generator of the dynamics averaged over intervals τ''. If the MM is approximately a simplex (i.e., containing no DFSs or NSSs) the motion generated by $\mathcal{L}_{\text{eff}}$ is that of classical transitions between macrostates described by the EMSs (see Ref. [38] for $m = 2$ and Ref. [52] for the general case). For class A when the MM contains coherent subsystems or subspaces, the motion preserves the structure of Eq. (6) and can be shown to be trace preserving and approximately completely positive [38,53,54]. Note that the decoupling of the (slower) classical dynamics from the (faster) quantum evolution in the MM requires further separation in low-lying eigenvalues of $\mathcal{L}$. This is illustrated in Fig. 2(c) for example II.

In practice, metastability can be accessed through the connected autocorrelation [14] of the measurement $\mathcal{M}$ of a system observable, even in the stationary state, $C(t) \coloneqq \mathrm{Tr}(\mathcal{M}e^{t\mathcal{L}}\mathcal{M}\rho_{SS}) - \mathrm{Tr}(\mathcal{M}\rho_{SS})^2$ [55]; see Figs. 1(e) and 2(d). The first measurement $\mathcal{M}$ perturbs ρ_{SS}, and the state conditioned on the result partially relaxes towards the MM for $t \lesssim \tau''$. In the metastable regime correlations will persist as the different blocks in Eq. (6) do not communicate, and for the case where all low-lying eigenvalues are real, $C(t) \approx \mathrm{Tr}(\mathcal{M}\mathcal{P}\mathcal{M}\rho_{SS}) - \mathrm{Tr}(\mathcal{M}\rho_{SS})^2$. When the low-lying eigenvalues are complex, oscillations of $C(t)$ can occur in the metastable regime, as in Fig. 2(d). When $t \gtrsim \tau'$, the dynamics begins to relax back towards ρ_{SS}, erasing all information about the initial result $[C(t) \approx 0]$ for $t \gg \tau$.

Outlook.—The next steps in the development of the theory of quantum metastability presented here include the following. (i) For many-body systems, where the direct diagonalization of $\mathcal{L}$ is impractical, it should be possible to use dynamical large-deviation methods [56] to identify dynamically the different blocks in Eq. (6) by biasing ensembles of quantum trajectories [44]. This approach could be implemented numerically by generalizing classical path sampling [57] and/or cloning techniques [58]. (ii) In order to reveal the structure of the MM, one needs to find a general computational scheme that can identify the basis in which metastable states look explicitly as in Eq. (6). Such a method would be useful to uncover DFSs and NSSs more generally. Also, it would be interesting to consider more broadly DFSs that do not arise as a consequence of symmetry, cf. example II above. (iii) We have considered here metastability in the case of Markovian dynamics generated by a Lindbladian $\mathcal{L}$. Metastability occurs also when the dynamics is non-Markovian, see, e.g., Ref. [59]. It should be possible to generalize the method introduced above to the non-Markovian case of a time-dependent generator $\mathcal{L}(t)$. (iv) A significant challenge is to extend the ideas presented here to study metastability in closed quantum systems. This would be relevant to the fundamental problems of thermalization [5] and many-body localization [8].

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[34] The calligraphic font denotes superoperators, such as the generator $\mathcal{L}$, while the Roman font denotes normal operators, such as the Hamiltonian H or the jump operators J_i.
[35] R_k and L_k are the right and left eigenmatrices of $\mathcal{L}$ for eigenvalue λ_k, i.e., $\mathcal{L}(R_k) = \lambda_k R_k$ and $\mathcal{L}^\dagger(L_k) = \lambda_k L_k$. In principle $R_k \neq L_k^\dagger$ since in general $\mathcal{L} \neq \mathcal{L}^\dagger$. The left and right eigenmatrices form a complete basis, which we normalize as $\mathrm{Tr}(L_k R_{k'}) = \delta_{k,k'}$. We assume there are no Jordan blocks in the part of the spectrum relevant for our analysis; see, e.g., Ref. [27].
[36] The norm $\|\cdot\|$ of a superoperator $\mathcal{S}$ is the norm induced by the trace norm $\|A\| \coloneqq \mathrm{Tr}\sqrt{A^\dagger A}$ of complex matrices A on which $\mathcal{S}$ acts: $\|\mathcal{S}\| \coloneqq \sup_{\|A\|=1} \mathrm{Tr}\|\mathcal{S}A\|$.
[37] For real eigenvalues, R_k and L_k can be chosen Hermitian. Note that while $R_1 = \rho_{ss}$, $R_{k>1}$ are not positive. Complex eigenvalues come in conjugate pairs, $\lambda_{k,1} = \lambda_{k,2}^*$ and if so we have $R_{k,1} = R_{k,2}^\dagger$, $L_{k,1} = L_{k,2}^\dagger$.
[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.116.240404, for derivations of (i) case $m = 2$: approximate disjointness of two EMSs and the effective classical dynamics. (ii) class A systems: the structure of the metastable manifold from Eq. (6), the metastable regime and the effective dynamics. (iii) class B systems: a comment on the conjecture, which includes Refs. [39–43].
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[51] This latter classical motion was not considered in Ref. [27], as the low-lying eigenvalues of $\mathcal{L}$ were assumed all real. One can show [52] that the set of $(c_2{}', \ldots, c_m{}')$ is again a simplex. Points within the simplex evolve within it. Because of exponential shrinking, this evolution can allow for rotations, but with limited frequencies that decrease with decreasing gap.

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