

## Energy Exchange in Driven Open Quantum Systems at Strong Coupling

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The time-dependent energy transfer in a driven quantum system strongly coupled to a heat bath is studied within an influence functional approach. Exact formal expressions for the statistics of energy dissipation into the different channels are derived. The general method is applied to the driven dissipative two-state system. It is shown that the energy flows obey a balance relation, and that, for strong coupling, the interaction may constitute the major dissipative channel. Results in analytic form are presented for the particular value  $K = \frac{1}{2}$  of strong Ohmic dissipation. The energy flows show interesting behaviors including driving-induced coherences and quantum stochastic resonances. It is found that the general characteristics persists for  $K$  near  $\frac{1}{2}$ .

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*Introduction.*—A deep understanding and precise control of energy exchange in strongly coupled systems at the quantum level may have profound impact both from a fundamental and a practical point of view. On one side, it can advance the formulation of a consistent nonequilibrium thermodynamics for strong coupling. In fact, the question of how to conceive a consistent thermodynamics for nonequilibrium scenarios with strong system-reservoir coupling has attracted recent interest [1–5], and the issue is currently a subject of controversy. Substantial progress has recently been made for classical systems [6]. On the applicative side, it can have great impact on the active field of quantum optimal control with pioneering applications in biology and information technology [7–9].

The natural setting to study energy exchange and dissipation is to consider an externally driven quantum system coupled to one or several heat reservoirs [10–15]. This might also serve as a building block in the study of more complex systems. Usually, the system-reservoir entity is studied in the weak-coupling limit. This is technically advantageous, since perturbative methods can be used. The weak coupling assumption is well justified in the theoretical analysis of experiments in which one would like to have long coherence times. There are, however, many physical systems for which the interaction energy is comparable with the system energy. Among all, the most striking ones are, e.g., a biomolecule in a solvent and photosynthetic systems, in which energy is absorbed and efficiently transported over long distances. It is now well established that the vast efficiency of this process cannot be explained with concepts of classical charge transport. Rather it exhibits not only quantum features [16–18] but strong interaction with a structured non-Markovian reservoir seems to be crucial for the high transport efficiency [9,19–21].

With such motivations, it is natural to ask if similar spectacular effects may arise in the energy transfer characteristics of small and well-controlled quantum systems,

when they are externally driven and strongly coupled to a heat reservoir. The time-dependent force is continuously pumping energy into the system, and thus drives it out of equilibrium [22,23]. Coincidentally, the strong coupling with the environment makes the dynamics more intricate, since the system can dissipate energy through different channels. The convoluted interplay between all these competitive contributions and the identification of the dissipative channels involved is the subject of this Letter.

First, we set the stage by facing the problem from a very general perspective. To this end we establish a functional integral approach for the characteristic function of the energy transfer statistics, in which the dissipative channels to the reservoir and to the interaction are identified. Then we apply this method to the versatile spin-boson system. We establish the energy balance relation and analyze the inherent dissipative channels. Results in analytic form are presented for the particular value  $K = \frac{1}{2}$  of strong Ohmic dissipation [24–27]. It is shown that the system-reservoir coupling can be the dominant dissipative channel in particular regions of the parameter space. It is found that the drive-induced coherences and quantum stochastic resonance features of the model [28–31] are reflected in the energy transfer characteristics. Finally, it is shown that the characteristic features also hold for  $K$  near  $\frac{1}{2}$ .

*General approach to energy exchange.*—The Hamiltonian of the system plus reservoir is  $H(t) = H_S(t) + H_R + H_I$  with the system part  $H_S(t) = H_0 + V(q, t)$ , the reservoir part  $H_R = \sum_\alpha [(p_\alpha^2/2m_\alpha) + \frac{1}{2}m_\alpha\omega_\alpha^2x_\alpha^2]$ , and the translational-invariant coupling  $H_I = -q\sum_\alpha c_\alpha x_\alpha + \frac{1}{2}\sum_\alpha (c_\alpha^2/m_\alpha\omega_\alpha^2)$  [25,27,32,33]. The driving is carried by the time-dependent potential  $V(q, t)$ . All effects of the reservoir coupling on the system are captured by the spectral density  $J(\omega) = (\pi/2)\sum_\alpha (c_\alpha^2/m_\alpha\omega_\alpha^2) \times \delta(\omega - \omega_\alpha)$ .

We are interested in the energy flow of the driven system coupled strongly to the heat bath. In the usually studied

weak coupling limit, it is assumed that all the energy is dissipated into the reservoir [12,13,34–38]. In this case, the dissipated energy is determined by a double projective measurement of the reservoir energy at the beginning and at the end of the evolution.

If the weak coupling limit is not met, the system-bath interaction represents an additional dissipative channel. In the sequel, we present an approach which allows us to analyze the time dependence of the various energy transfer contributions.

In a complete description of the energy transfer, knowledge of both the variations of the energy dissipated into the reservoir,  $\langle \mathcal{E}_R(t) \rangle \equiv \langle H_R(t) \rangle - \langle H_R(0) \rangle$ , and into the reservoir plus interaction,  $\langle \mathcal{E}_{RI}(t) \rangle \equiv \langle H_R(t) \rangle + \langle H_I(t) \rangle - \langle H_R(0) \rangle - \langle H_I(0) \rangle$ , is essential. Here the average of the observable  $O$  is defined as  $\langle O(t) \rangle = \text{Tr}[O\rho(t)]$ , where  $\rho(t)$  is the density operator. Moreover the variation of energy related to the interaction can be inferred simply by the difference  $\langle \mathcal{E}_I(t) \rangle \equiv \langle \mathcal{E}_{RI}(t) \rangle - \langle \mathcal{E}_R(t) \rangle$ . It is worth underlining that these observables can be related to specific measurement protocols. Assume that at time  $t \leq 0$  the system and the bath are decoupled and the system dwells in a diagonal state of the reduced density matrix (RDM) with probability distribution  $p_S(q(0))$ . Initially we have  $\langle q(0)|\rho(0)|q(0) \rangle = p_S(q(0))e^{-\beta H_R}/\text{Tr}[e^{-\beta H_R}]$ , where  $\beta = 1/T$  is the inverse temperature (throughout we put  $k_B = \hbar = 1$ ). Immediately before both the coupling with the reservoir and driving force are switched on, at time  $t = 0$ , the bath energy is measured. At a later time  $t$ , we either switch off the coupling and measure the bath energy again, or we coevally measure the energy of the bath and the coupling. These measurements may be projective [36,39,40] or performed with a full quantum detector [41]. The moment generating function (MGF)  $G_\lambda(\nu, t)$  embodies the entire statistics of these measurement protocols. Importantly, it can be written as the trace of a generalized density operator [38,39],

$$G_\lambda(\nu, t) = \text{Tr}[e^{i\nu(H_R + \lambda H_I)} U_{t,0} e^{-i\nu H_R} \rho(0) U_{t,0}^\dagger]. \quad (1)$$

Here the operator  $U_{t,0}$  implements the unitary time evolution of the composite system. Given the MGF  $G_\lambda(\nu, t)$ , the probability distribution  $P(\mathcal{E}_\lambda, t)$  for the energy transfer amount  $\mathcal{E}_\lambda$  is  $P(\mathcal{E}_\lambda, t) = \int d\nu G_\lambda(\nu, t) e^{-i\nu \mathcal{E}_\lambda}$ . The  $n$ th derivative of the MGF taken at  $\nu = 0$  yields the  $n$ th moment of the energy,  $\langle \mathcal{E}_\lambda^{(n)}(t) \rangle = (-i)^n d^n G_\lambda(\nu, t) / d\nu^n |_{\nu=0}$ . Here we focus on the first moment, i.e., the energy transferred on average,  $\langle \mathcal{E}_\lambda(t) \rangle = \langle \mathcal{E}_\lambda^{(1)}(t) \rangle$ . Finally, the control parameter  $\lambda$  serves to treat both protocols on equal footing. For  $\lambda = 0$ , we meet the reservoir measurement  $\langle \mathcal{E}_R(t) \rangle$  [36,39]. For  $\lambda = 1$ , we are actually probing  $\langle \mathcal{E}_{RI}(t) \rangle$ .

The expression Eq. (1) can be conveniently processed using a functional integral approach [27,33]. This procedure is a generalization of the one exposed in Ref. [36] and the details can be found in the Supplemental Material [42]. We readily get for  $\langle \mathcal{E}_\lambda(t) \rangle$  the path sum representation in terms of the quasiclassical path  $\eta(\tau) = [q(\tau) + q'(\tau)]/q_0$

and fluctuation path  $\xi(\tau) = [q(\tau) - q'(\tau)]/q_0$ , where  $q_0$  is a length unit, as

$$\langle \mathcal{E}_\lambda(t) \rangle = \int d\eta_i p_S(\eta_i) \int d\eta_f \times \int D\eta \int D\xi e^{iS_S[\eta, \xi]} \mathcal{F}_{\text{FV}}[\eta, \xi] \varepsilon_\lambda[\eta, \xi], \quad (2)$$

with  $\eta_i = \eta(0)$  and  $\eta_f = \eta(t)$ . Here  $\mathcal{F}_{\text{FV}}[\eta, \xi]$  is the standard Feynman-Vernon influence functional accounting for friction and quantum thermal noise, and  $S_S[\eta, \xi] = S_S[q] - S_S[q']$  is the system's action related to the double path  $\{\eta, \xi\}$ . The energy functional is found to read

$$\begin{aligned} \varepsilon_\lambda[\eta, \xi] = & -\frac{1}{2} \int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1 \dot{\eta}(\tau_2) \\ & \times [W_1(\tau_2 - \tau_1) \dot{\xi}(\tau_1) + W_2(\tau_2 - \tau_1) \dot{\eta}(\tau_1)] \\ & + \int_0^t d\tau_1 \{-\dot{\eta}(\tau_1) W_2(\tau_1) \eta_i \\ & + (1 - \lambda) \eta_f [W_1(t - \tau_1) \dot{\xi}(\tau_1) + W_2(t - \tau_1) \dot{\eta}(\tau_1)]\}. \end{aligned} \quad (3)$$

Here  $W_1(\tau) = \dot{W}(\tau)$ ,  $W_2(\tau) = \dot{W}''(0) - \dot{W}''(\tau)$ , in which  $W(\tau) = W'(\tau) + iW''(\tau)$  is the bath correlation function.

The expression Eq. (2), with Eq. (3), is generally valid for any composite quantum system with bilinear system-bath coupling. It may serve as a starting point for the implementation of powerful numerical schemes [43–47].

*Spin-boson model.*—To explore the powerfulness of our approach, consider the dissipative two-state system with two localized states at a distance  $q_0$ . In spin representation, the system Hamiltonian reads [25,27]

$$H_S(t) = -\frac{1}{2} \Delta \sigma_x - \frac{1}{2} \epsilon(t) \sigma_z, \quad (4)$$

where  $\Delta$  represents the transfer amplitude and  $\epsilon(t)$  the time-dependent bias. The path sum in Eq. (2) can be written in terms of all possible sequences of visiting the diagonal states (sojourns)  $\eta_j = \pm 1$  and off-diagonal states (blips)  $\xi_j = \pm 1$  of the RDM [25,27].

$$\begin{aligned} \langle \mathcal{E}_\lambda(t) \rangle = & \sum_{m=1}^{\infty} \left( \frac{-\Delta^2}{4} \right)^m \int_0^t \mathcal{D}_{2m} \{t_j\} \\ & \times \sum_{\{\xi_j = \pm 1\}} B_m \sum_{\{\eta_j = \pm 1\}'} \mathcal{F}_m \mathfrak{G}_m(\lambda). \end{aligned} \quad (5)$$

Here,  $\int_0^t \mathcal{D}_{2m} \{t_j\}$  denotes time-ordered integrations of the  $2m$  flip times  $t_j$ . The sums  $\sum_{\{\xi_j = \pm 1\}}$  and  $\sum_{\{\eta_j = \pm 1\}'}$  account for all possible sequences of the  $m$  off-diagonal and  $m - 1$  internal diagonal states of the RDM, respectively. For a given  $\{\xi, \eta\}$  configuration of the  $2m$  flips, the factor  $B_m$  represents the bias weight, and the influence factor  $\mathcal{F}_m$  administers quantum noise and friction (see Supplemental Material [42]). For simplicity, we have implied the initial state  $p_S(\eta_i) = \delta_{\eta_i, 1}$ .

The terms governing the energy transfer to the reservoir plus interaction, the reservoir, and the interaction alone are  $\mathfrak{G}_{\text{RI},m} \equiv \mathfrak{G}_{\lambda=1,m}$ ,  $\mathfrak{G}_{\text{R},m} \equiv \mathfrak{G}_{\lambda=0,m}$ , and  $\mathfrak{G}_{\text{I},m} = \mathfrak{G}_{\text{RI},m} - \mathfrak{G}_{\text{R},m}$ . We readily get

$$\begin{aligned}\mathfrak{G}_{\text{I},m} &= \sum_{k=0}^{m-1} \mathcal{U}_k(t) \eta_k + i \sum_{j=1}^m \mathcal{V}_j(t) \xi_j, \\ \mathfrak{G}_{\text{RI},m} &= \sum_{k=0}^{m-1} \mathcal{U}_k(t_{2m}) \eta_k + i \sum_{j=1}^m \mathcal{V}_j(t_{2m}) \xi_j,\end{aligned}\quad (6)$$

where  $\mathcal{U}_k(\tau) = \dot{W}'(\tau - t_{2k}) - \dot{W}'(\tau - t_{2k+1})$ , and where  $\mathcal{V}_j(\tau) = \dot{W}'(\tau - t_{2j}) - \dot{W}'(\tau - t_{2j-1})$ .

Interestingly, the expression Eq. (5) for  $\langle \mathcal{E}_{\text{RI}}(t) \rangle$  can be converted, upon integration by parts, into an energy balance relation. Together with the series expressions analogous to Eq. (5) for the spin functions  $\langle \sigma_x(t) \rangle$  and  $\langle \sigma_z(t) \rangle$  [27], we get

$$\langle \mathcal{E}_{\text{RI}}(t) \rangle \equiv \langle \mathcal{E}_{\text{R}}(t) \rangle + \langle \mathcal{E}_{\text{I}}(t) \rangle = -\langle \mathcal{E}_{\text{S}}(t) \rangle + \langle \mathcal{E}_{\text{exc}}(t) \rangle. \quad (7)$$

Here,  $\langle \mathcal{E}_{\text{S}}(t) \rangle = -\frac{1}{2} \Delta \langle \sigma_x(t) \rangle - \frac{1}{2} [\epsilon(t) \langle \sigma_z(t) \rangle - \epsilon(0)]$  is the mean energy transferred to the system, and the contribution  $\langle \mathcal{E}_{\text{exc}}(t) \rangle = -\frac{1}{2} \int_0^t dt' \dot{\epsilon}(t') \langle \sigma_z(t') \rangle$  is the excess energy pumped into the composite system by the work spent by the driving. These results are valid for general linear dissipation.

Next we turn to Ohmic coupling  $q_0^2 J(\omega)/\pi = 2K\omega e^{-\omega/\omega_c}$ . Here,  $K$  is a dimensionless friction strength, and  $\omega_c$  a cutoff. The Ohmic bath correlation function for  $\omega_c \tau \gg 1$  reads [27]

$$W(\tau) = 2K \ln [(\beta\omega_c/\pi) \sinh(\pi|\tau|/\beta)] + i\pi K \text{sgn}(\tau). \quad (8)$$

With this,  $\Delta^2$  is combined with  $\omega_c$  in the form  $\Delta^2 e^{-W(\tau)}$  as  $\Delta_r^{2-2K} = \Delta^2/\omega_c^{2K}$ . The scaling limit is  $\omega_c \rightarrow \infty$  with  $\Delta_r$  fixed.

For general  $K$ , the series for the individual terms of the relation Eq. (7) cannot be summed in analytic form. However, if  $K$  is near  $\frac{1}{2}$ , say  $0.3 \lesssim K \lesssim 0.7$ , the path sums are feasible using techniques reported in Refs. [26,27,48]. Following these lines,  $\langle \sigma_{x/z}(t) \rangle$  can be written in closed form as

$$\begin{aligned}\langle \sigma_z(t) \rangle &= F_{\text{S}}(t) + \int_0^t d\tau R(\tau) F_{\text{B}}(\tau) \int_0^{t-\tau} ds \\ &\quad \times F_{\text{S}}(s) \sin[\varphi(t-s, t-s-\tau)], \\ \langle \sigma_x(t) \rangle &= \frac{1}{\Delta} \int_{1/\omega_c}^t d\tau R(\tau) F_{\text{B}}(\tau) \cos[\varphi(t, t-\tau)],\end{aligned}\quad (9)$$

where  $R(\tau) = \Delta^2 \sin(\pi K) e^{-W(\tau)}$ , and  $\varphi(t_2, t_1) = \int_{t_1}^{t_2} d\tau \epsilon(\tau)$  is the bias phase accumulated between  $t_1$  and  $t_2$ . The form factors  $F_{\text{B}}(\tau)$  and  $F_{\text{S}}(s)$  dress the intervals  $\tau$  and  $s$ , in which the system dwells in an off-diagonal and diagonal state of the RDM, respectively. They obey  $F_{\text{B}}(0) = F_{\text{S}}(0) = 1$  and drop to zero on the time scale set by the inverse of the Kondo frequency, which is  $\Delta_K = [\Gamma(1-K)/2^K]^{1/(1-K)} \Delta_r$  for  $K$  near  $\frac{1}{2}$ .

In the same way the series for  $\langle \mathcal{E}_{\text{I}}(t) \rangle$  is summed to

$$\begin{aligned}\langle \mathcal{E}_{\text{I}}(t) \rangle &= \frac{1}{2} \int_0^t d\tau R(\tau) F_{\text{B}}(\tau) \int_{1/\omega_c}^{t-\tau} ds F_{\text{S}}(s) \\ &\quad \times \cos[\varphi(t-s, t-s-\tau)] [\dot{W}'(s) - \dot{W}'(\tau+s)].\end{aligned}\quad (10)$$

Together with the expressions for  $\langle \mathcal{E}_{\text{S}}(t) \rangle$  and  $\langle \mathcal{E}_{\text{exc}}(t) \rangle$  obtained with Eq. (9), all constituents of the relation Eq. (7) are given.

For harmonic driving,  $\epsilon(t) = \epsilon_0 + \epsilon_1 \cos(\omega t)$ , the initially transient dynamics vanishes on the time scale set by the system's relevant relaxation time  $\tau_0 = \min(1/\Delta_K, \beta/\pi)$ . In the subsequent stationary regime, the energy transfer contribution  $\langle \mathcal{E}_j(t) \rangle$ , where  $j = S, R, I$ , exc, is found, upon Fourier expansion of the respective bias factor, to read

$$\langle \mathcal{E}_j(t) \rangle = P_j t + \sum_m E_{j,m} e^{-im\omega t}. \quad (11)$$

The first term represents the energy draining into channel  $j$  at constant power  $P_j$  in the period  $t$ . In particular, we obtain

$$\begin{aligned}P_S &= P_I = 0, \\ P_R &= P_{\text{exc}} = \epsilon_1 \frac{\omega}{2} \int_0^\infty d\tau R(\tau) F_{\text{B}}(\tau) \cos(\epsilon_0 \tau) J_1[z(\tau)] \\ &\quad \times \int_0^\infty ds F_{\text{S}}(s) \sin \left[ \omega \left( \frac{\tau}{2} + s \right) \right],\end{aligned}\quad (12)$$

where  $z(\tau) = 2\epsilon_1 \sin(\omega\tau/2)/\omega$ , and  $J_1(x)$  is a Bessel function. The term  $P_{\text{exc}}$  is the constant part of the power injected into the composite system by the drive in the stationary state. Evidently, this amount is fully absorbed by the reservoir.

With the results [Eq. (12)] one directly sees from Eq. (11), that in the long run the reservoir contribution  $\langle \mathcal{E}_{\text{R}}(t) \rangle$  dominates over the interaction one, and lastly the energy is predominantly dissipated in the environment. Nevertheless, there are intermediate stationary time regimes in which the reservoir and interaction contributions are of the same order of magnitude.

Let us now look more closely at the particular case  $K = \frac{1}{2}$ , in which the form factors can be calculated exactly in analytic form, yielding  $F_{\text{B}}(\tau) = e^{-\gamma\tau/2}$  and  $F_{\text{S}}(s) = e^{-\gamma s}$  with the Kondo frequency  $\gamma \equiv \Delta_K (K = \frac{1}{2}) = \frac{1}{2} \pi \Delta^2 / \omega_c$  [26,27].

Consider the energy flow until time  $t$ . Taking in Eq. (11) the average over period  $2\pi/\omega$ , we get with Eq. (7)  $\bar{E}_{\text{I}}(t) = E_{\text{I},0}$  and  $\bar{E}_{\text{R}}(t) = P_R t - E_{\text{I},0} - E_{\text{S},0} + E_{\text{exc},0}$ . In practice, this corresponds to the case, in which the oscillating terms are averaged out during the measurement, i.e., by using a detector unable to resolve energy variation within period  $2\pi/\omega$ . In the scaling regime, the leading asymptotic behaviors are  $E_{\text{I},0} = (\gamma/2\pi) [\log(\omega_c/\gamma)]^2$  and  $E_{\text{S},0} = -(\gamma/2) \ln(\omega_c/\gamma)$ , while  $E_{\text{exc},0}$  does not depend on  $\omega_c$ . Hence, we have in the scaling regime  $E_{\text{I},0} \gg |E_{\text{S},0}| \gg E_{\text{exc},0}$ , and thus  $\bar{E}_{\text{R}}(t) \simeq P_R t - E_{\text{I},0}$ .

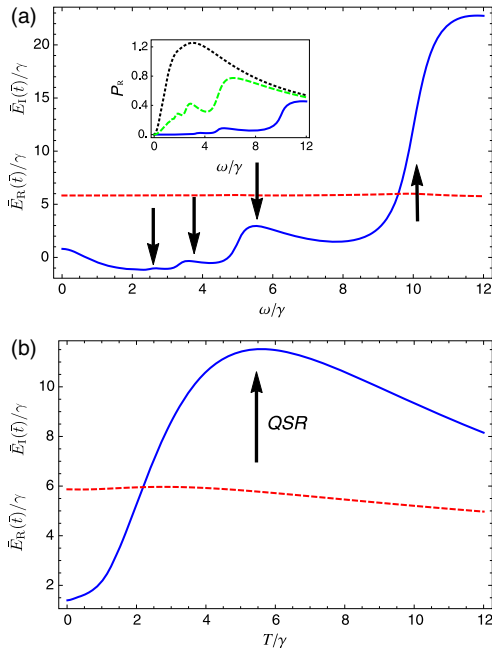


FIG. 1. A snapshot of  $\bar{E}_R(\bar{t})$  at time  $\bar{t} \equiv 50/\gamma$  (solid curve) and  $\bar{E}_I(\bar{t})$  (dashed curve) are plotted versus frequency  $\omega$  in (a) and versus temperature  $T$  in (b). The parameters are  $\epsilon_0 = 10\gamma$ ,  $\epsilon_1 = 5\gamma$ ,  $\omega_c = 5000\gamma$ , plus  $T = 0.1\gamma$  in (a), and  $\omega = 5\gamma$  in (b). In the inset the constant-in-time contribution to the power  $P_R$  versus  $\omega$  is plotted for  $\epsilon_0 = 2\epsilon_1$  (solid blue line),  $\epsilon_0 = \epsilon_1$  (dashed green line), and  $\epsilon_0 = 0.4\epsilon_1$  (dotted black line). The arrows denote the positions of the ground and side frequencies in (a) and the quantum stochastic resonance in (b).

To understand how the energy is dissipated into the various channels, we plot in Fig. 1 a snapshot of  $\bar{E}_R(\bar{t})$  and  $\bar{E}_I(\bar{t})$  at time  $\bar{t} \equiv 50/\gamma$  as functions of frequency  $\omega$  and of temperature  $T$ . The behavior of the reservoir channel is particularly interesting as it shows a sequence of plateaus combined with sudden ascents around  $\omega = \epsilon_0/n$ ,  $n = 1, 2, 3, 4$ . This resembles the driving-induced coherences (DICs) and resonances inherent in strongly coupled system-reservoir entities [28–31]. The resonances fade away into the continuum when  $\epsilon_0 < \epsilon_1$  is small [see inset in Fig. 1(a)], or when the temperature is increased.

The behavior of  $\bar{E}_R(\bar{t})$  reflects the one of the power  $P_R$ , as depicted in the inset of Fig. 1(a). The solid curve, reporting  $\bar{E}_R(\bar{t})$ , is effectively an amplification of the solid curve in the inset, showing  $P_R$ , by a factor  $50/\gamma$  with a displacement along the ordinate by roughly  $-E_{I,0}$ . We see that in a region around  $\omega = 2\gamma$  there holds  $\bar{E}_R(\bar{t}) < 0$ ; i.e., the energy  $E_{I,0}$  drained away into the interaction still outstrips the amount  $P_R\bar{t}$ . Since  $\bar{E}_R(\bar{t})$  depends strongly on  $\omega$  and  $\epsilon_0$ , and  $\bar{E}_I(\bar{t})$  is virtually constant, the case, in which the energy dissipated into the bath is small or large, can easily be adjusted by tuning these external parameters.

The energies  $\bar{E}_R(\bar{t})$  and  $\bar{E}_I(\bar{t})$  are depicted versus temperature in Fig. 1(b). While the latter depends weakly on  $T$ , the energy deposited in the bath has a peaked structure signifying that the respective energy is amplified

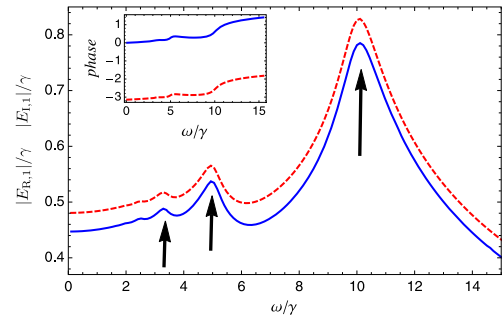


FIG. 2. The absolute values of the amplitudes of the first harmonic  $E_{R,1}$  (solid curve) and  $E_{I,1}$  (dashed curve) are plotted versus driving frequency  $\omega$ . The parameters are the same as in Fig. 1(a). Both curves show resonances at  $\epsilon_0/n$  (indicated by the arrows). Inset: phases of  $E_{R,1}$  (solid curve) and  $E_{I,1}$  (dashed curve) versus  $\omega$ . The two contributions are always in antiphase.

because of the noise. This reflects the quantum stochastic resonance characteristics of the driven damped quantum system [28]. Side resonances are absent because of the relatively high temperatures. At low  $T$ , the curve for  $\bar{E}_R(\bar{t})$  falls again below  $\bar{E}_I(\bar{t})$ .

Finally, we turn to the oscillatory part in Eq. (11). The individual terms of the  $m$ th component represent the oscillatory energy exchange between the four entries of the balance relation Eq. (7) at frequency  $m\omega$ . These features can be observed with a detector having a time resolution smaller than  $2\pi/\omega$ . The amplitudes of the  $m = \pm 1$  components are the largest ones. The absolute values of the amplitudes  $E_{R,1}$  and  $E_{I,1}$  are depicted versus  $\omega$  for a particular parameter set in Fig. 2. The amplitudes show distinct resonances at  $\omega = \epsilon_0/n$  with  $n = 1, 2$ , and  $3$  which are again related to the DIC. Since the phases of the amplitudes differ by nearly  $\pi$ , as shown in the inset of Fig. 2, and the absolute values are close, the major energy transfer is between the bath and the interaction. The surplus of energy put in and out of the reservoir oscillates with amplitude  $E_{\text{exc},1} - E_{S,1}$ , and the absolute value of it is just the difference of the two curves in Fig. 2. This is small since the leading contributions to  $E_{\text{exc},1}$  and  $E_{S,1}$  cancel each other.

For completeness, consider the asymptotic behaviors of the various energy contributions in the scaling regime for  $K \neq \frac{1}{2}$ . For  $K < \frac{1}{2}$ , we obtain from Eq. (10)  $E_{I,0} \propto \ln(\omega_c/\Delta_r)$ , while  $\bar{E}_{S,0}$  and  $E_{\text{exc},0}$  are independent of  $\omega_c$ . For  $K > \frac{1}{2}$ , we get from Eq. (9)  $E_{S,0} = -c_{S,0}\Delta_r(\omega_c/\Delta_r)^{2K-1}$ , and from Eq. (10) the same functional form,  $E_{I,0} = c_{I,0}\Delta_r(\omega_c/\Delta_r)^{2K-1}$  with the ratio  $c_{I,0}/c_{S,0} = 2\pi K/\sin[2\pi(K - \frac{1}{2})]$ . Interestingly, this term is very large for  $K$  slightly above  $\frac{1}{2}$ . Thus for  $K$  around  $\frac{1}{2}$ , we generally have  $E_{I,0} \gg |E_{S,0}|, E_{\text{exc},0}$ . Hence the characteristic behaviors depicted in Fig. 1 qualitatively hold for  $K$  near  $\frac{1}{2}$ . However, the time at which the energy drained into the bath exceeds the energy  $E_{I,0}$  drained into the interaction may sensitively depend on whether  $K < \frac{1}{2}$  or  $K > \frac{1}{2}$ . Finally, since all the oscillatory terms in Eq. (11) are independent of  $\omega_c$ , the coefficients  $E_{j,m}$ , where  $j = S, R, I$ , and exc, smoothly vary with  $K$  near  $K = \frac{1}{2}$ .

*Conclusions.*—We presented a general method to analyze the time-resolved energy transfer to the various dissipative channels of a driven open quantum system strongly coupled to a heat bath. The exact formal solution in path sum representation was given in Eqs. (2) and (3). It may form a firm basis for the implementation of efficient numerical tools [43–47]. The method was applied to the spin-boson model, with explicit results in the Ohmic scaling limit at strong coupling, for the special case of  $K = \frac{1}{2}$ . We showed that the interaction channel is a relevant dissipative drain which even may dominate in particular regimes of the parameters. For harmonic driving, the energy flowing into the reservoir shows distinct resonant behavior, thereby reflecting quantum stochastic resonance features of the model [28,29]. The findings may open new directions in the study of energy transfer in complex quantum systems, and pave the way to control energy dissipation into the reservoir by tuning the system's parameters.

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