Quantum Enhancement of the Index of Refraction in a Bose-Einstein Condensate

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We study the index of refraction of an ultracold bosonic gas in the dilute regime. Using phase-contrast imaging with light detuned from resonance by several tens of linewidths, we image a single cloud of ultracold atoms for 100 consecutive shots, which enables the study of the scattering rate as a function of temperature and density using only a single cloud. We observe that the scattering rate is increased below the critical temperature for Bose-Einstein condensation by a factor of 3 compared to the single-atom scattering rate. We show that current atom-light interaction models to second order of the density show a similar increase, where the magnitude of the effect depends on the model that is used to calculate the pair-correlation function. This confirms that the effect of quantum statistics on the index of refraction is dominant in this regime.

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The interaction of light and matter is the subject of intensive study in many disciplines, from cosmology to quantum information and photovoltaics. The crucial parameter describing the interaction between light and matter is the index of refraction $n = c/v$, where c is the speed of light and v the phase velocity of the light in the medium. For dilute Bose gases the refractive index is in most cases determined by the polarizability of a single atoms through the Lorentz model [1]. However, in a seminal paper by Morice *et al.* [2] the effect of dipoles surrounding a scatterer has been studied theoretically in the case of ultracold atoms. In the paper it is shown that the index of refraction is strongly modified around the phase transition to Bose-Einstein condensation (BEC). This effect depends on the pair-correlation function and thus on the quantum statistics of the atoms, either bosonic of fermionic. Bosonic atoms tend to bunch in space, whereas fermionic atoms tend to antibunch, as has recently been observed in an experiment [3]. Although the prediction by Morice *et al.* [2] has been in the literature for 2 decades, it has not been observed experimentally. In this Letter we observe that the index of refraction of matter is strongly modified by the bosonic nature of the atoms that constitute the matter.

The understanding of the interaction between atoms and light is very important in the field of ultracold atoms. Laser cooling and trapping plays a crucial role in the achievement of Bose-Einstein condensation. Resonant and near-resonant light are used for imaging purposes, such as fluorescence, absorption, and phase-contrast imaging (PCI) [4]. Light fields are used to outcouple atoms from ultracold clouds using, for instance, Bragg pulses [5], Raman coupling [6], etc. Here we exploit PCI to investigate the index of refraction of a cloud of ultracold atoms. In PCI the real part of the refractive index is detected [7–9] and subsequently used to determine the density distribution of atoms in the trap. In Ref. [2] it is shown that for small detunings of the light from resonance both the real and imaginary parts of the refractive index can be modified. For large detunings the real part is nearly unaltered, but the imaginary part, although small at these detuning, is strongly enhanced by the spatial correlation of the atoms, as we will show. Since we are using an open transition in PCI, we can detect the imaginary part very sensitively due to atomic losses and this provides us with a tool to investigate the refractive index.

In PCI the detuning can be chosen such that the scattering of the light is small, although the phase shift can be substantial. This enables nondestructive, *in situ* imaging and thus the multishot sampling of images on a single atomic cloud. It facilitates the reconstruction of the dynamics of the system using a single cloud eliminating cloud-to-cloud fluctuations in the preparation of the sample. However, the imaginary part of the refractive index can never be exactly zero for a real part not equal to one, so the cloud will still scatter some photons during a single shot. In PCI this is a detrimental effect, which should be minimized as much as possible. Here we exploit this property. The pulse of light from which the phase shift is determined in the phase-contrast technique creating an image is simultaneously used to prepare new conditions for the atomic cloud, since the light induces a small loss of particles accompanied by a slight heating. By taking a series of images of a single cloud we detect the index of refraction over a range of densities and temperatures. Since PCI is used by us at such a large detuning, the index of refraction of the cloud is close to one, only single scattering in the medium plays a role, and there are no nonlinear effects in the light intensity. The saturation parameter is much smaller than one, so light-assisted collisions can be neglected.

Using standard laser cooling and evaporative cooling schemes, we produce a cloud of 250×10^6 sodium atoms at

3 μK with a central density of 0.15×10^{20} atoms/m^3 in a cigar-shaped magnetic trap (MT) with trap frequencies $\omega_{\mathrm{ax}}/2\pi = 15.1$ Hz and $\omega_{\mathrm{rad}}/2\pi = 96.0$ Hz. This is still well above the critical temperature for Bose-Einstein condensation of ~ 1 μK. A series of 100 consecutive images with a frame rate of 200 Hz is detected on the CCD camera using light detuned -350 MHz from the $F = 1 \to F' = 1$ resonance. The probe pulses are 80 μs long as controlled by switching an acoustic optical modulator and have an intensity of 50–200 μW/cm^2, corresponding to a saturation parameter of 0.01–0.04. The time between the images (5 ms) is chosen such that the cloud rethermalizes in a few shots due to elastic collisions, since the elastic collision rate is 100/s. Since the rate of change of the temperature is small, the sample is close to thermal equilibrium at all times. We extract the temperature and the chemical potential by fitting the data using the Popov approximation [10], which includes interaction effects in the cloud above $T = 0$. The number of particles can be calculated from the chemical potential and the temperature, and the experimental data of such a measurement are shown in Fig. 1. During the exposure of the atoms to the light, the cloud is heated and the number of atoms decreases due to losses induced by the light.

We also produce a BEC with 35×10^6 atoms at a temperature of 500 nK with a central density of 3.0×10^{20} atoms/m^3 in the same trap and take 100 images with PCI using the same probe intensity. The results are also shown in Fig. 1. The vertical scale is logarithmic, so the slope is directly proportional to the loss rate. The loss rate

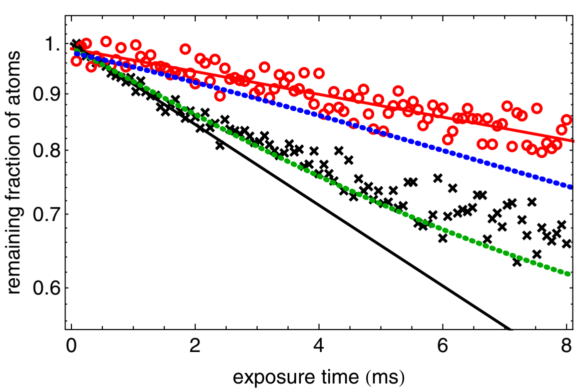

FIG. 1. A plot of the remaining fraction of particles on a logarithmic scale as a function of exposure time for a cloud above (red circles) and below (black crosses) the critical temperature T_C. The exposure time is the total time that the cloud is illuminated with light for PCI and the probe intensity is 67.7 μW/cm^2. The solid lines are linear fits assuming an exponential decay at $t = 0$. The statistical error can be inferred from the spread in the data points. The dotted lines are independent calculations (as discussed in the text) of the loss rate for the condensed cloud based on the intensity of the probe light and the enhancement induced by the pair-correlation function for an ideal Bose gas (dotted green line) and Hartree-Fock model (dotted blue line).

in the BEC is clearly larger than the loss rate in the thermal cloud, since the initial slope is 3 times larger in the BEC. Also, one can see that the loss rate decreases gradually as the BEC is depleted and approaches the value for thermal atoms above T_C. Many of these measurements are done for different light intensities and initial conditions both above and below T_C and show a comparable increase of the scattering rate below T_C.

In a spontaneous Raman transition an atom can be excited by the probe light followed by spontaneous emission back to the ground state. In our experiment the ^{23}Na atoms are initially in the state $|F = 1, m_F = -1\rangle$ of the $3^2S_{1/2}$ level because only atoms in that state are trapped in the MT. Using π-polarized light they can be excited close to the states $|F' = 1, m_{F'} = -1\rangle$ and $|F' = 2, m_{F'} = -1\rangle$ of the $3^2P_{3/2}$ level. The spontaneous decay can be to the initial state $|F = 1, m_F = -1\rangle$, but also to the states $|F = 1, m_F = 0\rangle$, $|F = 2, m_F = -2\rangle$, $|F = 2, m_F = -1\rangle$, and $|F = 2, m_F = 0\rangle$. All of these magnetic states are non-low-field seeking states, so atoms that decay to these states will no longer be trapped by the MT and thus are lost from the cloud. At the same time atoms will also gain on average an amount of energy of $\hbar^2k^2/m$, which is twice the recoil energy of a single photon. This energy will be redistributed due to elastic collisions among the remaining atoms, which leads to the heating of the cloud.

The Lorentz oscillator model [1,11] provides the simplest picture of the interactions between atoms and light. Electromagnetic radiation induces a dipole moment in the atom proportional to the polarizability, which has a real and imaginary part. For a single atom the polarizability α for an electromagnetic field interacting with a single ground state g and e excited states is given by [9]

$$\alpha = \imath \frac{\varepsilon_0 c \sigma_\lambda}{\omega} \sum_e \frac{C_{eg}}{1 - 2\imath \frac{\delta_{eg}}{\gamma}}, \tag{1}$$

where $\sigma_\lambda = 3\lambda^2/2\pi$ is the cross section for light absorption in a two-level system, C_{eg} is the relative strength for the transition from $g \to e$, $\delta_{eg} = \omega - \omega_{eg}$ is the detuning, ω_{eg} is the transition frequency, and γ is the natural linewidth. For a low density cloud the index of refraction n is given by $n^2 = 1 + \rho\alpha/\varepsilon_0$, where ρ is the atomic density. Since an atom has no internal degrees of freedom to dissipate energy, absorption is always followed by scattering and the scattering rate R depends on the imaginary part of n as

$$R = \frac{2I}{\hbar c} \int d\vec{r}\, \mathrm{Im}(n(\vec{r})), \tag{2}$$

where I is the intensity of the light and the integration is over the cloud. For sufficiently low densities this reduces to N times the single-atom scattering rate $\Gamma_{\mathrm{sc}} = R/N = I\mathrm{Im}(\alpha)/\hbar c \varepsilon_0$, with N the number of atoms. The accumulated phase shift $\phi(x, y)$, which is measured in PCI, depends on the real part as

$$\phi(x,y) = k\int dz\mathrm{Re}(n(\vec{r})-1), \tag{3}$$

where the integration is over the line of sight of the light.

Using these relations we make a prediction for the loss of particles as a function of exposure time. This prediction comprises calculating during a certain time interval the probability of scattering a photon and the probability of decaying back into an untrapped ground state. This gives us the fraction of atoms that is lost during the exposure time of a single image. In our case the atoms are nearly at rest and the Doppler shifts are much smaller than the linewidth and the detuning. The intensity of the light is determined from the number of electron counts on the CCD camera with a calibrated quantum efficiency. At a detuning larger than the hyperfine splitting in the excited state, the polarizability becomes insensitive to the angle of the linear polarization of the probe light [9]. Using these parameters in Eq. (2), assuming the density is sufficiently low we calculate a loss rate of 26.5 s^{-1}. In Fig. 1 the data of the thermal cloud yield a slope of $23.8 \pm 1.0\ \mathrm{s}^{-1}$, which differs by 10% from the theoretical value. The deviation from the theoretical value can be due to systematic uncertainties, such as the determination of the probe intensity in the vacuum chamber at the location of the atoms, since the probe beam is not fully homogeneous. This shows that within the error bars we can successfully describe the scattering of noncondensed atoms using the single-particle scattering model. However, the Lorentz model deviates up to a factor of 3 from the loss rate measured in a condensate.

The interaction of light with degenerate atoms has been the subject of many studies [2,12–17]. Here, we will focus on the case where the light is sufficiently detuned and the intensity sufficiently low that multiple scattering can be neglected. If we consider the propagation of light through a medium of dipoles, the incoming light field acts on the dipoles inducing a dipole moment. These dipoles act back on the field and the resulting equations have to be solved self-consistently. It can be shown that to second order in the polarizability the refractive index can be written as [2,18]

$$n^2 = 1 + \frac{\alpha\rho/\varepsilon_0}{1+\alpha\rho C/\varepsilon_0}, \tag{4}$$

where the factor C can be written as

$$C \equiv -\frac{1}{3} - \int d\vec{r}e^{-ikz}\tilde{g}_{xx}(\vec{r})h(\vec{r}) \tag{5}$$

when the polarization of the incoming light field is in the x direction. Here, the factor $1/3$ stems from the Lorentz-Lorenz correction [1,11], which has been measured in atomic vapors [19] and more recently in atomic vapors of nanometer thickness [20]. The second term on the right-hand side is the correlation integral, which describes the effect of the field $\tilde{g}_{xx}(\vec{r})$ of the surrounding dipoles assuming that the distribution of those dipoles is given by the second-order correlation function $h(\vec{r})$. Modifications of the Lorentz-Lorenz correction have recently been observed by Jennewein *et al.* [21], but their work is close to resonance, where multiple scattering dominates.

The pair-correlation function has been calculated in the Bogoliubov approach in Refs. [22,23], but their results only apply for temperatures far below T_C. In Refs. [24–26] the pair-correlation function for a Bose gas is calculated over the whole temperature range, but in these approaches the anomalous density-density contribution to the pair-correlation function is neglected. Thus, these methods cannot be used for the Bogoliubov or Popov approach. Here we will use the results of Ref. [25] for the ideal Bose gas and Hartree-Fock approximation, where in the first case the interaction between the condensate and thermal cloud is not taken into account and in the last case only the interaction of the condensate on the thermal cloud is accounted for. Measurement of second-order correlations have been performed for ultracold Bose gases using spatially or temporally resolved detection of particles [27–31]. Here, due to the correlation integrand of Eq. (5), which oscillates over a length scale of the optical wavelength, we are sensitive to the long-range behavior of the pair-correlation function. It is interesting to note around T_C that the correlation length, the thermal de Broglie wavelength, and the optical wavelength all have approximately the same length scale.

In Fig. 2 the real and imaginary part of the index of refraction are shown for typical experimental conditions. Note that the real part of $(n-1)$ is small and nearly independent of temperature. Therefore, the change in the real part has no effect on the PCI and its value from the Lorentz model can be used to determine the density from the accumulated phase. The imaginary part is nearly 2 orders of magnitude smaller than the real part minus one, which is a requirement for the use of PCI as a nearly nondestructive imaging technique. However, it has a very strong dependence on temperature and is strongly increased just below T_C, although the detailed shape of the increase on temperature is different for the different methods. It is interesting to note that for the imaginary part the effect of the Lorentz-Lorenz term of $1/3$ is much smaller than the effect of the correlation integral, as can be seen in Fig. 2(b). For our experimental conditions, the Lorentz-Lorenz term is not important.

It is interesting to observe that both for low and high temperatures the index of refraction asymptotically converges to the Lorentz model, although for very different reasons. For high temperatures the gas is thermal and the correlation length of the cloud is proportional to the de

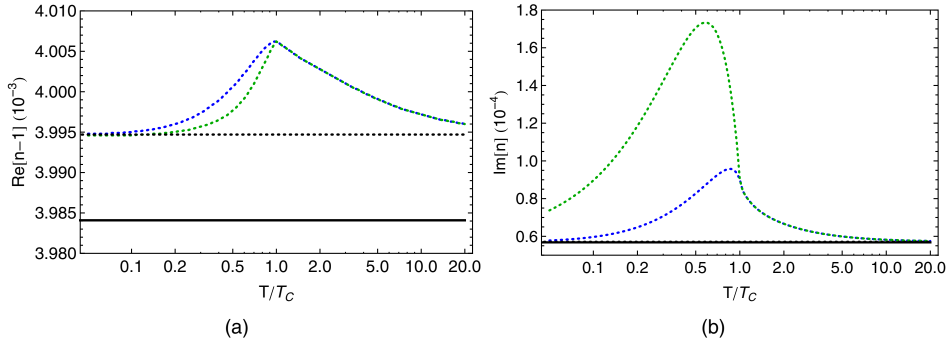

FIG. 2. The (a) real and (b) imaginary part of the index of refraction n calculated as a function of temperature for a homogeneous density of 0.54×10^{20} atoms/m^3 and a detuning of −350 MHz. The results are for the Lorentz model (solid black line), the Lorentz model including the Lorentz-Lorenz correction (dotted black line), and Eq. (4), using the pair-correlation function of an ideal Bose gas (dotted green line) or the Hartree-Fock model (dotted blue line). The refractive index already starts to increase above T_C, acting as a precursor for Bose-Einstein condensation.

Broglie wavelength, which decreases for increasing temperature. For low temperatures the correlation length increases strongly for decreasing temperature. However, the value of the pair correlation at $r = 0$ depends only on the densities and is given below T_C by $h(0) = 1 - n_c^2/(n_c + n_{th})^2$, where $n_{c,th}$ are the density of the condensed and thermal atoms, respectively [25]. For an ideal Bose gas, $h(0) = (T/T_C)^{3/2}[2 - (T/T_C)^{3/2}]$ in the condensed phase and thus decreases for decreasing temperature. In both cases it leads to a vanishing contribution of the correlation integral in Eq. (5). The index of refraction strongly increases in the intermediate temperature range just below T_c and acts as a precursor for Bose-Einstein condensation.

In order to compare the results of the calculation with our experiments, we average the refractive index over the density distribution in the trap using the local density approximation. Using the imaginary part of n we determine for each experimental data point (number of particles, temperature) the enhancement of the scattering rate. Figure 3(a) shows a contour plot for the enhancement for the ideal Bose gas as a function of the fit parameters μ and T after averaging over the trap. The data points show the conditions of the cloud, which change after each shot due to the losses and heating induced by the probe light. Figure 3(b) shows the results for the correction factor as a function of exposure time.

Based on this result we can make an independent prediction of the number of particles as a function of time with no free parameters, where we assume the loss rate of the Lorentz model to be multiplied by the enhancement factor as shown in Fig. 3(b), which is now a function of exposure time. The result is given by the dotted lines in Fig. 1. Both methods (ideal Bose gas and Hartree-Fock) show an increase of the scattering rate of the BEC compared to the thermal cloud, as indicated by the increased slope for zero exposure time, even though the calculated loss rate is slightly overestimated for the ideal Bose gas and underestimated for the Hartree-Fock method. Including anomalous density-density correlations in the pair-correlation function might modify its long-range behavior and thus determine the magnitude of the enhancement of the scattering rate. We hope that the current

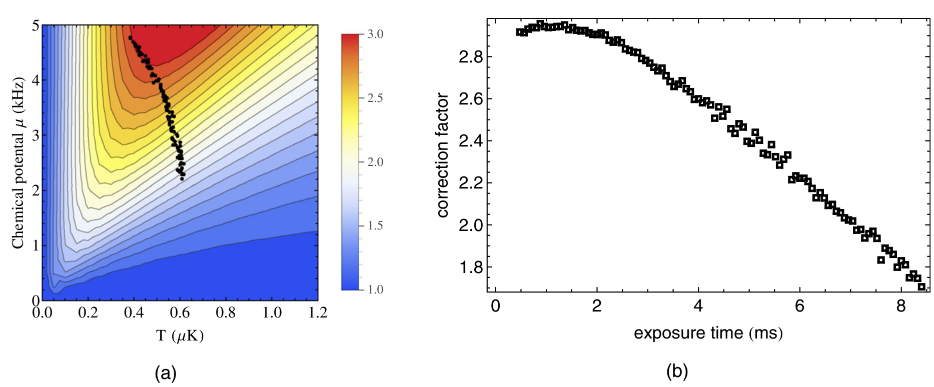

FIG. 3. (a) The enhancement of the scattering rate as a function of μ and T after averaging over the trap for the ideal, condensed Bose gas. The experimental conditions are shown by the black trace. The conditions change during the measurement due to the losses and heating induced by the probe light. (b) The enhancement as a function of the exposure time for our experimental conditions. The values follow the black trace in (a).

experimental results will stimulate theoreticians to derive the pair-correlation function in the Popov approach.

We conclude that the effect of the pair-correlation function significantly changes the refractive index of the gas in this regime and the experimental data can be explained well by taking this effect into account. In the case of absorption imaging the change is negligible, since the gas is usually allowed to expand before imaging. Our imaging method of measuring the number of particles and temperature, and simultaneously inducing small changes in these parameters, promises to be valuable to measure other processes, such as the heat capacity of the gas and of a BEC. Preferably this should be done in an optical dipole trap to prevent the loss of particles due to scattering into other states.

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