## Electroweak Corrections to  $pp \to \mu^+\mu^-e^+e^- + X$  at the LHC: A Higgs Boson Background Study

B. Biedermann, <sup>1</sup> A. Denner, <sup>1</sup> S. Dittmaier, <sup>2</sup> L. Hofer, <sup>3</sup> and B. Jäger<sup>4</sup>

<sup>1</sup> Julius-Maximilians-Universität Würzburg, Institut für Theoretische Physik und Astrophysik, D-97074 Würzburg, Germany<br><sup>2</sup> Albert Ludwigs Universität Freiburg, Physikalisches Institut, D 70104 Freiburg, Germany

 $^{2}$ Albert-Ludwigs-Universität Freiburg, Physikalisches Institut, D-79104 Freiburg, Germany

 ${}^{3}$ Department d'Estructura i Constituents de la Matèria (ECM), Institut de Ciències del Cosmos (ICCUB),

Universitat de Barcelona (UB), Martí Franquès 1, E-08028 Barcelona, Spain <sup>4</sup>

Eberhard-Karls-Universität Tübingen, Institut für Theoretische Physik, D-72076 Tübingen, Germany

(Received 6 February 2016; published 21 April 2016)

The first complete calculation of the next-to-leading-order electroweak corrections to four-lepton production at the LHC is presented, where all off-shell effects of intermediate Z bosons and photons are taken into account. Focusing on the mixed final state  $\mu^+\mu^-e^+e^-$ , we study differential cross sections that are particularly interesting for Higgs boson analyses. The electroweak corrections are divided into photonic and purely weak corrections. The former exhibit patterns familiar from similar W- or Z-boson production processes with very large radiative tails near resonances and kinematical shoulders. The weak corrections are of the generic size of 5% and show interesting variations, in particular, a sign change between the regions of resonant Z-pair production and the Higgs signal.

DOI: [10.1103/PhysRevLett.116.161803](http://dx.doi.org/10.1103/PhysRevLett.116.161803)

Introduction.—The investigation of pair production processes of electroweak (EW) gauge bosons  $W$ , Z, and  $\gamma$  is of great importance at the CERN Large Hadron Collider (LHC). These processes have sizable cross sections and provide experimentally clean signatures via the leptonic decay modes of the W or Z bosons. On the one hand, they offer an indirect window to potential new-physics effects through their sensitivity to the self-interactions among the EW gauge bosons; on the other hand, these reactions represent sources of irreducible background to many direct searches for new particles (e.g., additional heavy gauge bosons  $W', Z'$ ) and<br>to precision studies of the Higgs boson discovered in 2012 to precision studies of the Higgs boson discovered in 2012, in particular.

In order to optimally exploit and interpret LHC data, theoretical predictions to weak-gauge-boson pair production have to be pushed to an accuracy at the level of percent, a task that requires the inclusion of higher-order corrections of the strong and EW interactions and of decay and offshell effects of the W or Z bosons. In this Letter, we focus on the reaction  $pp \to \mu^+\mu^-e^+e^- + X$ , which does not only include doubly resonant ZZ production but also interesting regions in phase space where at least one of the Z bosons is far off shell, as, for example, observed in the important Higgs decay channels  $H \rightarrow 4$  leptons.

Precision calculations for Z-boson pair production with leptonic decays have been available for a long time including next-to-leading-order (NLO) QCD corrections [\[1](#page-4-0)–3]. They have even been pushed to next-to-next-toleading-order (NNLO) accuracy recently [\[4,5\],](#page-4-1) with a significant contribution from gluon-gluon fusion calculated already before [6–[8\].](#page-4-2) Beyond fixed perturbative orders, NLO QCD corrections were matched to a parton shower in Refs. [9–[13\];](#page-4-3) in Ref. [\[14\]](#page-4-4), even different jet multiplicities were merged at NLO QCD. Electroweak corrections at NLO are only completely known for stable Z bosons [\[15,16\]](#page-4-5) and in some approximation including leptonic decays of on-shell Z bosons [\[17\].](#page-4-6) The EW corrections to Z-pair production with off-shell Z bosons, on the other hand, are not yet known. In this Letter, we fill this gap and present results of the first full NLO EW calculation for the process  $pp \rightarrow \mu^+\mu^-e^+e^- + X$  in the standard model, including all off-shell contributions. This allows us, in particular, to investigate EW corrections in the yet unexplored kinematic region below the ZZ threshold, where direct Z-pair production is an important background to Higgs boson analyses.

General setup of the calculation.—At leading order (LO), the production of  $\mu^+\mu^-e^+e^-$  final states almost exclusively proceeds via quark-antiquark annihilation. Contributions from γγ collisions are extremely small (they contribute only at the level of a few per mille to the total cross section) owing to the suppression of the photon density in the proton; we, therefore, do not consider  $\gamma\gamma$ contributions in this Letter.

The LO amplitude for  $q\bar{q}$  annihilation involves contributions containing two, one, or no Z-boson propagators that may become resonant. At NLO, the same is true for  $q\bar{q}$ amplitudes with EW loop insertions and the corresponding amplitudes with real photonic bremsstrahlung. Since no couplings to W bosons are involved at LO, we can divide the EW corrections into separately gauge-independent photonic and purely weak contributions. By definition, the former comprise all contributions with real photons and all loop and counterterm diagrams with photons in the loop coupling to the external fermions, while the latter are furnished by the remaining EW corrections. Actually, the NLO EW corrections include contributions from  $q\gamma$ ,  $\bar{q}\gamma$ , and  $\gamma\gamma$  channels as well, but those contributions turn out to be phenomenologically unimportant.

Apart from the algebraic complexity, a major complication in the NLO EW calculation arises from the appearance of resonances which require at least a partial Dyson summation of the potentially resonant self-energy corrections, a procedure that jeopardizes the gauge invariance of the result if no particular care is taken. We employ the complex-mass scheme [\[18,19\]](#page-4-7), which provides a gaugeinvariant solution to this problem at NLO by replacing the real W- and Z-boson masses by complex quantities, including also the corresponding complexification of EW couplings. To evaluate all one-loop integrals with complex W or Z masses with sufficient numerical stability in the four-body phase space, we apply the library COLLIER, which is mainly based on the results of Refs. [20–[22\]](#page-4-8) and briefly described in Ref. [\[23\].](#page-4-9)

Infrared (soft and/or collinear) singularities in the realemission amplitudes are extracted via dipole subtraction, as formulated in Refs. [\[24,25\]](#page-4-10) for photon radiation. The infrared-singular contributions are alternatively treated in dimensional or in mass regularization. We have checked numerically that the sum of all (virtual and real) corrections is infrared finite and independent of the regularization scheme used.

We have performed two independent calculations of all contributions and found results that are in mutual agreement within statistical uncertainties of the final Monte Carlo phase-space integration. One calculation closely follows the strategy described in Refs. [\[19,26\]](#page-4-11), where NLO EW corrections to  $e^+e^- \rightarrow 4$  fermions via W-boson pairs were calculated in the loop part and builds on Ref. [\[27\]](#page-4-12) in the real correction and the Monte Carlo integration. The other calculation has been carried out with the program RECOLA [\[28,29\]](#page-4-13) facilitating the automated generation of the NLO EW amplitudes, in combination with an in-house Monte Carlo generator. Additional checks have been performed employing the MATHEMATICA package POLE [\[30\].](#page-4-14)

Input and event selection.—For the numerical analysis, we consider the LHC running at center-of-mass (c.m.) energies of 7, 8, 13, and 14 TeV and choose the input parameters as follows. The on-shell values for the masses and widths of the gauge bosons,

$$
M_W^{\rm OS} = 80.385 \text{ GeV}, \qquad \Gamma_W^{\rm OS} = 2.085 \text{ GeV},
$$
  

$$
M_Z^{\rm OS} = 91.1876 \text{ GeV}, \qquad \Gamma_Z^{\rm OS} = 2.4952 \text{ GeV} \quad (1)
$$

are first translated into the pole scheme according to

$$
M_V = M_V^{OS}/c_V, \qquad \Gamma_V = \Gamma_V^{OS}/c_V, \n c_V = \sqrt{1 + (\Gamma_V^{OS}/M_V^{OS})^2}, \qquad V = W, Z,
$$
\n(2)

and subsequently combined to complex mass parameters  $\mu_V^2 = M_V^2 - iM_V \Gamma_V$ , as demanded by the complex-mass<br>scheme Since no other particles show up as resonances in scheme. Since no other particles show up as resonances in our calculation, their decay widths can be neglected and their masses taken as on-shell parameters. In detail, we set the Higgs boson and the top-quark masses to

$$
M_H = 125 \text{ GeV}, \qquad m_t = 173 \text{ GeV}. \tag{3}
$$

We work in the  $G_u$  scheme where the electromagnetic coupling  $\alpha$  is derived from the Fermi constant

$$
G_{\mu} = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \tag{4}
$$

according to

$$
\alpha_{G_{\mu}} = \sqrt{2} G_{\mu} M_W^2 (1 - M_W^2 / M_Z^2) / \pi.
$$
 (5)

This choice absorbs the effect of the running of  $\alpha$  to the electroweak scale into the LO cross section and, thus, avoids mass singularities in the charge renormalization. Moreover,  $\alpha_{G_u}$  partially accounts for the leading universal renormalization effects originating from the  $\rho$  parameter. The fine-structure constant

$$
\alpha(0) = 1/137.035\,999\,679\tag{6}
$$

is only used as a coupling parameter in the relative photonic corrections because those are strongly dominated by real photon emission naturally coupling with  $\alpha(0)$ . The relative genuine weak corrections, however, are parametrized with  $\alpha_{G_u}$ .

The renormalization and factorization scales  $\mu_{\text{ren}}$  and  $\mu_{\text{fact}}$  are set equal to the pole mass of the Z boson,  $\mu_{\text{ren}} = \mu_{\text{fact}} = M_Z$ . Since we focus on EW corrections, we consistently employ the set NNPDF2.3QED [\[31\]](#page-4-15) of parton distribution functions (PDFs), which are the only up-to-date PDFs including QED corrections. Following the arguments of Ref. [\[32\]](#page-4-16), we employ a DIS-like factorization scheme in the QED corrections (inspired by the corresponding QCD factorization scheme used in deep inelastic scattering) because EW corrections are not taken into account in the fit of the PDFs to data.

<span id="page-1-0"></span>In the event selection, we apply a set of phase-space cuts that are optimized for Higgs studies, inspired by the CMS and ATLAS analyses [\[33,34\]](#page-4-17). For each lepton  $\ell_i$ , we exclude too low transverse momentum and too large rapidity demanding

$$
p_T(\mathcal{E}_i) > 6 \text{ GeV}, \qquad |y(\mathcal{E}_i)| < 2.5,\tag{7}
$$

and any pair of charged leptons is required to be well separated in the rapidity–azimuthal-angle plane,

$$
\Delta R(\ell_i, \ell_j) = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2} > 0.2.
$$
 (8)

Photons are recombined with the closest  $\ell_i$  if

$$
\Delta R(\gamma, \ell_i) < 0.2. \tag{9}
$$

<span id="page-2-2"></span>To these basic selection criteria, we add cuts on the invariant masses  $M_{\ell_i^+\ell_i^-}$  of the  $\ell_i^+\ell_i^-$  pairs,

40 GeV 
$$
M_{\ell_1^+\ell_1^-} < 120 \text{ GeV},
$$
\n12 GeV 
$$
M_{\ell_2^+\ell_2^-} < 120 \text{ GeV},
$$
\n(10)

with  $\ell_1^+ \ell_1^- (\ell_2^+ \ell_2^-)$  referring to the  $\ell^+ \ell^-$  pair that is closer to (further away from) the nominal mass of the Z boson. Moreover, we impose a cut on the invariant mass  $M_{4\ell}$  of the four-lepton system,

$$
M_{4\ell} > 100 \text{ GeV}.\tag{11}
$$

Numerical results.—In the following, we discuss the LO cross section  $\sigma_{qq}^{LQ}$  for  $pp \rightarrow \mu^+ \mu^- e^+ e^- + X$  and the corresponding full EW and purely weak relative corrections corresponding full EW and purely weak relative corrections  $\delta_{qqq}^{\text{EW}} = \delta_{qq}^{\text{photonic+weak}}$  and  $\delta_{qqq}^{\text{weak}}$ , which are normalized to  $\sigma_{q}^{LO}$ . The label  $\bar{q}q$  indicates that only quark-antiquark<br>annihilation channels are taken into account. The relative annihilation channels are taken into account. The relative corrections  $\delta_{qq}^{\text{EW}}$  and  $\delta_{qq}^{\text{weak}}$  are rather insensitive to the PDF set and, thus, can be used to promote QCD-based cross sections to state-of-the-art predictions via reweighting.

Table [I](#page-2-0) shows  $\sigma_{\bar{q}q}^{\text{LO}}$ ,  $\delta_{\bar{q}q}^{\text{EW}}$ , and  $\delta_{\bar{q}q}^{\text{weak}}$  for various LHC energies. The integrated cross sections are, of course, dominated by resonant Z-boson pair production, i.e., by dominated by resonant Z-boson pair production, i.e., by<br>partonic c.m. energies  $\sqrt{\hat{s}} > 2M_Z$ . Accordingly, the EW<br>corrections largely resemble the size of the known correccorrections largely resemble the size of the known corrections to on-shell ZZ production [\[15,16\]](#page-4-5), which amount to  $\sim$  − 4.5%. The remaining ~1% can be attributed to the EW corrections to nonresonant contributions and the acceptance effects on leptonic Z-boson decays. Although the corrections to Z-boson decays are known to be small in an inclusive setup (at the level of few per mille), in the presence of the applied acceptance cuts they are enhanced to few percent, mainly as a result of the sensitivity to finalstate radiation (FSR). In summary, our results confirm that the NLO EW corrections to the cross section of Z-pair production (including Z decays and off-shell effects) are at the 5% level at the LHC and, thus, have to be taken into account in the confrontation of data with theory. The major

<span id="page-2-0"></span>TABLE I. LO cross section  $\sigma_{qq}^{\text{LO}}$  for  $pp \to \mu^+ \mu^- e^+ e^- + X$  for various I HC energies and corresponding EW ( $\delta^{\text{EW}}$ ) and purely various LHC energies and corresponding EW  $(\delta_{\bar{q}q}^{\text{EW}})$  and purely weak relative corrections ( $\delta_{\bar{q}q}^{\text{weak}}$ ).

$\sqrt{s}$ (TeV)	$\sigma_{\bar{q}q}^{\text{LO}}$ (fb)	$\delta_{\bar{q}q}^{\text{EW}}$ (%)	$\delta_{\bar{q}q}^{\text{weak}}(\%)$
7	7.3293(4)	$-3.4$	$-3.3$
8	8.4704(2)	$-3.5$	$-3.4$
13	13.8598(3)	$-3.6$	$-3.6$
14	14.8943(8)	$-3.6$	$-3.6$

part of the EW corrections is due to genuine weak effects, while photonic corrections remain below the 1% level.

We turn to differential distributions at 13 TeV, focusing on kinematical variables that are particularly sensitive to the off shellness of the intermediate Z bosons, i.e., to distributions that are not accessible by previous calculations based on on-shell Z bosons. Figure [1](#page-2-1) shows the invariant-mass distribution of the  $\mu^+\mu^-$  system and the relative full EW and weak corrections. Note that the distribution is dominated by resonant  $e^+e^-$  pairs throughout ( $M_{e^+e^-} \sim M_Z$ ). The tiny threshold structure at  $M_{\mu^+\mu^-} =$ 40 GeV is a result of the invariant-mass cuts [\(10\)](#page-2-2), which require  $M_{e^+e^-} > 40$  GeV for  $M_{\mu^+\mu^-} < 40$  GeV. In the vicinity of the Z-boson resonance ( $M_{\mu^+\mu^-} \sim M_Z$ ), the weak corrections, thus, mainly result from the following two contributions of different origin: First, there is a constant offset of  $\sim$  − 5% stemming mainly from weak corrections to the dominant  $pp \to Z(\to e^+e^-)Z^*$  production with a<br>resonance in  $M_{++}$ : second, there are the weak corrections resonance in  $M_{\mu^+\mu^-}$ ; second, there are the weak corrections to the interference of the resonant and nonresonant contributions to the amplitude. These second contributions are proportional to  $(M_{\mu^+\mu^-}^2 - M_Z^2)$  and, thus, change sign at the  $\overline{Z}$  means are  $\overline{Z}$  means at the means of the  $\overline{E}$   $\overline{W}$  and means Z resonance. The pronounced shapes of the EW and weak corrections, in fact, largely resemble the structures known from single-Z production (see, e.g., Fig. 12 of Ref. [\[32\]](#page-4-16)), with the large radiative tail for  $M_{\mu^+\mu^-} \lesssim M_Z$  originating from FSR. While FSR effects can be reproduced by photonic parton showers quite well, the genuine weak corrections cannot be approximated easily. As in the case of single-Z production, the weak corrections exhibit a sign change near the resonance (shifted to smaller  $M_{\mu^+\mu^-}$  in Fig. [1](#page-2-1) because of the negative offset mentioned above). Far

<span id="page-2-1"></span>

FIG. 1. Invariant-mass distribution of the  $\mu^+\mu^-$  system in  $pp \rightarrow \mu^+\mu^-e^+e^- + X$  including NLO EW corrections (upper panel) and relative EW and purely weak corrections at NLO (lower panel).

<span id="page-3-0"></span>

FIG. 2. Four-lepton invariant-mass distribution in  $pp \rightarrow$  $\mu^+\mu^-e^+e^- + X$  including NLO EW corrections (upper panel) and relative EW and purely weak corrections at NLO (lower panel).

below the Z-boson resonance, the relative EW corrections do not show large variations. This fact is interesting in view of the Higgs boson signal resulting from  $p p (q q) \rightarrow H \rightarrow$  $\mu^+\mu^-e^+e^- + X$  (not shown here), whose  $M_{\mu^+\mu^-}$  distribution shows a shoulder for  $M_{\mu^+\mu^-} \lesssim M_H - M_Z \approx 34 \text{ GeV}$ sensitive to the quantum numbers of the Higgs boson [\[35\]](#page-4-18).

In Fig. [2,](#page-3-0) we show the invariant-mass distribution of the full four-lepton system, which features the Higgs resonance from gg fusion at  $M_{4\ell} \sim M_H \approx 125$  GeV (not included here). The steep shoulder at the Z-pair threshold at  $M_{4\ell} =$  $2M_Z \approx 182$  GeV creates a radiative tail at smaller invariant masses, similar to the case of the  $M_{\mu^+\mu^-}$  distribution, since  $M_{4\ell}$  can be strongly decreased by FSR effects. A similar effect, though reduced, is observed below the second shoulder near  $M_{4\ell} = 110$  GeV, which is a result of the  $p_T$  and invariant-mass cuts [\(7\)](#page-1-0) and [\(10\).](#page-2-2) In the region of the Higgs boson resonance, the EW corrections are at the level of a few percent. While photonic corrections might again be well approximated by parton showers, this does not apply to the weak corrections. Interestingly, the weak corrections change their size from  $-3\%$  to about  $+6\%$  when  $M_{4\ell}$  drops below the Z-pair threshold. The sign change can be understood from the fact that below the ZZ threshold, one of the two Z bosons is forced to be far off shell. For the corresponding  $l^+\ell^-$  pair, this means that  $M_{\ell^+\ell^-}$  drops below  $M_{Z}$ , so that the weak corrections turn positive, as can be seen from Fig. [1](#page-2-1). The sign change of the weak corrections near the ZZ threshold is quite interesting phenomenologically, since it renders their inclusion via a global rescaling factor impossible. Globally reducing differential cross sections by 3.6%, as deduced from the integrated cross section, would have the opposite effect on

<span id="page-3-1"></span>

FIG. 3. Distribution in the angle  $\phi$  between the two Z-boson decay planes in  $pp \rightarrow \mu^+\mu^-e^+e^- + X$  including NLO EW corrections (upper panel) and relative EW and purely weak corrections at NLO (lower panel).

the  $M_{4\ell}$  distribution near the Higgs signal as the true weak correction.

Finally, in Fig. [3](#page-3-1) we show the distribution in the angle  $\phi$ between the two Z-boson decay planes, which are each spanned by the two lepton momenta of the respective  $\ell^+ \ell^$ pair [\[36\].](#page-4-19) The distribution is sensitive to possible deviations of the Higgs boson coupling structure from the standard model prediction so that any distortion of the distribution induced by higher-order corrections, if not properly taken into account, could mimic nonstandard effects. Figure [3](#page-3-1) reveals a distortion by about 2% due to weak loop effects. The contribution of photonic corrections is negligible in our setup, similar to their contribution to the integrated cross section. This is due to the fact that photonic corrections mainly influence the absolute size of the lepton momenta via collinear FSR but not the directions of the leptons.

In summary, the NLO EW corrections to four-lepton production consist of photonic and purely weak contributions displaying rather different features. Photonic corrections can grow very large, to several tens of percent, in particular, in distributions where resonances and kinematic shoulders lead to radiative tails. While those corrections might be well approximated with parton showers, this is not the case for the remaining weak corrections, which are typically of the size of 5% and, thus, non-negligible. The weak corrections, in particular, distort distributions that are important in Higgs boson analyses. In the four-lepton invariant mass, even the signs of the weak corrections in the Higgs signal region and the region of resonant Z-boson pairs are different.

As long as higher-order EW or mixed QCD EW corrections are not known, an estimate of the uncertainties

due to missing higher-order corrections can be obtained from the powers or products of the respective NLO contributions, i.e., from  $(\delta^{\text{weak}})^2$  for the missing weak NNLO corrections, from  $|\delta^{\text{weak}}\delta^{\text{QCD}}|$  for the missing mixed QCD-weak NNLO corrections, etc. If the leading effects beyond NLO are known to be reproduced correctly in the combination of our results with other calculations, as, e.g., the dominant photonic corrections by a QED parton shower, those known effects have to be excluded from the uncertainty estimate. This task depends on the details of the combination and, thus, goes beyond the scope of this Letter.

B. J. gratefully acknowledges L. Salfelder for useful discussions. The work of S. D. is supported by the Research Training Group GRK 2044 of the German Science Foundation (DFG). A. D. and B. B. acknowledge support by the DFG under Reference No. DE 623/2-1. The work of L. H. was supported by Grants No. FPA2013-46570-C2-1- P and No. 2014-SGR-104, and partially by the Spanish MINECO under Project No. MDM-2014-0369 of ICCUB (Unidad de Excelencia "María de Maeztu"). The work of B. J. is supported in part by the Institutional Strategy of the University of Tübingen (DFG, ZUK 63) and in part by the German Federal Ministry for Education and Research (BMBF) under Contract No. 05H2015.

- <span id="page-4-0"></span>[1] J. Ohnemus, Phys. Rev. D **50**[, 1931 \(1994\)](http://dx.doi.org/10.1103/PhysRevD.50.1931).
- [2] J. M. Campbell and R. K. Ellis, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.60.113006) 60, 113006 [\(1999\).](http://dx.doi.org/10.1103/PhysRevD.60.113006)
- [3] L. J. Dixon, Z. Kunszt, and A. Signer, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.60.114037) 60, [114037 \(1999\).](http://dx.doi.org/10.1103/PhysRevD.60.114037)
- <span id="page-4-1"></span>[4] F. Cascioli, T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, and E. Weihs, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2014.06.056) 735, 311 (2014).
- [5] M. Grazzini, S. Kallweit, and D. Rathlev, *[Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2015.09.055)* 750, [407 \(2015\)](http://dx.doi.org/10.1016/j.physletb.2015.09.055).
- <span id="page-4-2"></span>[6] T. Matsuura and J. J. van der Bij, Z. Phys. C 51[, 259 \(1991\).](http://dx.doi.org/10.1007/BF01475793)
- [7] C. Zecher, T. Matsuura, and J. J. van der Bij, [Z. Phys. C](http://dx.doi.org/10.1007/BF01557393) 64, [219 \(1994\)](http://dx.doi.org/10.1007/BF01557393).
- [8] T. Binoth, N. Kauer, and P. Mertsch, [arXiv:0807.0024.](http://arXiv.org/abs/0807.0024)
- <span id="page-4-3"></span>[9] P. Nason and G. Ridolfi, [J. High Energy Phys. 08 \(2006\)](http://dx.doi.org/10.1088/1126-6708/2006/08/077) [077.](http://dx.doi.org/10.1088/1126-6708/2006/08/077)
- [10] S. Höche, F. Krauss, M. Schönherr, and F. Siegert, [J. High](http://dx.doi.org/10.1007/JHEP04(2011)024) [Energy Phys. 04 \(2011\) 024.](http://dx.doi.org/10.1007/JHEP04(2011)024)
- [11] K. Hamilton, [J. High Energy Phys. 01 \(2011\) 009.](http://dx.doi.org/10.1007/JHEP01(2011)009)
- [12] T. Melia, P. Nason, R. Röntsch, and G. Zanderighi, [J. High](http://dx.doi.org/10.1007/JHEP11(2011)078) [Energy Phys. 11 \(2011\) 078.](http://dx.doi.org/10.1007/JHEP11(2011)078)
- [13] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, and P. Torrielli, [J. High Energy Phys. 02 \(2012\) 099.](http://dx.doi.org/10.1007/JHEP02(2012)099)
- <span id="page-4-4"></span>[14] F. Cascioli, S. Höche, F. Krauss, P. Maierhöfer, S. Pozzorini, and F. Siegert, [J. High Energy Phys. 01 \(2014\) 046.](http://dx.doi.org/10.1007/JHEP01(2014)046)
- <span id="page-4-5"></span>[15] A. Bierweiler, T. Kasprzik, and J. H. Kühn, [J. High Energy](http://dx.doi.org/10.1007/JHEP12(2013)071) [Phys. 12 \(2013\) 071.](http://dx.doi.org/10.1007/JHEP12(2013)071)
- [16] J. Baglio, L. D. Ninh, and M. M. Weber, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.88.113005) 88, [113005 \(2013\).](http://dx.doi.org/10.1103/PhysRevD.88.113005)
- <span id="page-4-6"></span>[17] S. Gieseke, T. Kasprzik, and J. H. Kühn, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-014-2988-y) 74, [2988 \(2014\)](http://dx.doi.org/10.1140/epjc/s10052-014-2988-y).
- <span id="page-4-7"></span>[18] A. Denner S. Dittmaier, M. Roth, and D. Wackeroth, [Nucl.](http://dx.doi.org/10.1016/S0550-3213(99)00437-X) Phys. B560[, 33 \(1999\)](http://dx.doi.org/10.1016/S0550-3213(99)00437-X).
- <span id="page-4-11"></span>[19] A. Denner, S. Dittmaier, M. Roth, and L. H. Wieders, [Nucl.](http://dx.doi.org/10.1016/j.nuclphysb.2005.06.033) Phys. B724[, 247 \(2005\).](http://dx.doi.org/10.1016/j.nuclphysb.2005.06.033)
- <span id="page-4-8"></span>[20] A. Denner and S. Dittmaier, Nucl. Phys. **B658**[, 175 \(2003\).](http://dx.doi.org/10.1016/S0550-3213(03)00184-6)
- [21] A. Denner and S. Dittmaier, [Nucl. Phys.](http://dx.doi.org/10.1016/j.nuclphysb.2005.11.007) **B734**, 62 (2006).
- [22] A. Denner and S. Dittmaier, Nucl. Phys. **B844**[, 199 \(2011\).](http://dx.doi.org/10.1016/j.nuclphysb.2010.11.002)
- <span id="page-4-9"></span>[23] A. Denner, S. Dittmaier, and L. Hofer, Proc. Sci., LL2014 (2014) 071 [[arXiv:1407.0087\].](http://arXiv.org/abs/1407.0087)
- <span id="page-4-10"></span>[24] S. Dittmaier, A. Kabelschacht, and T. Kasprzik, [Nucl. Phys.](http://dx.doi.org/10.1016/j.nuclphysb.2008.03.010) B800[, 146 \(2008\).](http://dx.doi.org/10.1016/j.nuclphysb.2008.03.010)
- [25] S. Dittmaier, [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(99)00563-5) **B565**, 69 (2000).
- [26] A. Denner, S. Dittmaier, M. Roth, and L. H. Wieders, [Phys.](http://dx.doi.org/10.1016/j.physletb.2005.03.007) Lett. B 612[, 223 \(2005\).](http://dx.doi.org/10.1016/j.physletb.2005.03.007)
- <span id="page-4-12"></span>[27] M. Billoni, S. Dittmaier, B. Jäger, and C. Speckner, [J. High](http://dx.doi.org/10.1007/JHEP12(2013)043) [Energy Phys. 12 \(2013\) 043.](http://dx.doi.org/10.1007/JHEP12(2013)043)
- <span id="page-4-13"></span>[28] S. Actis, A. Denner, L. Hofer, A. Scharf, and S. Uccirati, [J. High Energy Phys. 04 \(2013\) 037.](http://dx.doi.org/10.1007/JHEP04(2013)037)
- [29] S. Actis et al., Proc. Sci., LL2014 (2014) 023.
- <span id="page-4-14"></span>[30] E. Accomando, A. Denner, and C. Meier, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s2006-02521-y) 47, [125 \(2006\)](http://dx.doi.org/10.1140/epjc/s2006-02521-y).
- <span id="page-4-15"></span>[31] R. D. Ball, V. Bertone, S. Carrazza, L. D. Debbio, S. Forte, A. Guffanti, N. P. Hartland, and J. Rojo (NNPDF Collaboration), Nucl. Phys. B877[, 290 \(2013\).](http://dx.doi.org/10.1016/j.nuclphysb.2013.10.010)
- <span id="page-4-16"></span>[32] S. Dittmaier and M. Huber, [J. High Energy Phys. 01 \(2010\)](http://dx.doi.org/10.1007/JHEP01(2010)060) [060.](http://dx.doi.org/10.1007/JHEP01(2010)060)
- <span id="page-4-17"></span>[33] S. Chatrchyan et al. (CMS Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.89.092007) 89, [092007 \(2014\).](http://dx.doi.org/10.1103/PhysRevD.89.092007)
- [34] G. Aad et al. (ATLAS Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.91.012006) 91, [012006 \(2015\).](http://dx.doi.org/10.1103/PhysRevD.91.012006)
- <span id="page-4-18"></span>[35] S. Y. Choi, D. J. Miller, M. M. Mühlleitner, and P. M. Zerwas, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(02)03191-X) 553, 61 (2003).
- <span id="page-4-19"></span>[36] A. Bredenstein, A. Denner, S. Dittmaier, and M. M. Weber, Phys. Rev. D 74[, 013004 \(2006\)](http://dx.doi.org/10.1103/PhysRevD.74.013004).