Speeding up Adiabatic Quantum State Transfer by Using Dressed States

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We develop new pulse schemes to significantly speed up adiabatic state transfer protocols. Our general strategy involves adding corrections to an initial control Hamiltonian that harness nonadiabatic transitions. These corrections define a set of dressed states that the system follows exactly during the state transfer. We apply this approach to stimulated Raman adiabatic passage protocols and show that a suitable choice of dressed states allows one to design fast protocols that do not require additional couplings, while simultaneously minimizing the occupancy of the "intermediate" level.

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Introduction.—The general goal of moving quantum states between two different systems finds numerous applications in quantum information processing [1,2]. It has generated intense theoretical interest, with numerous approaches developed to allow high fidelity state transfer that are robust against dissipation and noise. Among the more powerful and interesting strategies are adiabatic transfer protocols [3]. These generically involve adiabatically evolving an eigenstate of a composite quantum system, such that the state is initially localized in the "source" system and ends up being localized in the "target" system [see Fig. 1(a)]. The adiabatic evolution thus corresponds to a state transfer, with the initial state of the source system "riding" the adiabatic eigenstates, and ending up in the target system. The most famous examples of such approaches are the stimulated Raman adiabatic passage (STIRAP) [4] and coherent tunnelling by adiabatic passage [5] protocols, well known in atomic physics.

There are two main advantages in using transfer protocols based on adiabatic passage instead of resonant techniques. First, adiabatic passage is inherently more robust against pulse area and timing errors. Second, it is useful in situations where the source and target only interact via a lossy "intermediate" system, as it allows one to use the mediated coupling without being harmed by the noise. This is of particular relevance in optomechanical state transfer schemes, where a dissipative mechanical resonator is the intermediate system [6–9].

Despite these advantages, adiabatic schemes are necessarily slow, and hence can suffer from dissipation and noise in the target and/or source system. Therefore, several approaches have been put forward to speed up adiabatic passage [10,11]. Among the known methods, counterdiabatic control [12], also referred to as transitionless driving [13], or its higher-order variants [14,15] are analytical methods that allow one to construct a modification of an original Hamiltonian to compensate for nonadiabatic errors. While in principle transitionless driving would allow a perfect state transfer, it suffers from two major flaws: it sometimes requires either a direct coupling of the source and target systems [16–19] or a coupling not available in the original Hamiltonian [20]. The higher-order variants overcome the first flaw of transitionless driving, but do not allow one to control the population in the intermediate system [14,15]. A related approach based on constructing dynamical invariants [21] has also been applied to STIRAP, but it leads to pulse schemes that either need an infinite energy gap to be perfect [22], or do not smoothly turn on or off [22,23] and are thus extremely challenging to implement experimentally. Finally, one could use the general framework of optimal quantum control [24], but as we will show there is no need to use such a complex procedure.

In this Letter, rather than constructing perfect protocols from scratch, we present an approach that corrects existing efficient adiabatic protocols such that they allow for a perfect state transfer even in the nonadiabatic regime. Moreover, the high flexibility of this approach allows one to engineer and reduce the population in the intermediate lossy level. The main idea of our approach is sketched in Fig. 1(b). We work with a basis of dressed states whose very definition incorporates the nonadiabatic processes. Then, by introducing additional control fields, we can ensure these dressed states coincide with the desired adiabatic eigenstate at the initial and final time of the

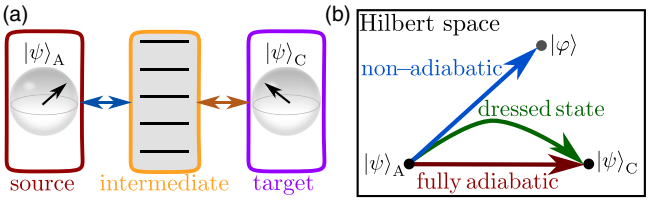

FIG. 1. (a) Schematic of a composite quantum system where the source and the target systems (qubits in this schematic) are coupled via some intermediate system. (b) Schematic of the possible evolutions: (red line) perfect adiabatic evolution; (blue line) speeding up the evolution results in nonadiabatic errors leading to an imperfect state transfer; (green line) by dressing the adiabatic eigenstates it is possible to design an evolution that leads to a perfect state transfer.

protocol. It is thus possible to do a state transfer by having the exact dynamics follow these new dressed states, even if the protocol is too fast to allow a naive adiabatic evolution. We illustrate this general idea by developing simple and effective pulses for speeding up adiabatic state transfer in generic Λ-system setups.

General problem.—We consider a general composite quantum system, composed of source, intermediate, and target subsystems, respectively labeled A, B, and C. The goal is to transfer some initial quantum state $|\psi\rangle$ (e.g., a qubit state) from subsystem A to the target subsystem C. Adiabatic transfer achieves this goal by constructing a time-dependent Hamiltonian whose instantaneous eigenstates evolve in a way that facilitates the transfer. We start by assuming that one has constructed such a protocol. The instantaneous eigenstates (hereafter referred to as adiabatic eigenstates) and corresponding adiabatic energies (both indexed by k) are defined via

$$\hat{H}(t)|\varphi_k(t)\rangle = E_k(t)|\varphi_k(t)\rangle. \tag{1}$$

A subset of eigenstates has been engineered to form a basis of the A subsystem at the initial time t_i and a basis of the target system at the final time t_f. In other words the eigenstates $\{|\varphi_{m_j}(t)\rangle\}_{j=0}^{n}$ will serve as "medium" states and have the following properties:

$$\begin{aligned} |\varphi_{m_j}(t_i)\rangle &= |\beta_j\rangle_A \otimes |\chi_i\rangle_{B,C}, \\ |\varphi_{m_j}(t_f)\rangle &= |\chi_f\rangle_{A,B} \otimes |\gamma_j\rangle_C, \end{aligned} \tag{2}$$

where $\{|\beta_j\rangle\}_{j=0}^{n}$ and $\{|\gamma_j\rangle\}_{j=0}^{n}$ span the subspaces A and C, respectively. The states $|\chi_i\rangle_{B,C}$ and $|\chi_f\rangle_{A,B}$ are not necessarily equal.

It follows that if the evolution is perfectly adiabatic (i.e., happens on a time scale $\tau \gg 1/\Delta E$, where ΔE is the smallest instantaneous energy gap of the system), the initial source state will be mapped on the final target state. However for $\tau \lesssim 1/\Delta E$, the evolution will not be perfectly adiabatic. It is convenient to move to the adiabatic frame where the adiabatic eigenstates are time independent. The relevant unitary is $\hat{U}(t) = \sum_k |\varphi_k\rangle\langle\varphi_k(t)|$. At each instant in time, $\hat{U}(t)$ maps the adiabatic eigenstate $|\varphi_k(t)\rangle$ onto the time-independent state $|\varphi_k\rangle$. In the adiabatic frame, the Hamiltonian becomes

$$\begin{aligned} \hat{H}_{\rm ad}(t) &= \hat{H}_0(t) + \hat{W}(t) \\ &= \sum_k E_k(t)|\varphi_k\rangle\langle\varphi_k| + i\frac{d\hat{U}(t)}{dt}\hat{U}^\dagger(t). \end{aligned} \tag{3}$$

The operator $\hat{W}(t)$ generically has off-diagonal matrix elements connecting the various adiabatic eigenstates. The magnitude of these matrix elements increases as τ decreases, leading to an imperfect state transfer.

Correcting nonadiabatic errors.—In order to correct the nonadiabatic errors, we look for a correction Hamiltonian $\hat{H}_c(t)$ such that the modified Hamiltonian $\hat{H}_{\rm mod}(t) = \hat{H}(t) + \hat{H}_c(t)$ leads to a perfect state transfer. For this scheme to be reasonable, we require that $\hat{H}_{\rm mod}(t)$ has no unattainably large coupling strengths and that $\hat{H}_c(t)$ does not involve couplings that cannot be experimentally implemented.

Our strategy is based on the observation that the corrected dynamics only needs to evolve the system from the correct state at t_i to the correct state at t_f [cf. Fig. 1(b)]. This suggests a strategy whose crucial ingredients are: (I) a new basis of dressed states $|\tilde{\varphi}_k(t)\rangle$ formally defined by a time-dependent unitary transformation $V(t)$ as

$$|\tilde{\varphi}_k(t)\rangle \equiv \hat{V}^\dagger(t)|\varphi_k\rangle, \tag{4}$$

and (II) a control field $\hat{H}_c(t)$ that is added to the original Hamiltonian.

The additional control Hamiltonian $\hat{H}_c(t)$ and dressed-state basis [i.e., $\hat{V}(t)$] must be chosen as to satisfy the following constraints. (i) The dressed medium states coincide with the medium states at time t_i and t_f

$$\hat{V}^\dagger(t_f)|\varphi_{m_j}\rangle = \hat{V}^\dagger(t_i)|\varphi_{m_j}\rangle = |\varphi_{m_j}\rangle. \tag{5}$$

(ii) For all j, the evolution of $|\tilde{\varphi}_{m_j}(t)\rangle$ is trivial in the basis defined by $\hat{V}(t)$ (i.e. $|\tilde{\varphi}_{m_j}(t)\rangle\langle\tilde{\varphi}_{m_j}(t)|$ is a conserved quantity).

If both these conditions are satisfied, then the perfect desired state transfer will occur. A sketch of the general idea is shown in Fig. 1(b). Condition (ii) is better defined by moving in the frame defined by $\hat{V}$ in which the Hamiltonian takes the form

$$\hat{H}_{\rm new}(t) = \hat{V}\hat{H}_{\rm ad}(t)\hat{V}^\dagger + \hat{V}\,\hat{U}\,\hat{H}_c(t)\hat{U}^\dagger\hat{V}^\dagger + i\frac{d\hat{V}}{dt}\hat{V}^\dagger. \tag{6}$$

We have omitted the explicit time dependence of $\hat{U}$ and $\hat{V}$ for clarity. In this new frame the dressed states of Eq. (4) have no intrinsic time dependence, and condition (ii) becomes

$$\langle\tilde{\varphi}_{m_j}|\hat{H}_{\rm new}|\tilde{\varphi}_k\rangle = 0 \quad \text{for } 1 \le k \le n, \quad k \ne m_j. \tag{7}$$

In other words, $\hat{H}_c(t)$ has to be designed such that it cancels the unwanted off-diagonal elements in $\hat{H}_{\rm new}(t)$.

To summarize, the general method involves picking an appropriate pair of operators $[\hat{V}(t), \hat{H}_c(t)]$: the unitary $\hat{V}(t)$ selects a (time-dependent) basis of dressed states, while the additional control Hamiltonian $\hat{H}_c(t)$ ensures the correct dynamics. The net result is that the desired state transfer dynamics occurs perfectly despite not being in the adiabatic limit.

Transitionless driving [12–14] is a special case of this approach and is retrieved by choosing $\hat{V}(t) = \hat{\mathbb{1}}$ and $\hat{H}_c = -\hat{U}^\dagger\hat{W}\,\hat{U}$. The alternative schemes described in Refs. [14,15] are also recovered from our approach, by choosing the dressed states as the superadiabatic states [25–27] (instantaneous eigenstates of $\hat{H}_{\rm ad}$) or its higher order counterparts. In what follows, we use our method to derive truly new protocols.

Application: STIRAP.—We apply our general approach to the problem of STIRAP [3,4] in a three-level Λ-type

system. For concreteness, each of the subsystems A, B, and C are qubits such that A and C only interact with B via the so-called pump and Stokes pulses (Ω_p and Ω_S, respectively). The Hamiltonian reads

$$\hat{H}(t) = \Omega_p(t)|B\rangle\langle A| + \Omega_S(t)|B\rangle\langle C| + \text{H.c.} \quad (8)$$

with $|A\rangle = |100\rangle$, $|B\rangle = |010\rangle$, $|C\rangle = |001\rangle$. The pulses are parametrized by the frequency $\Omega(t)$ and the angle $\theta(t)$

$$\Omega_p(t) = -\Omega(t)\sin\theta(t), \qquad \Omega_S(t) = \Omega(t)\cos\theta(t). \quad (9)$$

The adiabatic eigenstates (see the Supplemental Material [28]) consist of two "bright" states $|\varphi_\pm(t)\rangle$ with energy $E_\pm(t) = \pm\Omega(t)$, a "dark" state $|\varphi_D(t)\rangle$ with $E_D(t) = 0$, and $|\varphi_0(t)\rangle = |000\rangle$ with $E_0(t) = 0$. A general adiabatic state transfer from qubit A to C can be performed using the medium states

$$|\varphi_D(t)\rangle = \cos\theta(t)|A\rangle + \sin\theta(t)|C\rangle \quad (10)$$

and $|\varphi_0(t)\rangle$, which operates a state transfer from $|A\rangle$ to $|C\rangle$ by using the counterintuitive pulse sequence $\theta(t_i) = 0$ and $\theta(t_f) = \pi/2$. As mentioned before, as the protocol time is reduced, the perfect adiabatic transfer will be more and more corrupted. This is described by going in the adiabatic basis where the Hamiltonian (8) becomes

$$\hat{H}_{\text{ad}}(t) = \Omega(t)\hat{M}_z + \dot{\theta}(t)\hat{M}_y, \quad (11)$$

where $\hat{M}_z = |\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|$, $\hat{M}_x = (|\varphi_-\rangle - |\varphi_+\rangle)\langle\varphi_D|/\sqrt{2} + \text{H.c.}$, and $\hat{M}_y = i(|\varphi_+\rangle + |\varphi_-\rangle)\langle\varphi_D|/\sqrt{2} + \text{H.c.}$ are spin 1 operators, obeying the commutation relation $[M_p, M_q] = i\varepsilon^{pqr}M_r$. The second term of the adiabatic Hamiltonian (11) corresponds to the nonadiabatic couplings coming from the inertial term in Eq. (3).

Thanks to the analogy between the adiabatic Hamiltonian (11) and a spin 1 in a magnetic field, ingredient (I) (i.e., the construction of dressed states) of our approach can be parametrized as a rotation of the spin with Euler angles $\xi(t)$, $\mu(t)$, and $\eta(t)$

$$\hat{V}_g = \exp[i\eta(t)\hat{M}_z]\exp[i\mu(t)\hat{M}_x]\exp[i\xi(t)\hat{M}_z]. \quad (12)$$

In order to satisfy condition (i), the angle $\mu(t)$ has to satisfy $\mu(t_i) = \mu(t_f) = 0(2\pi)$ and the two other angles can have arbitrary values. It can be shown that by choosing the ingredient (II) of our method to have the general form

$$\hat{H}_c(t) = \hat{U}^\dagger_{\text{ad}}(t)(g_x(t)\hat{M}_x + g_z(t)\hat{M}_z)\hat{U}_{\text{ad}}(t) \quad (13)$$

we find a control Hamiltonian $\hat{H}_c$ that does not directly couple the states $|A\rangle$ and $|C\rangle$. The corrected protocol will consist in a simple modification of the original STIRAP angle and amplitude

$$\theta(t) \to \tilde{\theta}(t) = \theta(t) - \arctan\left(\frac{g_x(t)}{\Omega(t) + g_z(t)}\right), \quad (14)$$

$$\Omega(t) \to \tilde{\Omega}(t) = \sqrt{(\Omega(t) + g_z(t))^2 + g_x^2(t)}. \quad (15)$$

Moreover, in order to satisfy Eq. (7), the control parameters have to be chosen as

$$g_x(t) = \frac{\dot{\mu}}{\cos\xi} - \dot{\theta}\tan\xi, \quad (16)$$

$$g_z(t) = -\Omega + \dot{\xi} + \frac{\dot{\mu}\sin\xi - \dot{\theta}}{\tan\mu\cos\xi}, \quad (17)$$

and are independent of $\eta(t)$. Within our framework, it can be shown that the population in the intermediate level $|B\rangle$ is given by

$$|\langle\psi(t)|B\rangle|^2 = \sin^2\mu(t)\cos^2\xi(t). \quad (18)$$

From now on, in order to keep the discussion simple, we focus on the $\xi(t) = 0$ case.

Application to Vitanov-style pulses.—We apply these dressed-state protocols to the optimal STIRAP pulses discussed by Vitanov *et al.* in Ref. [29] and defined by

$$\Omega(t) = \Omega_0, \qquad \theta(t) = \frac{\pi}{2}\frac{1}{1 + e^{-t/\tau}}, \quad (19)$$

where the time scale τ controls the effective duration of the protocol. The simplest nontrivial choice of the dressed-states basis is the superadiabatic basis, for which

$$\mu = -\arctan\left(\frac{\dot{\theta}(t)}{\Omega(t)}\right), \qquad g_x(t) = \dot{\mu}, \qquad g_z(t) = 0. \quad (20)$$

This choice will be referred to as superadiabatic transitionless driving (SATD). With this choice the only way to reduce the population in the intermediate level [cf. Eq. (18)] is to decrease the magnitude of $\dot{\theta}(t)$, and hence slow down the protocol (i.e., longer τ). Interestingly, SATD represents a nonperturbative version of the derivative removal by adiabatic gate approach to leakage errors [30,31] applied to this problem (see the Supplemental Material [28]).

Our approach allows one to construct alternatives to SATD (based on alternate dressed states) that reduce the intermediate-level occupancy. This can be extremely beneficial in systems where the intermediate state is lossy, but where adiabatic evolution is impossible, as the protocol must be fast to avoid dissipation of the source and/or target system, or because of slow drifts of system parameters. A concrete example with all these features is optomechanical state transfer [6–9]. By generalizing Eq. (20) to

$$\mu = -\arctan\left(\frac{\dot{\theta}(t)}{f(t)\Omega(t)}\right), \qquad g_x(t) = \dot{\mu},$$
$$g_z(t) = -\Omega - \frac{\dot{\theta}(t)}{\tan\mu} \quad (21)$$

we can chose the auxiliary function $f(t)$ to reduce μ (and hence the amount of state dressing) to avoid unnecessary B-state population. Here, we choose to consider the simple class of functions $f(t) = 1 + A\exp(-t^2/T^2)$ [$f(t) \geq 1 \quad \forall\, t$] with $A > 0$ and $T > 0$ two parameters

that can be optimized for each τ to minimize the population in B. As we show below, this intuitive and physically motivated choice allows for a sizeable reduction of the occupancy of the intermediate level without having to rely on more complex methods (e.g., control theory).

To compare protocols, we look at the relevant case where the fidelity is limited both by a nonzero τ in Eq. (19) and by the protocol starting and ending at a finite time. In theory, the protocol should start at $t_i = -\infty$ and end at $t_f = +\infty$ in order to achieve the requirement $\theta(t_i) = 0$, $\theta(t_f) = \pi/2$, and $\mu(t_i) = \mu(t_f) = 0(2\pi)$. To simulate pulses with a finite duration, we have chosen $t_f = -t_i = 15\tau$ such that $\Omega_p(t_i) = \Omega_S(t_f) < 10^{-6}\Omega_0$. With our choices of correction, the shorter the protocol time is, the bigger the amplitude $\tilde{\Omega}(t,\tau)$ is. We consider the case where each corrected pulse cannot exceed its original maximal amplitude $\Omega_0\{\max_t[\tilde{\Omega}(t,\tau)\sin\tilde{\theta}(t,\tau),\tilde{\Omega}(t,\tau)\cos\tilde{\theta}(t,\tau)] \leq \Omega_0, \forall t\}$. This constraint implies that we can only correct protocols with an effective protocol time $\tau > \tau_{\min} \simeq 1/2.63\Omega_0$.

$$\varepsilon = 1 - F = 1 - |_C\langle\psi(t_f)|\psi(t_i)\rangle_A|^2. \quad (22)$$

Since we are interested in a qubit state transfer and $|000\rangle$ has a trivial dynamics, only the transfer of state $|A\rangle$ to $|C\rangle$ gives rise to errors. Thus, we plot the fidelity for transferring the $|A\rangle$ state only, which sets an upper bound for the error when transferring a superposition of an arbitrary qubit state (see the Supplemental Material [28]). In Fig. 2(a), we plot the residual error ε as a function of τ for SATD [Eq. (20)] and the modified SATD [Eq. (21)] with optimized parameters. Both choices reduce the residual error by the same amount and lead to a several orders of magnitude reduction as compared to the protocol defined by Eq. (19). The oscillatory behavior is a direct consequence of having finite-time pulses (see the Supplemental Material [28]).

To illustrate the additional advantage of our choice of correction, we consider the time integral over the full protocol duration of the population in $|B\rangle$. In Fig. 2(b), we plot this quantity for both SATD and the modified SATD: the integrated population is reduced between $\approx 21\% - 25.5\%$ with the modified SATD [Eq. (21)] as compared to SATD [Eq. (20)]. In Figs. 2(c) and (d), we plot the corrected pump pulse for SATD and the modified SATD for different values of τ. The Stokes pulse is the symmetric reflection of the pump pulse with respect to $(t_f - t_i)/2$. The SATD pulses rapidly converge to the Vitanov style pulses [Eq. (19)] when τ increases, while the modified SATD pulses converge more slowly. This is due to the fact that the modified SATD pulses have been designed not only to reduce the residual error, but also to reduce the population in the mechanics, which slowly converges to 0 as $\tau \to \infty$.

Application to Gaussian pulses.—An additional advantage of our approach is that it allows one to correct protocols for which the correction (20) does not work.

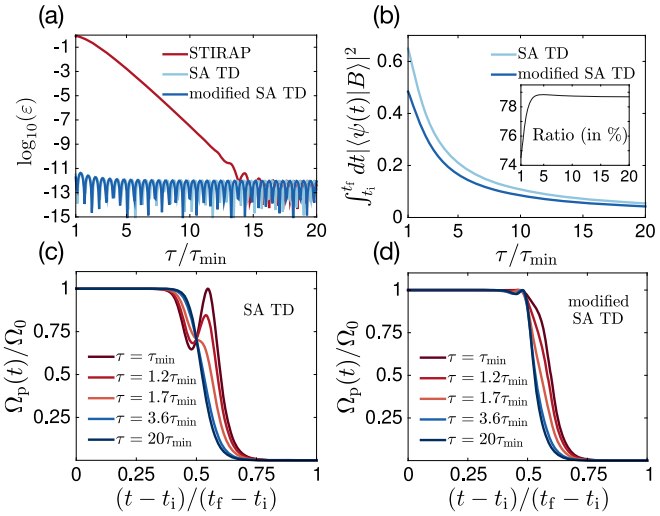

FIG. 2. (a) Comparison of the residual error between STIRAP [Eq. (19)], SATD [Eq. (20)], and the modified SATD [Eq. (21)] as a function of the effective protocol duration τ in units of $\tau_{\min}$. (b) Comparison of the integrated population in $|B\rangle$ over the whole protocol time between SATD [Eq. (20)] and our new dressed state approach [Eq. (21)] as a function of τ in units of $\tau_{\min}$. Inset: ratio of those two quantities. The integrated population is reduced by at least 21% and at most 26% with our new protocol. Plot of the corrected pump pulse for SATD (c) and the modified SATD (d) for different values of τ as a function of time $(t - t_i)$ in units of the total protocol time $(t_f - t_i)$.

In particular, the most common approach to STIRAP uses Gaussian pulses [3,4] $\Omega_p(t) = \Omega_0 \exp[-(t - t_0/2)^2/\tau^2]$ and $\Omega_S(t) = \Omega_0 \exp[-(t + t_0/2)^2/\tau^2]$ with t_0 the delay time between the two pulses. Using the parametrization defined in Eq. (9), we have

$$\theta(t) = \arctan[\exp(2tt_0/\tau^2)],$$
$$\Omega(t) = \Omega_0 \exp\left(-\frac{t^2 + t_0^2/4}{\tau^2}\right)\sqrt{2\cosh(tt_0/\tau^2)}. \quad (23)$$

For this particular case, we cannot use the SATD prescription to construct a control Hamiltonian as the condition $\mu(t_i) = \mu(t_f) = 0(2\pi)$ is not satisfied (for this choice of pulse $\dot{\theta}(t)/\Omega(t) \to +\infty$ as $t \to \pm\infty$). However, our dressed state approach allows one to find a control Hamiltonian using Eq. (17) ($\xi = 0$) and

$$\mu(t) = -\arctan\left(\frac{\dot{\theta}(t)}{g(t)/\tau + \Omega(t)}\right). \quad (24)$$

Here, $g(t)/\tau$ is used to regularize $\mu(t)$: it has to be chosen such that it tends to zero at t_i and t_f slower than $\dot{\theta}$. In Fig. 3, we have plotted the residual error for STIRAP with Gaussian densities [Eq. (23)] and for the modified SATD [Eq. (24)]. We have chosen $t_0 = 6/5\tau$ and $g(t) = A/\cosh\zeta t$ with $A = 1/40$ and $\zeta = 9/10\tau$, which gives $\tau_{\min} \approx 1/1.27\Omega_0$. Under the condition $\Omega_p(t_i) = \Omega_S(t_f) <$

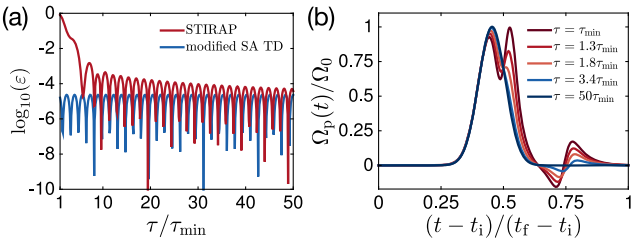

FIG. 3. (a) Comparison of the residual error for STIRAP with Gaussian densities [Eq. (23)] and the modified SATD [Eq. (24)] as a function of the effective protocol duration τ in units of $\tau_{\min}$. The residual error is reduced by several orders of magnitude in the nonadiabatic regime. (b) Corrected pump pulse for different values of τ as a function of time $(t - t_i)$ in units of the total protocol time $(t_f - t_i)$.

$10^{-6}\Omega_0$, we have $t_f = -t_i = 6\tau$. This new pulse scheme leads to a reduction of the residual error by several orders of magnitude [see Fig. 3(a)] in the nonadiabatic regime while SATD [Eq. (20)] fails. In Fig. 3(b), we plot the corrected pump pulse for different values of τ. The Stokes pulse is the symmetric reflection of the pump pulse with respect to $(\Omega_p, t) = [0, (t_f - t_i)/2]$.

Conclusion.—We have developed a general method to achieve a perfect state transfer between two quantum systems coupled via an intermediate lossy system. In contrast to previous schemes, our approach is both physically transparent and extremely flexible, allowing application to a wide variety of realistic experimental situations.

In future work, it could be interesting to investigate the resilience of the generated pulse sequences with respect to experimental imperfections of the system and of the control fields as in Refs. [32,33]. It would also be interesting to investigate the implementation of our method in more complicated systems, where analytical diagonalization is not possible. In particular one could study perturbative variants of our approach as well as numerical diagonalization allowing one to look for transitionless driving and higher order variants corrections [13–15].

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