

## Fractional Quantum Hall States in a Ge Quantum Well

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Measurements of the Hall and dissipative conductivity of a strained Ge quantum well on a SiGe/(001)Si substrate in the quantum Hall regime are reported. We analyze the results in terms of thermally activated quantum tunneling of carriers from one internal edge state to another across saddle points in the long-range impurity potential. This shows that the gaps for different filling fractions closely follow the dependence predicted by theory. We also find that the estimates of the separation of the edge states at the saddle are in line with the expectations of an electrostatic model in the lowest spin-polarized Landau level (LL), but not in the spin-reversed LL where the density of quasiparticle states is not high enough to accommodate the carriers required.

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The strongest fractional quantum states have been found in delta-doped GaAs quantum wells and heterostructures. These have been the cleanest samples with the highest mobilities, and yet the strength of a fractional quantum Hall state does not appear to be connected in a simple way with the zero magnetic field mobility of the samples involved [1,2].

The strength of a quantum Hall state is associated with how rapidly the dissipative conductance drops with temperature. This is known to be affected by the long-range impurity potential, arising out of the ionized donors in a semiconductor heterostructure, although describing this effect quantitatively has proved difficult. The background potential leads to the formation of compressible regions, which then partially screen the background potential. Predicted theoretically in Refs. [3,4], and later verified in scanning electron transistor measurements [5], these compressible regions are separated from the percolating incompressible region by internal edges. The transfer between these internal edges is then the principal dissipative process at low temperatures [6,7]. Characterizing these internal edges is not just an important challenge for describing the quantum Hall response but also for the interpretation of Aharonov-Bohm interference experiments at filling factor  $\nu = 5/2$  [8]. It turns out, for example, that it is crucial to the interpretation of these experiments to know whether any charge redistribution associated with changing the field occurs internally (by shifts in the local occupation of compressible regions) or by the outer sample edges [9].

Here, we report conductivity measurements on a *p*-type strained germanium quantum well. There is a clear family of quantum Hall states in the composite fermion (CF) series centered on  $\nu = 1/2$  [10]. Our analysis of the data shows that the model can give a consistent fit across all filling fractions in the CF family for the gaps in these systems and explains why the zero field mobility is not simply connected with the strength of the quantized Hall states. The gaps are in line with the predicted dependence on the filling fraction. In addition, we estimate the typical separation at a saddle point  $a$  of two compressible regions occupied by the same type of carrier. As a function of the gap, these correlate well with the separation of neighboring compressible regions containing carriers of opposite charge, which can be predicted on the basis of an electrostatic model [11]. While the dependence of the width parameter  $a$  on the gap  $\Delta_s$  is the same for filling fractions in the lowest Landau level (LL) above and below  $\nu = 1/2$ , there is a systematic shift to higher values for the quasihole states ( $\nu > 1/2$ ). However, for states above  $\nu > 1$  where multiple LLs are occupied, the dependence does not coincide with the predictions of the electrostatic model. We attribute this to the need for a charge rearrangement across the incompressible region, which is too large for a simple edge to accommodate.

Our sample was grown by reduced pressure chemical vapor deposition at the University of Warwick (sample ID 11-289SQ1D). It is taken from the same wafer that was reported in Ref. [12]; it has a density of  $2.9 \times 10^{11} \text{ cm}^{-2}$

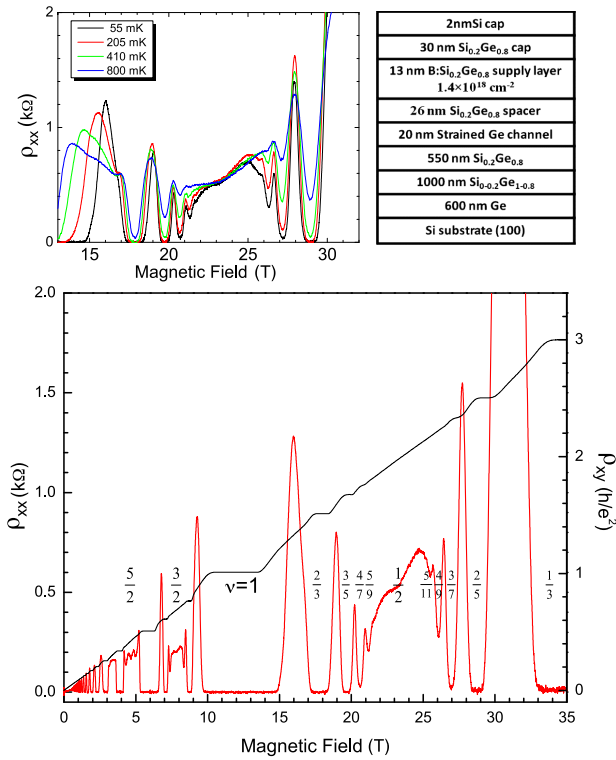


FIG. 1. The Hall resistance  $\rho_{xy}$  and dissipative response  $\rho_{xx}$  at 55 mK (main figure) and the dissipative response at different temperatures [14] in the high field regime (top left panel). The wafer structure is summarized in the schematic (top right panel).

and a mobility of  $1.3 \times 10^6 \text{ cm}^2/\text{Vs}$  (with current flow along the [110] direction). The hole effective mass of  $0.073(1)m_e$ , measured from the temperature dependence of the Shubnikov–de Haas oscillations at low field, is remarkably similar to that of electrons in GaAs ( $0.067m_e$ ) and much lower than for holes in GaAs. The Dingle ratio is  $78 \pm 2$  [13,14].

Figure 1 shows the structure of the wafer. A reverse linear-graded, strain tuning buffer [15] terminating in over-relaxed  $\text{Si}_{0.2}\text{Ge}_{0.8}$  is followed by the Ge quantum well, which is under 0.65% biaxial compressive strain. Holes are supplied from a boron-doped layer that is set back 26 nm above the quantum well. Magnetoresistance measurements were performed at the National High Magnetic Field Laboratory using static fields of up to 35 T and at temperatures down to 26 mK. We note that results on a similar sample grown in our laboratory have already been reported [16] but without any discussion of the CF family in the lowest Landau level.

Figure 1 shows field traces of the longitudinal resistance taken in a  $4 \times 4$ -mm square geometry at four different temperatures with the current along the [110] direction. The Hall resistance was measured separately in a Hall bar geometry. There are clear quantum Hall states at fractions in the main CF sequence around  $\nu = 1/2$  and  $\nu = 3/2$ . As seen in CdTe [17], there is some deviation from simple

activated behavior even when there are good quantum Hall states. Around  $\nu = 5/2$ , we find a minimum on  $\rho_{xx}$  but no clear plateau at  $\nu = 5/2$ . The mobility of our sample is high for a Ge sample, but it is still significantly less than for GaAs samples showing such clear FQH states. For example, in a  $p$ -doped GaAs sample, with  $\mu = 2.3 \times 10^6 \text{ cm}^2/\text{Vs}$  (close to twice that of our sample), there are precursor signals of states at  $\nu = 3/5$  and at  $\nu = 4/7$  but not quantized Hall states [18]. In  $n$ -doped GaAs, no sign of the  $\nu = 5/2$  state is visible for  $\mu \lesssim 6.7 \times 10^6 \text{ cm}^2/\text{Vs}$  [1,2].

While the zero field mobility reflects all scattering, we assume that it is the long-range potential of the ionized donors that controls the dissipation in quantum Hall states. Any short-range scattering centers, not located directly in the saddle points of the long-range impurity potential, will not affect the response of the quantum Hall state. This makes it clear why mobility is unlikely to be a good indicator of strong quantum Hall states [1].

We have analyzed the data using the model developed in Refs. [7,19], assuming that localized regions or puddles of compressible regions are nucleated within the incompressible quantum Hall fluid. The dissipative response is controlled by excitations, localized in one puddle, crossing to another via a saddle point in the impurity potential (see Fig. 2). The saddle points act as effective resistors, and the puddles of quasiparticles (QP) and quasiholes (QH) act as reservoirs in a resistor network [6]. The response is then that of the average saddle point (with barrier height  $\Delta_s/2$ ) [20,21].

If the energy required to create a QP-QH pair near a saddle point is  $\Delta_s$ , the average barrier height to traverse a saddle point is  $\Delta_s/2$  for both QPs ( $E_{sp}$ ) and QHs ( $E_{sh}$ ). In the absence of tunneling through the saddle point, the dissipative conductance per square is  $[2(qe)^2/h]e^{-\Delta_s/2kT}$  as predicted by Polyakov and Shklovskii [6]. Taking into account tunneling gives a dissipative conductance per square

$$\sigma_{xx}^{\square} = 2 \frac{(qe)^2}{h} F[\Delta_s, a/l_q], \quad (1)$$

where  $a$  is the typical saddle point width (see Fig. 2) and  $l_q$  is the magnetic length for QPs/QHs with fractional charge  $qe$  [7,14,22]. We fit the data to the computed form for  $F$  using the gap  $\Delta_s$  and the width parameter  $a$  as free parameters.

We convert the measured minimum values of  $\rho_{xx}$  at each filling fraction and for each temperature to equivalent conductivities  $\sigma_{xx}$  (the Hall resistance is taken as the corresponding quantized value). The results for  $\nu = 7/5$  are shown in Fig. 2. We also show the results of the more traditional method of assuming the Arrhenius form, drawing a tangent to the curves at the inflection point and identifying the gradient with an energy gap  $\Delta_i$ .

The aspect ratio (width/length) of the active region of the sample gives an overall additive constant to  $\ln \sigma_{xx}$  [14]. We use this as a consistency check on the model, as it should

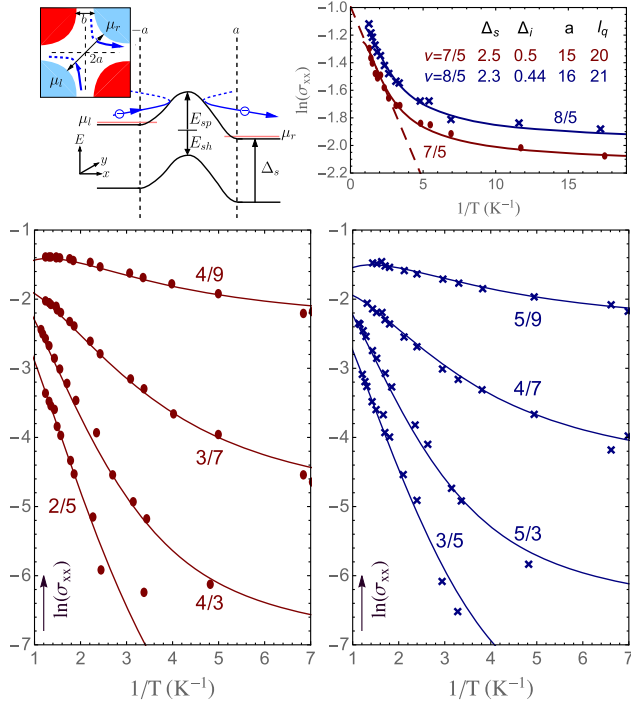


FIG. 2. Band alignment and particle flow across a typical saddle point in the potential (top left panel). The blue (red) regions are compressible with nucleated QPs (QHs), while the white region is incompressible. Dissipation occurs when excitations move across a saddle point from a compressible region with chemical potential  $\mu_l$  to a compressible region at  $\mu_r$ . The transfer proceeds by thermally activated tunneling across the energy barrier, which on average is  $\Delta_s/2$ . The resulting logarithmic conductance [with  $\sigma$  in units of  $2(qe^2)/h$ ] at different filling fractions is shown in the top right panel and below. For the  $\nu = 7/5$  and  $\nu = 8/5$  states, the theoretical prediction and model parameters used to describe the data are shown (the data at  $\nu = 8/5$  have all been raised by 0.1 for clarity).

not vary significantly between filling fractions. For the states at  $\nu = 2/3, 2/5, 3/5, 3/7$ , and  $4/7$ , the data in Fig. 2 are fitted assuming a constant aspect ratio of 1.8. At filling fractions with small gaps, we can expect that the effective width of the percolating incompressible region may reduce as the state is weaker. At  $\nu = 4/9$  and  $\nu = 5/9$ , the aspect ratio is reduced to 1.4 and 1.5, respectively. However, we should emphasize that at these filling fractions the states are weak (the ratio between low and high temperature conductance is less than 1.9).

In Fig. 3 we compare the results for  $\Delta_s$  with the predicted dependence on the magnetic field for the composite fermion model [23]. For spin-polarized states at filling fraction  $\nu_p = p/(2p+1)$ ,  $1 \pm \nu_p$  or  $2 - \nu_p$ , the theory predicts that the gap in a homogeneous system  $\Delta_h$  varies as

$$\Delta_h = \frac{C}{|2p+1|(\ln|2p+1| + C')} \frac{e^2}{\epsilon l_0}, \quad (2)$$

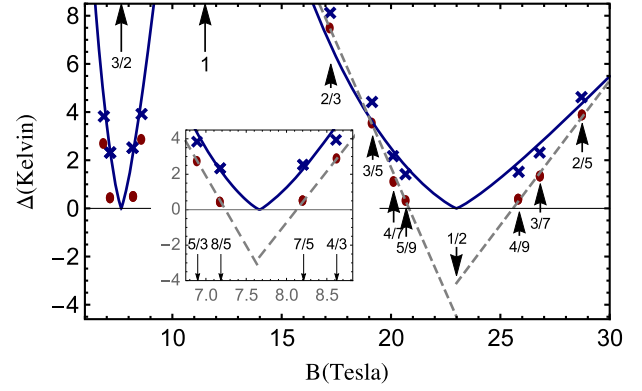


FIG. 3. Estimates of the gap as a function of magnetic field. The crosses denote the estimate of the gap at each  $\nu$ ,  $\Delta_s$ . The ovals show the energies  $\Delta_l$  estimated from the gradients in the Arrhenius plots taken at the point of inflection for each  $\nu$  (see Fig. 2). The solid lines are the dependence predicted in CF theory [23] with  $C' = 2$  and  $C = 0.25$  [see Eq. (2)]. The straight dashed lines are best linear fits. Inset: Results close to  $\nu = 3/2$  on an expanded scale.

where  $C$  and  $C'$  are dimensionless constants and  $l_0$  is the magnetic length. Analysis of the logarithmic divergences as  $p \rightarrow \infty$  for a homogeneous system with zero width and without LL-mixing corrections suggested  $C = 1.27$  [23], while comparison with exact diagonalization studies put  $C' = 3.0$  and, later,  $C' = 4.11$  [24]. We have treated  $C$  and  $C'$  as free parameters and compared with the values we obtain for  $\Delta_s$ . The results are shown as solid lines in Fig. 3 with  $C = 0.25$  and  $C' = 2$ .

The agreement between CF theory and the estimated gaps  $\Delta_s$  is good even at filling fractions above  $\nu = 1$ . The values of the constants  $C$  and  $C'$  are different from those estimated for the homogeneous zero width case and indicate a larger role for the logarithmic corrections to the CF effective mass. The result (2) can be rewritten as a formula for the effective mass via

$$m^*(p) = \hbar^2 \left( \frac{\epsilon l_0}{e^2} \right) \frac{\ln|2p+1| + C'}{C}. \quad (3)$$

As expected, the effective mass depends on the magnetic length—it varies between  $0.69 m_e$  at  $\nu = 5/3$  and  $1.09 m_e$  at  $\nu = 2/3$  (both have  $p = -2$ ) and on  $p$ . Our results also suggest that the dependence of the effective mass on  $p$  is stronger than predicted for the homogeneous 2DEG ( $C' = 2$  instead of 4.1 suggested by diagonalizations of the Hamiltonian of homogeneous systems [24]).

Although there is no microscopic model giving the typical saddle point width  $a$  as a function of  $\Delta_s$ , we assume that it should be related to  $b$ , the width of the incompressible region between two puddles (see upper left panel in Fig. 2). We estimate  $b$  using the theory developed for edges, which computes the electrostatic potential induced by fixing the charge density at the incompressible value [11]. The width  $b$

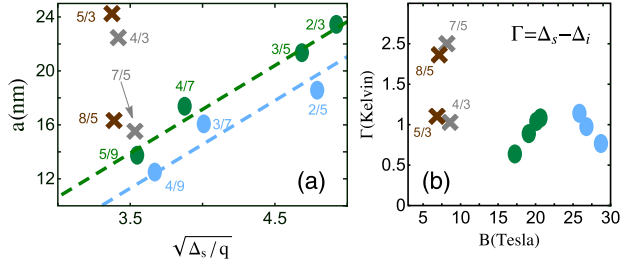


FIG. 4. (a) The saddle point width parameter  $a$  plotted against  $\sqrt{\Delta_s/q}$  (in units of  $\text{K}^{1/2}$ ), for the different CF families (left). Dashed lines are guides to the eye. (b)  $\Gamma = \Delta_s - \Delta_i$  for different filling fractions.  $\Gamma$  reflects the reduction in the apparent gap due to tunneling and has traditionally been assumed to be independent of the filling factor.

is determined by setting the energy to create a QP-QH pair equal to the electrostatic energy gained by transferring a QP from the low to the high density side of the incompressible region. This gives  $b^2 = \{[8e(e/q)\Delta]/[q\pi dn/dx|_c]\}$ , where  $dn/dx|_c$  is the gradient of the carrier density at the center of the incompressible strip if the system was fully screening the background impurity potential.

In Fig. 4(a) we show  $a$  as a function of  $\sqrt{\Delta_s/q}$ , where the  $\Delta_s$  are values from fits to the data. For the two families  $\nu^{qp} = p/(2p+1)$  and  $\nu^{qh} = 1 - p/(2p+1)$ , we see that a linear dependence on  $\sqrt{\Delta_s/q}$  works quite well. There is evidence of a systematic difference between the QP family and the QH family, with the width in the QH case slightly larger than in the QP case. A difference between the two cases is not surprising, given that the boundary of a compressible puddle is an internal edge and the fractionally charged QP/QH is an internal edge excitation. As the model does not factor in any details of the edge state reconstruction, a small systematic difference between the results for the two families [ $\nu = p/(2p+1)$  and  $\nu = 1 - p/(2p+1)$ ] is to be expected.

In the case of the partially occupied reverse spin LL (RSLL), Fig. 4(a) shows that  $a$  does not follow the simple CSG scaling [11]. The scaling assumes that the compressible regions behave like perfect metals, i.e., that the screening length in the puddles is short enough for the electrostatic model to be a good approximation. This turns out to be unlikely in the RSLL in our sample. The theory would require that the screening electron density changes between edges of an incompressible strip by  $\Delta n \sim bdn/dx|_c$ . At filling fraction  $\nu = 1 + p/(p+1)$ , the corresponding change in number density of QPs, with charge  $qe = e/(2p+1)$ , is  $\Delta n_{QP} = \Delta n/q$ . The average density of electrons in the RSLL is  $n_1 = np/(3p+1)$ . This gives

$$\frac{\Delta n_{QP}}{n_1} = \frac{\Delta n}{n} \frac{(2p+1)(3p+1)}{p}. \quad (4)$$

For uncorrelated ionized donors, the typical density gradient would be  $dn/dx|_c \sim \sqrt{n/8\pi/d^2}$  [4], which would

give  $(\Delta n_{QP}/n_1) \sim 0.2$  at  $\nu = 4/3$  ( $p = 1$ ) and  $0.3$  at  $8/5$  ( $p = 2$ ). These would be large density variations in a LL and would take the system close to (or into) neighboring CF states [in the CF model, a ratio  $1/(p+1)$  defines the next CF state in the hierarchy]. The assumption of fully screening metallic regions, which is implicit in the CSG model, applies to this sample at filling fractions above  $\nu = 1$ .

Our model is not consistent with an approach that assumes a  $\nu$ -independent broadening due to impurities. The difference,  $\Gamma = \Delta_s - \Delta_i$ , between the intrinsic gap and the slope measured in Arrhenius plots of the dissipative conductance depends on the strength of the incompressible state. Figure 4(b) shows the broadening parameter  $\Gamma$ , which, according to our model, varies between 2.5 K at  $\nu = 7/5$  and around 0.6 K at  $\nu = 2/3$ . The smaller  $\Gamma$  is, the larger the intrinsic gap. On the other hand, the dashed lines in Fig. 3(a) show that, if we fit  $\Delta_i$  by a linear function of  $B - B(\nu = 1/2)$  [10], we obtain different values for the broadening and effective masses: 4.5 K and  $0.65m_e$  for  $\nu < 1/2$ , and 3.1 K and  $1.1m_e$  for  $\nu > 1/2$ . We conclude that estimates of intrinsic gaps using states of significantly different strengths on the basis of a  $\nu$ -independent broadening parameter are likely to be unreliable (see also Ref. [25]).

We have reported measurements of the dissipative response and the Hall response of a Ge quantum well in the fractional quantum Hall regime. The results across all filling fractions, for which FQH states are found, fit well with the predictions of a model of thermally assisted quantum tunneling. The model demonstrates that the properties of the internal edge states around puddles control how a fractional quantum Hall state responds to any slowly varying background potential. Our sample has a high mobility for Ge but still significantly below that for GaAs samples (both  $p$  and  $n$  type); however, it shows much stronger quantum Hall states than would be expected given the mobility. Our model makes it clear why the mobility is not likely to be a reliable measure of the sample in the quantum Hall regime. The strength of the quantum Hall state is determined by the strength of the long-range impurity potential at saddle points in the potential.

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