## Chirality-Dependent Transmission of Spin Waves through Domain Walls

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(Received 4 August 2015; published 8 April 2016)

Spin-wave technology (magnonics) has the potential to further reduce the size and energy consumption of information-processing devices. In the submicrometer regime (exchange spin waves), topological defects such as domain walls may constitute active elements to manipulate spin waves and perform logic operations. We predict that spin waves that pass through a domain wall in an ultrathin perpendicularanisotropy film experience a phase shift that depends on the orientation of the domain wall (chirality). The effect, which is absent in bulk materials, originates from the interfacial Dzyaloshinskii-Moriya interaction and can be interpreted as a geometric phase. We demonstrate analytically and by means of micromagnetic simulations that the phase shift is strong enough to switch between constructive and destructive interference. The two chirality states of the domain wall may serve as a memory bit or spin-wave switch in magnonic devices.

DOI: 10.1103/PhysRevLett.116.147204

Motivated by the aim to reduce energy dissipation in electronic devices, spin waves are considered as an alternative information carrier in the field of magnon spintronics [1]. A spin wave acquires a phase shift when it passes through a magnetic domain wall (DW) [2]. In this Letter, we show that the Dzyaloshinskii-Moriya interaction (DMI) in ultrathin ferromagnetic films makes the phase shift dependent on the DW chirality, leading to constructive or destructive interference in a two-branch interferometer. The mechanism we identify raises the prospect of magnonic devices in which DW chirality acts as a spin-wave switch.

It is by now clear that the DMI plays a crucial role in the magnetization dynamics of ultrathin films [3–6], due to the broken inversion symmetry at the interfaces. The interfacial DMI favors, in a perpendicular-anisotropy film, Néel DWs with a fixed chirality [Fig. 1(d)] [5,7], in competition with the dipolar interaction, which tends to favor Bloch DWs [Fig. 1(b)]. The most interesting regime is when the two interactions have a comparable strength, yielding a DW intermediate between Bloch and Néel [7,8] with two stable minimum-energy configurations (chirality states) whose inplane orientations differ by ~90°, as shown in Fig. 1(c).

Recent experiments demonstrated that DWs can be brought into the intermediate regime and that the DMI strength can be fine-tuned by modifying the thicknesses of the adjacent nonmagnetic layers [9]. The internal orientation might be also tuned by an adjacent layer of a topological insulator; its surface states induce in the magnetic layer an interfacial-DMI-like effect that depends on the chemical potential and applied electric field [10–13].

Our main result is summarized in Fig. 2, where we consider an interferometer in which incoming spin waves are divided between two identical waveguides, each



FIG. 1. Effect of the interfacial DMI on the magnetization profile  $\mathbf{m}(x)$  of a DW in a thin film with perpendicular anisotropy. (a) Away from the DW, magnetization points out of the film ( $\hat{z}$  or  $-\hat{z}$ ). Near the DW, the DMI creates an effective field  $H_{DMI}$  in the  $-\hat{\mathbf{x}}$  direction. Depending on the competition between the dipolar and DMI interactions, the equilibrium configurations circle, circle prime, star, star prime, and square, shown in (b)-(d), are possible. (b) Without DMI, the minimum-energy configurations (flux closure) are two equivalent Bloch DWs (circle, in dark colors, and circle prime, in light colors), whose in-plane orientations differ by 180°. (c) For intermediate DMI, the minimum-energy configurations are intermediate between Bloch and Néel. There are two equivalent minimum-energy states (star and star prime), whose in-plane orientations differ by approximately 90° for an appropriately tuned DMI strength D. (d) For strong DMI, a single minimum-energy configuration (square) exists: a Néel DW with magnetization in the center pointing in the  $-\hat{\mathbf{x}}$  direction.

containing a DW. The two DWs are identical in every respect except possibly their chirality. When the spin waves rejoin, they are transmitted or reflected depending on the phase difference. While it is obvious that spin waves interfere constructively if the two DWs have the same chirality [Fig. 2(a)], we ask if it is possible to achieve destructive interference (spin wave blocked) by reversing the chirality of one DW [Fig. 2(b)]. Without DMI, the two



FIG. 2. Interferometer setup in a thin film with perpendicular anisotropy. The two DWs may have (a) identical or (b) opposite chiralities. Spin waves enter the device from the left. If the chiralities are identical, constructive interference is always obtained on the right-hand side. For opposite chiralities, the phase difference depends on the DMI strength, as shown in (c)-(e). (c) Without DMI, spin waves interfere constructively even if the chiralities are opposite (circle, circle prime). (d) For intermediate DMI, we find a phase difference  $\Delta \varphi$  of up to 180° (destructive interference) for opposite chirality states star, star prime. (e) For strong DMI, the configurations in both branches are the same (square), trivially resulting in constructive interference. In (c)-(e), large arrows represent the equilibrium magnetization direction  $\mathbf{m}(x)$ . On the left, spin-wave basis vectors  $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ are defined identically for all configurations circle, circle prime, star, star prime, square. Their orientation after parallel transportation, shown on the right, depends on the DW configuration. In (d), notice that the transported basis vectors for star and star prime are rotated by 180°. This geometric phase difference  $\Delta \varphi_{\text{geom}}$  is the dominant contribution to  $\Delta \varphi$ .

chirality states of the Bloch DW induce identical phase shifts, leading to constructive interference [Fig. 2(c)]. For strong DMI, the DW has a single stable (Néel) configuration and the phase shifts are obviously also identical [Fig. 2(e)]. However, for intermediate DMI, the two equilibrium orientations induce geometric phase shifts differing by as much as 180° [Fig. 2(d)]. In this regime, the interferometer can switch between constructive and destructive interference—transmission or reflection depending on whether the chiralities are identical or opposite.

The two chirality states are separated by an energy barrier  $\Delta E$  (an unfavorable Néel configuration). If  $\Delta E$  is high enough, spontaneous reversals of chirality due to thermal fluctuations are very rare (for the system in Fig. 3, we obtain  $\Delta E = 2.5 \times 10^{-12}$  erg =  $61k_BT_{room}$ ). We could consider the intermediate-DMI interferometer as a twostate memory device where the transmission of spin waves serves as a readout mechanism ("open" or "closed").

Switching does not require modifications of the material parameters, nor to insert or remove DWs [2], but only to reverse the chirality of one DW, for instance, by a field pulse normal to the plane of the film. (The "field pulse" might alternatively be generated through optomagnetic effects [14], provided the light can be focused onto a single branch.) The field causes the DW magnetization to



FIG. 3. Micromagnetic simulations of the propagation of spin waves in a ferromagnetic thin-film waveguide with perpendicular magnetization  $(2\pi M_S^2 = 0.907K, L = 3l)$ , where  $l = \sqrt{A/K}$ ) through DWs of the indicated chiralities (circle prime, star prime vs circle, star). (a) Without DMI (D = 0), spin waves experience the same phase shift regardless of DW chirality, leading to constructive interference (avg = average). (b) For intermediate DMI (D = 0.06A/l), there is a phase difference of almost 180° between spin waves that passed through DWs of different chirality, leading to destructive interference. We remark that the attenuation on the right-hand side is not the result of Gilbert damping (we take  $\alpha = 0.0030$ ) but merely represents the present location of the wave front [ $t = 89.6M_S/(|\gamma|K)$ ].

precess as shown in Fig. 4 until, when it is switched off, the DW relaxes to the nearest chirality state.

We have tested the results of Fig. 2, which we derive analytically below, by means of explicit micromagnetic simulations. The total energy *E* is given by the sum of the usual micromagnetic energy functionals for exchange  $E_{\text{ex}} = A \iint (\|\partial_x \mathbf{m}\|^2 + \|\partial_y \mathbf{m}\|^2) dx dy$ , uniaxial anisotropy  $E_{\text{ani}} = -K \iint m_z^2 dx dy$ , and dipolar energy [15], plus a functional

$$E_{\rm DMI} = -2D \iint \mathbf{m} \cdot (\nabla m_z) dx dy \tag{1}$$

describing the DMI induced near the interfaces of the ultrathin film [5]. The DMI strength *D* can be positive or negative (we take D > 0 in Figs. 1 and 2). We treat the film as effectively two-dimensional (magnetization is a function of *x* and *y* only), but we do consider the finite film thickness *L* in the *z* direction for the dipolar interactions [15]. Here we consider a waveguide made of a long strip of ultrathin film with perpendicular magnetization). The waveguide width *W* is at least so large that the dipolar interactions, in the absence of a DMI, favor a Bloch DW.

The interfacial DMI is qualitatively different from a DMI  $\propto \iiint \mathbf{m} \cdot (\nabla \times \mathbf{m}) dV$  present in isotropic bulk materials with a chiral crystal structure [15]. The effect of a bulk DMI on the interaction of spin waves with DWs was considered in Refs. [23,24]. Since a bulk DMI favors the Bloch DW (circle in Figs. 1 and 2), it does not, in the geometry considered here, provide the competition with dipolar interactions that is essential to obtain the intermediate DW with two equivalent minimum-energy orientations star, star prime differing by approximately 90°.

Figure 3 shows how spin waves, generated on the lefthand side of the strip, pass through a DW. We solve the Landau-Lifshitz-Gilbert equation, for relaxed initial states, on a square grid  $(0.33l \times 0.33l$  cells) using a self-developed C++ code [25] with implicit-midpoint time integration [26]. Spin waves are generated by a space-local, time-periodic in-plane applied field ( $\omega = 1.70 |\gamma| K/M_S$ ), switched on at t = 0. Each waveguide strip  $(267l \times 10l)$  is simulated in a vacuum-padded periodic box  $(333l \times 27l)$ . We calculate the difference  $\Delta \varphi = \varphi' - \varphi$  in the phase shift between the two chiralities at the right-hand side of the interferometer, comparing the intermediate DMI to the case without a DMI. A phase difference of up to  $180^{\circ}$  (destructive interference) is obtained for the intermediate DMI.

Fixing  $4\pi M_S = 3.8$  kG and  $A = 10^{-6}$  erg/cm [4], the other parameters of Fig. 3 become f = 9.9 GHz (frequency), W = 127 nm (waveguide width  $\approx$  wavelength), D = 0.047 erg/cm<sup>2</sup>, and L = 38 nm, assuming the free-electron gyromagnetic ratio. Spin waves of such frequencies and wavelengths can be experimentally generated, observed, and visualized [27–29]. Since the DMI is an interfacial effect, D is inversely proportional to film thickness L [30]. Extrapolation of the values in Ref. [4]  $(D=0.5 \text{ erg/cm}^2, L=3 \text{ nm})$  suggests that  $D \sim 0.04 \text{ erg/cm}^2$  is realistic for  $L \sim 38$  nm.

The phase difference  $\Delta \varphi$  between spin waves traveling along the two paths (star and star prime) has a geometric [31] origin. It is convenient to define  $\varphi = \varphi_{\text{geom}} + \varphi_{\text{rel}}$ . A spin wave causes magnetization to precess around its local equilibrium direction  $\mathbf{m}(x)$  [32]. In the limit of exchange spin waves  $(k_x \to \infty)$ , the dynamics induced by the wave is given by the real part of

$$\mathbf{m}(x) + \epsilon e^{i[\omega t + k_x x + k_y y + \varphi_{\rm rel}(x)]} [\hat{\mathbf{a}}(x) - i\hat{\mathbf{b}}(x)], \qquad (2)$$

where  $\epsilon > 0$  is the infinitesimal amplitude of the spin wave (linear regime). The orthonormal basis vectors  $\hat{\mathbf{a}}(x)$  and  $\hat{\mathbf{b}}(x)$  must be perpendicular to  $\mathbf{m}(x)$  for all x, so that their orientation continually changes across the DW. A natural choice is to define  $\hat{\mathbf{a}}(x)$  and  $\hat{\mathbf{b}}(x)$  according to parallel transport,  $(d\hat{\mathbf{a}}/dx) = -[\hat{\mathbf{a}} \cdot (d\mathbf{m}/dx)]\mathbf{m}$ , by which the basis vectors, at any given point x, match their orientation in an infinitesimal neighborhood of x as closely as possible.

The function  $\varphi_{rel}(x)$  in Eq. (2) determines the phase of the spin wave relative to the basis  $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ . However, the orientation of the basis  $\hat{\mathbf{a}}, \hat{\mathbf{b}}$  after parallel transportation across the DW strongly depends on the DW configuration (circle, circle prime, star, star prime, or square), as shown in



FIG. 4. Evolution of the magnetization during switching of DW chirality by means of an applied field  $H_z$  perpendicular to the film in a two-dimensional micromagnetic simulation. Only a part of the waveguide is shown. The arrows represent the direction of the in-plane component of magnetization  $\mathbf{m}(t, x, y)$  and the color the *z* component. We take  $H_z = 0.09K/M_S$ ,  $\alpha = 0.2$ ; other parameters are as in Fig. 3(b).

Figs. 2(c)–2(e). This reorientation of  $\hat{\mathbf{a}}, \hat{\mathbf{b}}$  implies an additional phase shift  $\varphi_{\text{geom}}$ , which is purely geometric in nature.

It is apparent from Figs. 2(c)-2(e) that the geometric contribution is approximately given by

$$\Delta \varphi_{\text{geom}} \approx 4\vartheta, \tag{3}$$

where  $\vartheta$  is the in-plane angle of the magnetization at the DW center, as shown in Fig. 2(d). For example, we have a geometric phase difference  $\Delta \varphi_{geom} \approx 180^{\circ}$  for intermediate DWs with  $\vartheta = 45^{\circ}$ . The value of  $\vartheta$  is determined by the competition between the DMI and the dipolar interaction, as shown in Fig. 5(a). While in principle  $\Delta \varphi_{geom}$  depends on the exact shape of the equilibrium profile  $\mathbf{m}(x)$ , we find that the deviation from Eq. (3) is at most a few degrees [15].

We derive [15] the relative contribution  $\Delta \varphi_{\rm rel}$  for  $k_x \to \infty$ , up to a correction of order  $|k_x|^{-1}$ , as

$$\Delta \varphi_{\rm rel} = \frac{D}{2A} \int_{-\infty}^{\infty} m'_y(x) dx - \frac{D}{2A} \int_{-\infty}^{\infty} m_y(x) dx, \quad (4)$$

where  $m'_y$  and  $m_y$  are the magnetization profiles star prime and star, respectively, calculated numerically, taking into account DMI and dipolar interactions. Notice that the exchange interaction does not contribute directly to Eq. (4) because the basis  $\hat{\mathbf{a}}(x)$ ,  $\hat{\mathbf{b}}(x)$  (parallel transport) absorbs such a contribution into  $\Delta \varphi_{\text{geom}}$ .



FIG. 5. (a) DW angle  $\vartheta$  as a function of the ratio of DMI strength *D* and dipolar interaction, for three film thicknesses *L*, taking  $\sqrt{2\pi M_S^2/K} = 0.8$  [33]. The angle  $\vartheta$  determines whether intermediate DWs (star prime, star) are closer to a Bloch or a Néel configuration. More "Néel-like" DWs (larger  $\vartheta$ ) are obtained for stronger *D*. Conversely, the dipolar interaction penalizes the Néel configuration (this effect is weaker in thinner films). (b) Phase difference  $\Delta \varphi$  between spin waves passing through DWs of opposite chiralities (star prime vs star), as a function of  $\vartheta$ , in the  $k_x \rightarrow \infty$  limit, for two values of  $\sqrt{2\pi M_S^2/K}$ , taking L = 3.0l. The dominant contribution  $\Delta \varphi_{\text{geom}}$  is separated out. The remaining contribution  $\Delta \varphi_{\text{rel}}$  lowers the value of  $\vartheta$  needed for destructive interference ( $\Delta \varphi = 180^\circ$ ). For  $\sqrt{2\pi M_S^2/K} = 0.9$ ,  $\Delta \varphi_{\text{rel}}$  is larger than for 0.5 because a relatively strong DMI  $D/(Aw_0^{-1})$  is then needed to obtain a given  $\vartheta$ .

Equation (4) gives, approximately,

$$\Delta \varphi_{\rm rel} = \varphi_{\rm rel}' - \varphi_{\rm rel} \approx \frac{D}{A} w_0 \cos \vartheta, \tag{5}$$

where  $w_0$  is a characteristic DW width ( $w_0 \approx \pi l$  for  $2\pi M_S^2 \ll K$ ). Notice that  $\Delta \varphi_{rel}$  vanishes for D = 0 (circle, circle prime) and for large D (square), where  $\vartheta = \pi/2$  (Néel wall). As shown in Fig. 5(b), the contribution of  $\Delta \varphi_{rel}$  enhances the effect of  $\Delta \varphi_{geom}$  and merely shifts the critical internal angle  $\vartheta$  for perfect destructive interference ( $\Delta \varphi = 180^\circ$ ) to a somewhat lower value (more Bloch-like DW). Therefore, the concept of the interferometer spinwave switch is robust: We can always find a DW angle  $0^\circ < \vartheta < 90^\circ$  such that the phase difference  $\Delta \varphi$  between opposite chiralities is exactly 180°. The desired value  $\vartheta$  could then be realized by fine-tuning the DMI strength D [Fig. 5(a)].

While Eq. (4) is derived in the short-wavelength limit, we have numerically solved the spin-wave normal-mode problem [25] for incoming waves of arbitrary wave number  $k_x$ . The phase shifts  $\varphi'$  and  $\varphi$  depend significantly on  $k_x$ , as in the case without a DMI [ $\varphi = 2 \arctan(k_x l)^{-1}$  [16]], but the difference  $\Delta \varphi = \varphi' - \varphi$  between the two chiralities, which is the relevant quantity in our interferometer, is weakly wavelength dependent for wavelengths comparable to (or shorter than) the DW width. The weak dependence of  $\Delta \varphi$  on  $k_x$  can, under certain approximations, also be derived analytically [15]. This observation justifies our approach  $k_x \to \infty$ .

In summary, we have shown that the interfacial Dzyaloshinskii-Moriya interaction in ultrathin magnetic films provides a new way of manipulation of spin waves. With this interaction, spin waves experience a different phase shift when passing through DWs of different chiralities, leading to either constructive or destructive interference in a two-branch interferometer. One can open or close the transmission of spin waves through the device by changing the DW chirality in one of the two branches. This opens the possibility of developing a memory element or transistor based on the manipulation of magnonic currents without charge transport.

The authors thank K. S. Novoselov for useful discussions. A. Qaiumzadeh provided some additional literature. This work is part of the research program of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO). Y. F. acknowledges support from the Spanish MECD Grant No. FIS2011-23713.

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- [33] Notice that the DMI strength *D* used in Fig. 3(b) is significantly lower than the value suggested by the plot in Fig. 5(b) for  $\vartheta \approx 45^\circ$ . The data in Fig. 5 assume that the waveguide is infinite in the *y* direction.