Few-Body Precursor of the Higgs Mode in a Fermi Gas

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We demonstrate that an undamped few-body precursor of the Higgs mode can be investigated in a harmonically trapped Fermi gas. Using exact diagonalization, the lowest monopole mode frequency is shown to depend nonmonotonically on the interaction strength, having a minimum in a crossover region. The minimum deepens with increasing particle number, reflecting that the mode is the few-body analogue of a many-body Higgs mode in the superfluid phase, which has a vanishing frequency at the quantum phase transition point to the normal phase. We show that this mode mainly consists of coherent excitations of time-reversed pairs, and that it can be selectively excited by modulating the interaction strength, using, for instance, a Feshbach resonance in cold atomic gases.

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The transition from few-body quantum physics to the thermodynamic limit with increasing particle number is a fundamental problem in science. A systematic investigation of this question is complicated by the fact that the few-body systems existing in nature, such as atoms and nuclei, have limited tunability. Artificially created clusters [1,2] or semiconductor quantum dots [3] offer more flexibility, but they are often strongly coupled to their surroundings making a study of pure quantum states difficult. The creation of highly controllable few-fermion systems using cold atoms in microtraps [4,5], however, has opened new perspectives. Tunneling experiments in the few-body limit demonstrated single-atom control [6,7]. One has already observed the formation of a Fermi sea [8], as well as pair correlations in one-dimensional (1D) few-body atomic gases [5] that have also been studied extensively theoretically [9–13]. The few- to many-body transition is arguably even more interesting in higher dimensions, where quantum phase transitions with varying degrees of broken symmetry are ubiquitous [14]. A key question concerns the few-body fate of the order parameter, which describes a broken symmetry phase in the thermodynamic limit.

Another fundamental problem concerns the properties of the Higgs mode, which corresponds to oscillations in the size of the order parameter for a given broken symmetry phase [15,16]. Elementary particles acquire their mass from the presence of a Higgs mode [17], which was famously observed at CERN [18,19]. The Higgs mode also leads to collective modes in condensed matter and nuclear systems [14,20]. Despite its fundamental importance, the list of table top systems where it has been observed is short, mainly because it is typically strongly damped, and because it couples only weakly to experimental probes [21–23]. Experimental evidence for the existence of a Higgs mode has been reported in disordered and niobium selenide superconductors [24–27]. Also, neutron scattering experiments for a quantum antiferromagnet [28] are consistent with the presence of a broad Higgs mode, and lattice experiments combined with theoretical models for bosonic atoms in an optical lattice, indicate that a threshold feature can be interpreted in terms of a strongly damped Higgs mode [29,30].

Here, we show how one can explore both these fundamental questions, the few- to many-body transition and the nature of the Higgs mode, using an atomic Fermi gas in a new generation of microtraps. We calculate the few-body spectrum using exact diagonalization and show that for closed-shell configurations, the lowest monopole excitation energy depends nonmonotonically on the interaction strength, having a minimum in a crossover region, which deepens with increasing particle number. By comparing with a many-body theory, we demonstrate that the mode is the few-body precursor of the Higgs mode in the superfluid phase, which exhibits a vanishing frequency at a quantum phase transition to a normal phase. The mode mainly consists of time-reversed pair excitations, and it can be selectively excited by modulating the interaction strength.

We consider $N/2$ fermions of mass m in each of two hyperfine (spin) states $\sigma = \uparrow, \downarrow$ in a 2D harmonic trap $m\omega^2 r^2/2$. Particles with opposite spin interact via an attractive delta function interaction (suitably regularized, see below) $g\delta(\mathbf{r}-\mathbf{r}')$ with $g<0$, whereas particles of the same spin do not interact. The Hamiltonian is

$$\hat{H} = \sum_{i=1}^{N}\left(-\frac{\hbar^2\nabla_i^2}{2m} + \frac{1}{2}m\omega^2\mathbf{r}_i^2\right) + g\sum_{k,l}\delta(\mathbf{r}_k - \mathbf{r}_l), \quad (1)$$

where $\mathbf{r}_i = (x_i, y_i)$ is the spatial coordinate of particle i, $\nabla_i^2 = \partial_{x_i}^2 + \partial_{y_i}^2$, and k and l in the second sum denote particles with spin $\uparrow$ and spin $\downarrow$, respectively.

In order to make rigorous predictions unbiased by any assumptions, we calculate the eigenstates of (1) by exact diagonalization using a basis of harmonic oscillator states with energy $(2n + |m| + 1)\hbar\omega$, where $n = 0, 1, 2, 3, \ldots$, and $m = 0, \pm 1, \pm 2, \ldots$ is the angular momentum. This method has been extensively applied to attractive fermion systems, mainly in one dimension [9–13] but also in two dimensions [31,32]. As explained in the Supplemental Material [33], we employ a two-parameter cutoff scheme for the basis states in order to reach maximum convergence. Using a sparse representation of the resulting matrix, we find the eigenvectors using the implicitly restarted Arnoldi iteration method [36]. This generally allows for a significantly larger number of basis states, $\sim 10^7$, as compared to other available diagonalization methods, which is crucial, since we need a very large basis set for an accurate calculation of the low-lying collective modes.

As it stands, the spectrum of $\hat{H}$ depends logarithmically on the energy cutoff $E_{\rm cut}$. To cure this UV divergence, we eliminate the coupling constant g and cutoff $E_{\rm cut}$ in favor of the two-body binding energy ϵ_b per particle. This is defined as $E_2 = 2\hbar\omega - 2\epsilon_b$, where E_2 is the ground state energy of one $\uparrow$- and one $\downarrow$-particle in the trap. In practice, we calculate ϵ_b and the many-body spectrum as a function of g for the same $E_{\rm cut}$, and then we plot the spectrum as a function of ϵ_b. Since the two-body problem contains the same logarithmic divergence as the many-body problem, this procedure yields a well-defined theory for $E_{\rm cut} \to \infty$ [31,37,38]. A similar UV divergence appears for the system in three dimensions, where it has been regularized using a variety of methods [39–45].

Figure 1 shows the lowest monopole (zero angular momentum) excitation spectrum as a function of the two-body binding energy ϵ_b for a $3 + 3$ system, consisting of three $\uparrow$-particles and three $\downarrow$-particles. The noninteracting ground state is a closed-shell configuration with the two lowest harmonic oscillator shells filled. For no interaction, the excitations all have the energy $2\hbar\omega$, and they are formed either by pair excitations taking two particles with opposite angular momenta one shell up, see Figs. 2(a)–2(b), or by single particle excitations taking one particle two shells up; see Fig. 2(c). We see that all excitation energies, except the lowest, increase with increasing attraction since the attractive mean-field interaction potential increases the effective trapping frequency, thereby increasing the single particle excitation energies. The lowest mode is, however, qualitatively different: The excitation energy first *decreases* reaching a minimum at a "critical" two-body binding energy ϵ_b^c

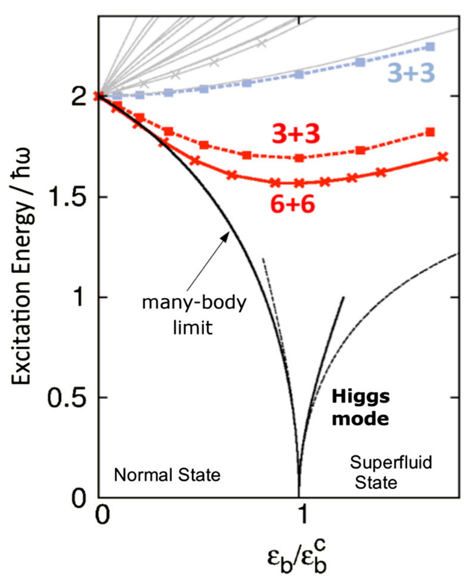

FIG. 1. The lowest monopole excitation for $3 + 3$ fermions (dashed red line) and for $6 + 6$ fermions (red solid line) obtained by numerical diagonalization of Eq. (1). The blue dashed line is the second excited state, and the gray solid lines are higher excitations for the $3 + 3$ system. The black solid (dotted) lines show the numerical (analytic) many-body Higgs-mode energy [46] (see Supplemental Material [33]).

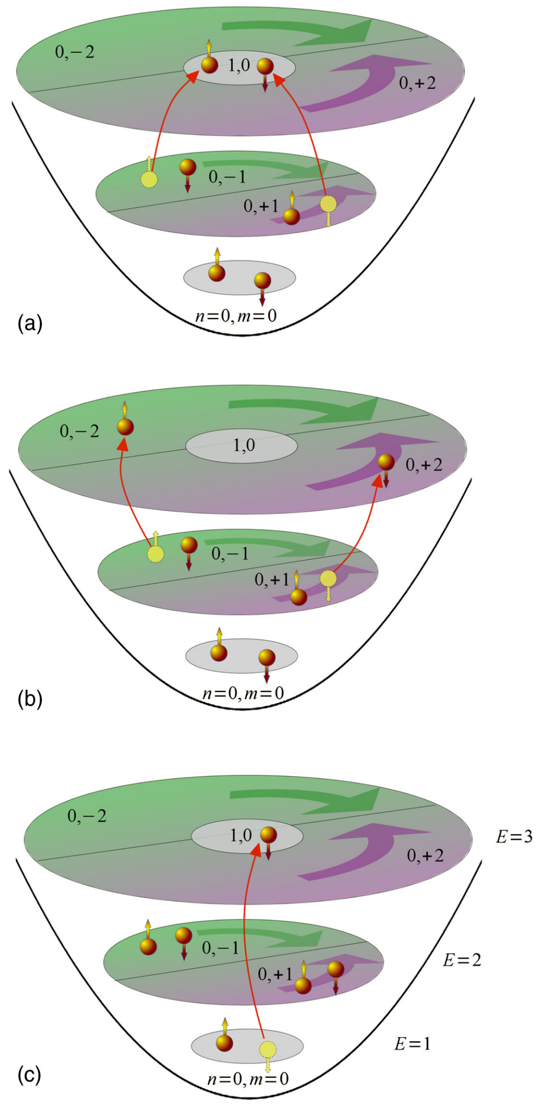

FIG. 2. Panels (a) and (b) show a schematic sketch of an excitation of a time-reversed pair $(n, m, \uparrow)$ and $(n, -m, \downarrow)$ one shell up. The energy of such excitations decreases with increasing attraction. Panel (c) shows an example of a single-particle monopole excitation two shells up. The energy of such excitations grows with increasing attraction.

(we will justify this name shortly), after which it increases for stronger attraction. This nonmonotonic behavior cannot be understood from a single-particle picture. Instead, it is due to pair correlations. The energy cost of exciting a pair of time-reversed states across the energy gap, as illustrated in Figs. 2(a)–2(b), initially decreases with increasing attraction, since the two excited particles can use the available states in the empty shell to increase their overlap. In Fig. 1, we normalize ϵ_b by ϵ_b^c, defined as the two-body binding energy that gives the minimum monopole excitation energy, so that we can compare results for different particle numbers and for the thermodynamic limit. Exact values of ϵ_b^c are given in the Supplemental Material [33].

To link the few-body spectrum to the thermodynamic limit, we also plot in Fig. 1 the lowest monopole mode obtained from a many-body calculation, which includes fluctuations around the Bardeen-Cooper-Schrieffer (BCS) solution [46] (see Supplemental Material [33]). Because of the energy gap in the single particle spectrum for a closed-shell configuration, there is a normal to superfluid quantum phase transition at a critical binding energy ϵ_b^c. The system is in the normal phase for $\epsilon_b < \epsilon_b^c$, and the lowest monopole mode corresponds to vibrations in the pairing energy $|\Delta|$ around the $\Delta = 0$ equilibrium value. The frequency of this mode decreases with increasing attraction and vanishes at ϵ_b^c, signaling a quantum phase transition to a superfluid phase. In the superfluid phase, the minimum energy is obtained for $|\Delta| > 0$, and the Higgs mode corresponds to vibrations in $|\Delta|$ around this minimum. Its energy is approximately given by $2|\Delta|$ (The deviation is due to the breaking of particle-hole symmetry), increasing from zero at the critical point. When $|\Delta| \ll \hbar\omega$, the Cooper pairs are predominantly formed by time-reversed states in the same shell [46]. Importantly, the Higgs mode is *undamped* in this regime due to the discrete nature of the trap level spectrum, which is in sharp contrast to the other tabletop systems mentioned above, where the damping is significant.

Comparing the $3+3$ and the many-body spectrum in Fig. 1 clearly shows that the lowest monopole mode for the $3+3$ system becomes the few-body precursor of the Higgs mode with increasing attraction. The nonmonotonic behavior of its energy is the smooth few-body analogue of the sharp thermodynamic normal to superfluid quantum phase transition with a vanishing Higgs mode frequency at the critical point. We also show in Fig. 1 the lowest monopole mode for the $6+6$ system corresponding to a closed-shell configuration with the three lowest shells filled. The nonmonotonic behavior of the lowest excitation energy is now even more pronounced with a deeper minimum, reflecting the gradual few- to many-body transition with increasing particle number.

In the Supplemental Material [33], we illustrate further the few- to many-body transition by calculating the spectrum for the closed shell configurations up to $15 + 15$ particles. Since it is numerically intractable to perform exact diagonalizations of Eq. (1) beyond $6+6$ particles, we use a simplified model, which includes only the highest filled and the lowest two empty shells. This calculation clearly shows a pronounced deepening of the minimum of the excitation energy with increasing particle number.

In Fig. 3, we plot the lowest monopole excitations for a $4+4$ system, which corresponds to an open-shell configuration where there is a pair of $\uparrow\downarrow$ particles in the third shell. Contrary to the closed-shell configuration, all excitation energies now increase monotonically with the attraction. This is because there is pairing for *any* attractive interaction so that the lowest excitations involve pair breaking, and it demonstrates that the nonmonotonic behavior of the lowest mode energy is characteristic of a closed-shell configuration, where there is a quantum phase transition in the thermodynamic limit.

In order to investigate further the connection between the few- and many-body physics, we quantify the amount of time-reversed pairing correlations in a given state by

$$P = \sum_i |C_i^{\mathrm{tr}}|^2. \tag{2}$$

Here, C_i are the expansion coefficients in the many-body basis for a given eigenstate. The sum runs over all basis states formed from the noninteracting ground state by excitations of time-reversed (tr) pairs. In Fig. 4, we plot P for the ground state and the two lowest excited states. Comparing the first excited state with the ground state and with the second excited states clearly shows that below the critical binding energy ϵ_b^c, the wave function of the lowest mode is mainly formed by coherent excitations of time-reversed pairs. It is consistent with the canonical many-body picture of

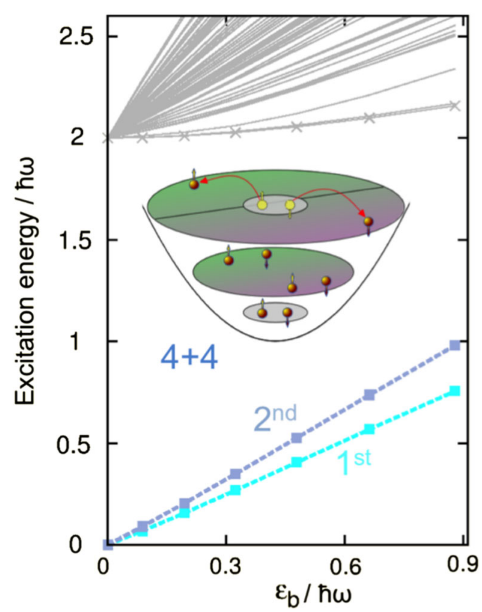

FIG. 3. Monopole excitations of an open-shell system. The lowest excitations are intrashell excitations, which do not exhibit a minimum. The gray lines show higher excitations (which correspond to inter-shell transitions). Inset: Sketch of time-reversed intrashell pair excitations.

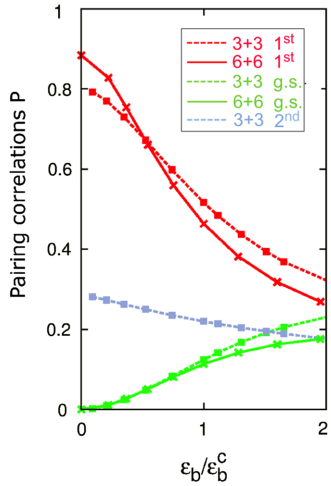

FIG. 4. Pairing correlations of the few-body states as defined by Eq. (2) for $N = 3 + 3$ fermions (dashed lines) and $N = 6 + 6$ fermions (solid lines). The green lines show the ground state and the red lines show the first excited state. The blue line shows the second excited state (only for $3 + 3$).

vibrations in $|\Delta|$, since such excitations give rise to fluctuations in the pairing field. The higher mode has a significantly smaller proportion of pair correlations, and it mainly consists of single-particle excitations two shells up. The pairing correlations in the ground state increase with increasing attraction, as it becomes more favorable to excite time-reversed pairs across the energy gap. This smooth increase of ground state pair correlations is the few-body analogue of the normal to superfluid quantum phase transition, where excitations of time-reversed pairs cost zero energy at the critical coupling strength, making the system spontaneously form Cooper pairs. The pair correlated part of the few-body Higgs mode decreases for $\epsilon_b > \epsilon_b^c$, since it is orthogonal to the ground state.

We now address how one can detect the few-body Higgs mode in atomic gas experiments using microtraps. Two experimental probes are widely used: Periodic modulations of the trapping frequency and of the interaction strength. From Fermi's golden rule, the transition rate from the ground state $|G\rangle$ to an excited state $|E\rangle$ is proportional to the transition matrix elements

$$\Gamma^E_{\rm trap} = |\langle G|\sum_i \mathbf{r}_i^2|E\rangle|^2,$$

$$\Gamma^E_{\rm int} = |\langle G|\sum_{k,l} \delta(\mathbf{r}_k - \mathbf{r}_l)|E\rangle|^2, \qquad (3)$$

for the two probes. In Fig. 5, we plot $\Gamma^E_{\rm trap}$ and $\Gamma^E_{\rm int}$ to the excited states of the $3 + 3$ and the $6 + 6$ systems. Figure 5 (left) shows that the transition rate into the lowest mode is much larger than the rate into the second excited state when the coupling strength is modulated. This is because the interaction operator $\Gamma_{\rm int}$ can excite time-reversed pairs, (see Supplemental Material [33]), which are precisely the excitations that give rise to pair vibrations. Thus, the Higgs mode can be selectively excited by modulating the interaction

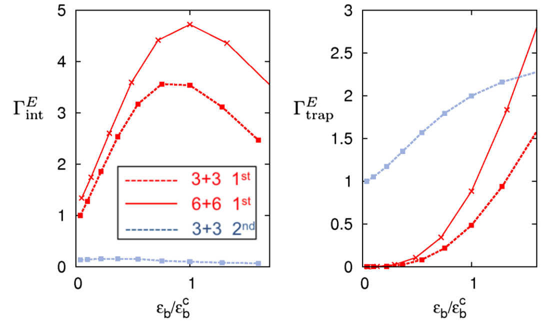

FIG. 5. Left: Transition matrix elements $\Gamma^E_{\rm int}$ corresponding to modulating the interaction strength for a $3 + 3$ and a $6 + 6$ system. The matrix elements are normalized by $\Gamma^E_{\rm int}$ calculated to the Higgs mode at a very low coupling strength for the $3 + 3$ system. Right: Transition matrix elements $\Gamma^E_{\rm trap}$ corresponding to modulating the trapping frequency. The matrix elements are normalized by $\Gamma^E_{\rm trap}$ to the second excited state calculated at a very low coupling strength.

strength, using, for instance, a Feshbach resonance. This fact, together with the nonmonotonic frequency behavior, can be used to experimentally identify the Higgs mode. On the other hand, Fig. 5 (right) shows that when the trapping potential is modulated, the transition rate into the second excited state is much larger than into the lowest mode for small attraction. The reason is that $\sum_i \mathbf{r}_i^2$ is a single particle operator, whereas the lowest mode mostly consists of time-reversed pair excitations. With increasing attraction, the transition rate into the lowest mode increases, consistent with the fact that the pair correlation P in the Higgs mode decreases with increasing coupling.

In conclusion, we demonstrated using exact diagonalization that the lowest monopole excitation energy of a two-component Fermi gas exhibits a nonmonotonic behavior with increasing attractive interaction for closed shell configurations. The mode frequency has a minimum in a crossover region, which deepens as the many-body limit is approached with increasing particle number. Comparing with a many-body calculation, we identified the few-body precursor of the Higgs mode, which has a vanishing frequency at the quantum phase transition point between a normal and a superfluid phase. We showed that the mode is mainly formed by coherent excitations of time-reversed pairs, and that it can be selectively excited by modulating the interaction strength. These results demonstrate how a new generation of cold atom experiments using microtraps can be used to explore two fundamental questions in physics: The nature of the Higgs mode and the crossover from few- to many-body physics. Our results are also relevant to the nuclear structure community, since we show how cold atoms can be used to probe pair correlations in finite systems much more systematically compared to what is possible in nuclei [47,48].

We end by noting that similar results hold for atoms in a 3D trap [49,50]. Focus was here on the 2D case, as it is closer to being experimentally realized. Indeed, the first experiment observing pairing correlations in two dimensions has already been reported [51].

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