Higher Spin Interactions from Conformal Field Theory: The Complete Cubic Couplings

Charlotte Sleight[1,*] and Massimo Taronna[2,†]
[1]Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 Munich, Germany
[2]Université Libre de Bruxelles and International Solvay Institutes, ULB– Campus Plaine CP231, 1050 Brussels, Belgium
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In this Letter we provide a complete holographic reconstruction of the cubic couplings in the minimal bosonic higher spin theory in $(d+1)$-dimensional anti– de Sitter space. For this purpose, we also determine the operator-product expansion coefficients of all single-trace conserved currents in the d-dimensional free scalar $O(N)$ vector model, and we compute the tree-level three-point Witten diagram amplitudes for a generic cubic interaction of higher spin gauge fields in the metriclike formulation.

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Introduction.—Despite the long existence of fully nonlinear equations of motion [1] for theories of higher spin gauge fields, a complete list of Lagrangian cubic couplings is unknown. The main difficulty has been in taking the Noether procedure beyond the cubic order. Indeed, cubic consistency leaves the relative coefficients of the possible cubic interactions arbitrary. So far only a small number of Lagrangian couplings have been determined, particularly those which are constrained by the higher spin algebra structure constants [2–6].

In this Letter we take an alternative route, and we find the complete list of cubic couplings for the type A minimal bosonic higher spin theory in anti– de Sitter (AdS) space as determined by holographic reconstruction. For higher spin symmetry preserving boundary conditions, the latter theory is conjectured [7,8] to be dual to (the singlet sector of) the free scalar $O(N)$ vector model. For the duality to hold, the cubic interactions of the bulk theory must reproduce the three-point correlators of the dual conformal field theory (CFT). From this basic requirement, we are able to determine all bulk cubic couplings of the minimal bosonic higher spin theory. As a nontrivial check of our holographic reconstruction, we relate the couplings obtained to the metriclike classification of gauge invariant $(d+1)$-dimensional anti– de Sitter (AdS_{d+1}) couplings in Refs. [9–11] and check their gauge invariance.

To reach our goal of a complete set of cubic couplings, we established two main intermediate results. (1) The explicit form of all two- and three-point functions of conserved currents in the singlet sector of the d-dimensional free scalar $O(N)$ vector model. (2) The amplitude of a three-point Witten diagram for an arbitrary triplet (s_1, s_2, s_3) of external higher spin gauge fields, generated by a generic consistent cubic interaction.

This work constitutes a key step of the holographic reconstruction program, which began with the extraction of the 0-0-s cubic couplings [12–15] and scalar quartic self-interaction [13,16,17]. See also the subsequent work [18], which gave a simple extension of the cubic 0-0-s result to the type B theory [19].

Single-trace operator-product expansion (OPE) coefficients.—Generating function for conserved currents: We first introduce a useful technology for dealing with conserved currents of arbitrary spin in CFT, and their correlation functions.

The conserved currents we consider in this Letter reside in the singlet sector of the free scalar $O(N)$ vector model, which are traceless bilinears in the fundamental scalar ϕ^a [22]:

$$\mathcal{J}_{\mu_1,\ldots,\mu_s} \sim \phi^a \partial_{\mu_1},\ldots,\partial_{\mu_s}\phi^a + \ldots, \qquad a = 1,\ldots,N. \quad (1)$$

In the above, the ellipses represent further singlet bilinear structures, which ensure conservation and tracelessness. Moreover, these currents are nontrivial only for even spins s.

It is convenient to use index-free notation, introducing a null polarization vector,

$$\mathcal{J}_s(x|z) \equiv \mathcal{J}_{\mu_1,\ldots,\mu_s}(x) z^{\mu_1},\ldots,z^{\mu_s}, \quad (2)$$

where tracelessness is encoded in the null condition $z^2 = 0$. They can then be packaged in the compact expression

$$\mathcal{J}_s = f^{(s)}(z\cdot\partial_{x_1}, z\cdot\partial_{x_2})\phi^a(x_1)\phi^a(x_2)\big|_{x_1,x_2\to x}, \quad (3)$$

where the function $f^{(s)}(x,y)$ is given in terms of a Gegenbauer polynomial,

$$f^{(s)}(x,y) = (x+y)^s C_s^{[(\Delta-1)/2]}\left(\frac{x-y}{x+y}\right), \quad (4)$$

and $\Delta/2$ denotes the scaling dimension of ϕ^a. This is an old trick [23], and it can easily be derived by demanding that the expression (3) is annihilated by the conformal boost operator. This gives the differential equation for $f^{(s)}(x,y)$,

$$\left[\left(\frac{\Delta}{2}+x\partial_x\right)\partial_x+\left(\frac{\Delta}{2}+y\partial_y\right)\partial_y\right]f^{(s)}(x,y)=0, \quad (5)$$

whose solution is expressed in terms of the Gegenbauer polynomials above [24]. For $\Delta = d-2$, which is twice the dimension of a free boson, the primary operator has the dimension $d-2+s$ and saturates the unitarity bound for $s>0$. For this scaling dimension we thus obtain conserved currents, which is straightforward to verify.

The form (3) of the conserved currents plays a crucial role in the following sections, as it allows for the seamless application of Wick's theorem to determine their two- and three-point functions.

Two-point functions: Conformal symmetry fixes two-point functions up to an overall coefficient, with those of spin-s conserved currents (3) taking the form

$$\langle \mathcal{J}_{s_1}\mathcal{J}_{s_2}\rangle = \mathsf{C}_{\mathcal{J}_{s_1}}\frac{\delta_{s_1,s_2}}{(x_{12}^2)^{\Delta+s}}\left(z_1\cdot z_2+\frac{2z_1\cdot x_{12}z_2\cdot x_{21}}{x_{12}^2}\right)^s. \quad (6)$$

We determine the overall coefficient $\mathsf{C}_{\mathcal{J}_s}$ for the currents (3) in the following.

Since the theory is free, we may simply apply Wick's theorem to express the two-point function in terms of that of the fundamental scalar. To extract the overall two-point coefficient, by conformal invariance it is sufficient to restrict our attention to terms with zero contractions of the null auxiliary vectors. We thus set to zero $z_1\cdot z_2$ and match it to the corresponding term in Eq. (6). The computation is drastically simplified using the Schwinger-parametrized form

$$\langle\phi^a(x_1)\phi^b(x_2)\rangle = \frac{\delta^{ab}}{(x_{12}^2)^{\Delta/2}} = \frac{\delta^{ab}}{\Gamma(\frac{\Delta}{2})}\int_0^\infty\frac{dt}{t}t^{\Delta/2}e^{-tx_{12}^2},$$

which gives

$$\mathsf{C}_{\mathcal{J}_s} = \left(\frac{1+(-1)^s}{2}\right)N2^{s+1}\frac{(\Delta-1)_s(\Delta-1)_{2s}}{\Gamma(s+1)}, \quad (7)$$

as a consequence of the Gegenbauer orthogonality relation.

Three-point functions and OPE coefficients: We now turn to the three-point functions of the conserved currents and employ the same approach as for the two-point functions above. A combination of Wick's theorem and Schwinger parametrization. For previous results in three dimensions, see Refs. [25–27].

In CFT one can parametrize the most general conformal structure built from three distinct points in terms of the following six basic objects:

$$\mathsf{Y}_i = \frac{z_i\cdot x_{i,j}}{x_{i,j}^2}-\frac{z_i\cdot x_{i,k}}{x_{i,k}^2}, \quad (8a)$$

$$\mathsf{H}_i = \frac{1}{x_{j,k}^2}\left(z_j\cdot z_k+\frac{2z_j\cdot x_{jk}z_k\cdot x_{kj}}{x_{jk}^2}\right), \quad (8b)$$

with i, j, and k ranging from now on from 1 to 3 and always, if not stated otherwise, cyclically ordered.

By demanding the correct spin structure and behavior under scale transformations, conformal symmetry dictates that the three-point function of a generic triplet of conserved currents is a sum of monomials of the type

$$\langle\mathcal{J}_{s_1}\mathcal{J}_{s_2}\mathcal{J}_{s_3}\rangle = \sum_{n_i}\mathsf{C}^{n_1,n_2,n_3}_{s_1,s_2,s_3} \times\frac{\mathsf{Y}_1^{l_1}\mathsf{Y}_2^{l_2}\mathsf{Y}_3^{l_3}\mathsf{H}_1^{n_1}\mathsf{H}_2^{n_2}\mathsf{H}_3^{n_3}}{(x_{12}^2)^{[(\tau_1+\tau_2-\tau_3)/2]}(x_{23}^2)^{[(\tau_2+\tau_3-\tau_1)/2]}(x_{31}^2)^{[(\tau_3+\tau_1-\tau_2)/2]}}, \quad (9)$$

with twists $\tau_i=\Delta_i-s_i$, $l_i=s_i-n_j-n_k$, i, j, and k cyclically ordered and theory-dependent coefficients $\mathsf{C}^{n_1,n_2,n_3}_{s_1,s_2,s_3}$. Each individual term is independently invariant under conformal transformations, and thus conformal symmetry alone does not determine the coefficients $\mathsf{C}^{n_1,n_2,n_3}_{s_1,s_2,s_3}$. Current conservation gives further constraints; however, in the following we simply determine their explicit form using Wick's theorem.

As with the two-point functions in the previous section, we may at first set $z_i\cdot z_j=0$ and evaluate all Wick contractions. By matching with the corresponding expansion at $z_i\cdot z_j=0$ of the general form for the correlator (9), one can then establish a recursion relation for the coefficients $\mathsf{C}^{n_1,n_2,n_3}_{s_1,s_2,s_3}$.

The final result can be resummed in terms of a Bessel function, giving the following compact form for the correlation function:

$$\langle\mathcal{J}_{s_1}\mathcal{J}_{s_2}\mathcal{J}_{s_3}\rangle = \frac{N}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}} \times\left[\prod_{i=1}^3\mathsf{c}_{s_i}q_i^{[(1/2)-(\Delta/4)]}\Gamma\left(\frac{\Delta}{2}\right)J_{[(\Delta-2)/2]}(\sqrt{q_i})\right]\mathsf{Y}_1^{s_1}\mathsf{Y}_2^{s_2}\mathsf{Y}_3^{s_3}. \quad (10)$$

Here, $q_i=2\mathsf{H}_i\partial_{\mathsf{Y}_j}\partial_{\mathsf{Y}_k}$ and c_{s_i} are given by

$$\mathsf{c}_{s_i}^2 = \frac{\sqrt{\pi}2^{-\Delta-s_i+3}\Gamma(s_i+\frac{\Delta}{2})\Gamma(s_i+\Delta-1)}{Ns_i!\Gamma(s_i+\frac{\Delta-1}{2})\Gamma(\frac{\Delta}{2})^2}, \quad (11)$$

for the canonically normalized current two-point function, i.e., by redefining $\mathcal{J}_{s_i}\to 1/\sqrt{\mathsf{C}_{\mathcal{J}_{s_i}}}\mathcal{J}_{s_i}$. A nice check at this

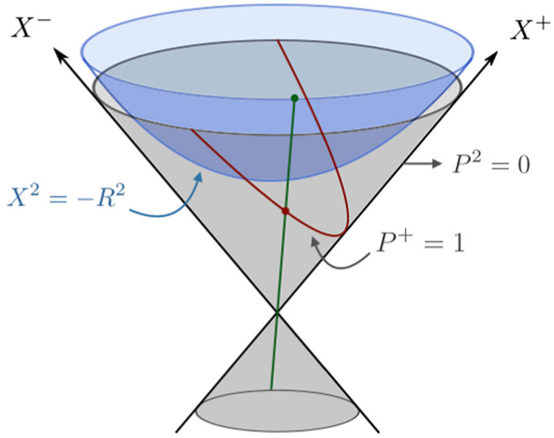

FIG. 1. Euclidean AdS space and its boundary in ambient space. This figure displays the AdS surface $X^2 = -R^2 = -1$ and the identification of a (green) boundary point with a (green) light ray of the light cone $P^2 = 0$, which intersects the Poincaré section (16) on a (red) point.

point is that the result for the s-0-0 correlator coincides with that already given in the literature [28].

A few remarks are in order. (i) The above result is nicely factorized, as expected from a free CFT. Notice that the overall coefficients are simply a product of s-0-0 OPE coefficients. This generalizes the result [28] for the latter to three-point functions of an arbitrary triplet (s_1, s_2, s_3) of single-trace operators. (ii) The Bessel function form (10) of the tensor structures generated are consistent with those observed in the literature for three-point functions of arbitrary spin conserved currents [29–32]. (iii) With the simple factorized result (10), it is tempting to conjecture that the form of any n-point function of such currents is given by simply extending the product from $i = 1, \ldots, n$:

$$\langle \mathcal{J}_{s_1} \mathcal{J}_{s_2} \ldots \mathcal{J}_{s_n} \rangle = \frac{N}{(x_{12}^2)^{\Delta/2} (x_{23}^2)^{\Delta/2} \ldots (x_{n1}^2)^{\Delta/2}} \times \left[\prod_{i=1}^{n} \mathsf{c}_{s_i} q_i^{[(1/2)-(\Delta/4)]} \Gamma\left(\frac{\Delta}{2}\right) J_{(\Delta-2)/2}(\sqrt{q_i}) \right] \times \mathsf{Y}_1^{s_1} \mathsf{Y}_2^{s_2} \ldots \mathsf{Y}_n^{s_n} + \text{perm}, \tag{12}$$

with $q_i = 2\mathsf{H}_i \partial_{\mathsf{Y}_{i-1}} \partial_{\mathsf{Y}_{i+1}}$ while summing over the inequivalent permutations (perm.) of the external legs.

Three-point Witten diagrams.—We now turn to the bulk side of the story. The three-point correlation functions of conserved currents computed in the previous section are dual to three-point Witten diagrams generated by cubic interactions of the corresponding bulk gauge fields. In this section we compute the three-point amplitudes for a generic such cubic interaction, thus providing the dictionary between bulk cubic couplings and the boundary CFT correlators.

Brief review of the ambient space formalism: It is convenient to employ the ambient space formalism (Fig. 1), in which AdS_{d+1} space is realized as a hyperboloid in an ambient $(d+2)$-dimensional Minkowski space:

$$X^2 + 1 = 0, \qquad X^0 > 0. \tag{13}$$

In this section we give a brief overview of the relevant aspects of this framework and direct the unfamiliar reader to, e.g., Refs. [33–37] for further details.

Fields intrinsic to AdS_{d+1} space can be represented by homogeneous fields in ambient space that are tangent to the hyperboloid (13). As for the CFT discussion above, it is useful to encode tensor structures in polynomials of auxiliary variables. Symmetric rank-s tensors can be described by

$$\Phi(X, U) = \frac{1}{s!} \Phi_{M_1,\ldots,M_s}(X) U^{M_1}, \ldots, U^{M_s}, \tag{14}$$

subject to the following homogeneity and tangentiality conditions:

$$(X \cdot \partial_X - \Delta)\Phi(X, U) = 0, \qquad X \cdot \partial_U \Phi(X, U) = 0. \tag{15}$$

Above the degree of homogeneity for ambient symmetric fields is chosen to be compatible with the AdS/CFT dictionary for fields dual to conserved currents [38] $\Delta = 2 - d - s$.

The boundary of AdS_{d+1} space is described by the hypercone $P^2 = 0$. It is then convenient to introduce auxiliary variables $Z_A(x)$ to be contracted with the CFT currents defined on the hypercone at the boundary point x. The explicit relation between the ambient and intrinsic variables can be obtained solving the constraints in some given coordinate system. It is usually given by employing ambient light cone coordinates $X^A = (X^+, X^-, X^a)$ as

$$Z^B(x) = (0, 2x \cdot z, z^b), \qquad P^B(x) = (1, x^2, x^b), \tag{16}$$

where $X^2 = -X^+X^- + \delta_{ab} X^a X^b$.

Cubic couplings and their Witten diagrams: In the ambient framework, it is straightforward to parametrize the most general on-shell cubic interaction up to integrations by parts. On shell, the most general cubic interaction involving fields of spins s_1-s_2-s_3 can be recast [9] as

$$\mathcal{V}_{s_1,s_2,s_3} = \sum_{n_i} g_{s_1,s_2,s_3}^{n_1,n_2,n_3} I_{s_1,s_2,s_3}^{n_1,n_2,n_3}(\Phi_i), \tag{17}$$

with

$$
\begin{aligned}
&I^{n_1,n_2,n_3}_{s_1,s_2,s_3}(\Phi_i)\\
&\quad = \mathcal{Y}_1^{s_1-n_2-n_3}\mathcal{Y}_2^{s_2-n_3-n_1}\mathcal{Y}_3^{s_3-n_1-n_2}\mathcal{H}_1^{n_1}\mathcal{H}_2^{n_2}\mathcal{H}_3^{n_3}\\
&\qquad \times \Phi_1(X_1,U_1)\Phi_2(X_2,U_2)\Phi_3(X_3,U_3)\Big|_{X_i=X}, \qquad (18)
\end{aligned}
$$

in terms of the minimal set of contractions $\mathcal{H}_i = \partial_{U_j}\cdot\partial_{U_k}$ and $\mathcal{Y}_i = \partial_{U_i}\cdot\partial_{X_{i+1}}$. In the next section, we determine the couplings $g^{n_1,n_2,n_3}_{s_1,s_2,s_3}$ in minimal bosonic higher spin theory by employing the holographic duality. We do so by matching the tree-level three-point Witten diagram generated by the vertex (17) with the corresponding three-point function (10) in the dual free scalar $O(N)$ vector model.

(i) Boundary to bulk propagators: The spin-s boundary-to-bulk propagator [39] is a linear solution to the bulk wave equation,

$$
(\Box - m_s^2)\Pi_{\Delta,s} = 0, \qquad m_s^2 = \Delta(\Delta-d)-s, \qquad (19)
$$

with a boundary delta-function source. In the ambient framework, it can be expressed in the form

$$
\begin{aligned}
&\Pi_{\Delta,s}(X,P|U,Z)\\
&= \frac{C_{\Delta,s}}{s!}\times\frac{[(-2P\cdot X)(U\cdot Z)+2(U\cdot P)(Z\cdot X)]^s}{(-2P\cdot X)^{\Delta+s}}, \qquad (20)
\end{aligned}
$$

with $Z^2=0$, $P^2=0$, $Z\cdot P=0$, which also ensures tracelessness of the propagator. We use the normalization [37]

$$
C_{\Delta,s} = \frac{(\Delta+s-1)\Gamma(\Delta)}{2\pi^{d/2}(\Delta-1)\Gamma(\Delta+1-\frac{d}{2})}. \qquad (21)
$$

Just as with the CFT results of the previous section, it is instrumental to express the scalar boundary-to-bulk propagator in the Schwinger-parametrized form

$$
\frac{1}{(-2X\cdot P)^{\Delta}} = \frac{1}{\Gamma(\Delta)}\int_0^\infty \frac{dt}{t}t^{\Delta}e^{-2tP\cdot X}. \qquad (22)
$$

In this way, the nth ambient derivative of the bulk-to-boundary propagator can be expressed in terms of a scalar propagator of dimension $\Delta+n$:

$$
\begin{aligned}
&(W\cdot\partial_X)^n\Pi_{\Delta,s}(X,P|U,Z)\\
&= \frac{C_{\Delta,s}}{s!}\sum_{i=0}^{s}\sum_{\omega=0}^{i}\binom{s}{i}\binom{i}{\omega}\frac{(-1)^s 2^n}{(n-\omega+1)_\omega}\frac{1}{\Gamma(\Delta+i)}\\
&\quad\times (U\cdot P)^i(U\cdot Z)^{s-i}(Z\cdot W)^{\omega}(P\cdot W)^{n-\omega}\\
&\quad\times (Z\cdot\partial_P)^{i-\omega}\int_0^\infty\frac{dt}{t}t^{\Delta+n}e^{-2tP\cdot X}. \qquad (23)
\end{aligned}
$$

(ii) Three-point bulk integrals: Employing the above Schwinger representations for the boundary-to-bulk propagators, the integral over AdS space generated by the generic tensor structure (17) can be reduced to that of a basic scalar cubic interaction,

$$
\begin{aligned}
&\int_{\mathrm{AdS}_{d+1}} dX\left(\prod_{i=1}^{3}\frac{dt_i}{t_i}t_i^{\Delta_i}\right)e^{2(t_1P_1+t_2P_2+t_3P_3)\cdot X}\\
&= \pi^{d/2}\Gamma\left(\frac{\sum_{i=1}^{3}\Delta_i - d}{2}\right)\int_0^\infty\prod_{i=1}^{3}\left(\frac{dt_i}{t_i}t_i^{\Delta_i}e^{-t_jt_kP_{jk}}\right), \qquad (24)
\end{aligned}
$$

where we defined $P_{ij} = -2P_i\cdot P_j = x_{ij}^2$ and it should be understood that i, j, and k range from 1 to 3 and are cyclically ordered. The integrations over the t_i's are simply integral representations of gamma functions.

The resulting amplitude of the s_1-s_2-s_3 three-point Witten diagram for the vertex (19) for a general n_i is lengthy, and we give it explicitly in the Supplemental Material [40]. The crucial observation is that the simplest structure with $n_1=n_2=n_3=0$ is singled out when demanding that we generate the precise combination of boundary structures of the dual free-boson CFT correlator:

$$
\begin{aligned}
\int_{\mathrm{AdS}_{d+1}} dX I^{0,0,0}_{s_1,s_2,s_3}(\Pi_i) &= \frac{\pi^{(3/2)-d}(-1)^{s_1+s_2+s_3}2^{-3d-s_1-s_2-s_1+8}\Gamma(d+s_1+s_2+s_1-3)}{(x_{12}^2)^{d/2-1}(x_{23}^2)^{d/2-1}(x_{31}^2)^{d/2-1}}\\
&\quad\times\frac{\Gamma(d-3+s_1)\Gamma(d-3+s_2)\Gamma(d-3+s_1)}{\Gamma(\frac{d-3}{2}+s_1)\Gamma(\frac{d-3}{2}+s_2)\Gamma(\frac{d-3}{2}+s_1)}\left(\prod_{i=1}^{3}q_i^{1-(d/4)}J_{(d/2)-2}(\sqrt{q_i})\right)\mathsf{Y}_1^{s_1}\mathsf{Y}_2^{s_2}\mathsf{Y}_3^{s_3}. \qquad (25)
\end{aligned}
$$

This identifies the bulk structure that reproduces the free scalar CFT correlator up to an overall coefficient.

A few remarks are in order. (i) The dependence on the spins s_i is completely factorized, but for the overall gamma function prefactor coming from the AdS integration. (ii) Remarkably, a single ambient structure resums, at fixed spins, all couplings, including the quasiminimal structures present in the higher spin theory [11,41,42]. In the Supplemental Material [40], we detail how the lower derivative terms are generated upon translating the result in terms of AdS covariant derivatives. We also provide a full recursive solution for the reduction. (iii) We have checked that the above couplings are gauge invariant in the bulk and match couplings classified in Ref. [9]. This shows that the holographic reconstruction at this order is compatible with the Noether procedure. More technical details on this point can be found in the Supplemental Material [40]. (iv) The couplings considered here were shown to induce deformations to the gauge transformations and to the gauge algebra compatible with the relevant higher spin algebras (see, e.g., Ref. [11]). It was, however, not yet possible to determine the relative coefficients (though some progress was made in Refs. [2,3,5,6]).

Holographic reconstruction.—It is now straightforward to combine the above bulk results with those for the three-point functions in the $O(N)$ model. This gives the complete holographic reconstruction of the cubic couplings for the minimal bosonic higher spin theory in AdS_{d+1} space.

Normalizing the two-point functions of the $\mathcal{J}_{s_i}$ to one in both the bulk and boundary computations, the complete bulk cubic coupling thus reads [43]

$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3}, \tag{26}$$

where we obtain the following coupling constants:

$$g_{s_1,s_2,s_3} = \frac{\pi^{[(d-3)/4]} 2^{(3d-1+s_1+s_2+s_3)/2}}{\sqrt{N}\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}.$$

The simplest form for the above coupling constants manifests itself in AdS_4 space, where the spin dependence remarkably coincides with the one obtained in Refs. [44,45] from a flat space quartic analysis (see also Ref. [46]):

$$g^{(\mathrm{AdS}_4)}_{s_1,s_2,s_3} = \frac{2^{[(s_1+s_2+s_3)/2]+4}}{\sqrt{N}\Gamma(s_1+s_2+s_3)}. \tag{27}$$

This is in accordance with the flat limit of the above AdS_4 vertices. Notice that the flat limit requires us to first fix the spin of the external states and keep the highest derivative term for each triple of spins.

Furthermore, that the above result reduces to the 0-0-s coupling obtained in Ref. [13] for any two pairs of spins s_i set to zero is a nice consistency check.

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*charlotte.sleight@gmail.com
†massimo.taronna@ulb.ac.be

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