Superglass Phase of Interaction-Blockaded Gases on a Triangular Lattice

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We investigate the quantum phases of monodispersed bosonic gases confined to a triangular lattice and interacting via a class of soft-shoulder potentials. The latter correspond to soft-core potentials with an additional hard-core onsite interaction. Using exact quantum Monte Carlo simulations, we show that the low temperature phases for weak and strong interactions following a temperature quench are a homogeneous superfluid and a glass, respectively. The latter is an insulating phase characterized by inhomogeneity in the density distribution and structural disorder. Remarkably, we find that for intermediate interaction strengths a *superglass* occurs in an extended region of the phase diagram, where glassy behavior coexists with a sizable finite superfluid fraction. This glass phase is obtained in the absence of geometrical frustration or external disorder and is a result of the competition of quantum fluctuations and cluster formation in the corresponding classical ground state. For high enough temperature, the glass and superglass turn into a floating stripe solid and a supersolid, respectively. Given the simplicity and generality of the model, these phases should be directly relevant for state-of-the-art experiments with Rydberg-dressed atoms in optical lattices.

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It is well established that bosonic and fermionic systems subjected to a disordered external potential feature localization phenomena [1,2]. The interplay between disorder, interactions, and many-body quantum effects such as superfluidity is now a subject of intense research [3–8], as, e.g., bosons in random environments occur in a variety of experimentally relevant systems ranging from cold atoms [9–13], to superconductors [14] and quantum liquids [15,16]. Usually, the combination of disorder and repulsive interactions inhibits the emergence of superfluidity and Bose-Einstein condensation (BEC) and leads to an insulating gapless phase, known as Bose glass [17–19].

Remarkably, results of quenched Monte Carlo simulations in the context of ^{4}He have shown that superfluidity and BEC may coexist with structural disorder and inhomogeneity (i.e., glassy physics) in the absence of any random external potentials [20]. The resulting out-of-equilibrium state was termed *superglass* (SG), as a disordered analog of the supersolid (SS) phase [21]. While experiments have so far remained inconclusive [22,23], this proposal has spurred considerable theoretical activity to derive possible microscopic models of a SG [24–30]. Exact numerical results for bosons on lattices have shown that a thermodynamic SG phase can indeed emerge as a result of a competition of quantum fluctuations and externally induced frustration. For attractive interactions the latter can be induced via a random chemical potential [27], while for repulsive ones a SG can occur in theoretical models where either a self-disordered environment is induced by geometrical frustration (e.g., on random graphs) [28] or where disorder is a consequence of properly chosen random interparticle interactions [29,30]. In this context, main open questions are whether it is possible to obtain a SG in any theoretical models where frustration is not artificially built in the Hamiltonian, and if this new phase of matter may be experimentally observable in any physical system.

Here, we show that the SG phase can exist for a large class of bosonic lattice Hamiltonians. The latter are of the extended Hubbard-type, featuring a soft-shoulder interaction potential. Surprisingly, glassy behavior is obtained in the absence of any externally imposed frustration, e.g., in the lattice geometry, or in the interactions. Rather, frustration is here induced by cluster formation for large particle density, similar to the conditions of SS formation in soft-core models [31–33]. As an example, we consider a simple triangular lattice with isotropic two-body interactions. We analyze the phases and, following a quench in the temperature T or in the interaction strength, demonstrate the existence of both a classical glass (G) and a SG at low enough T. The latter are the out-of-equilibrium counterparts of a floating stripe solid (S) and a SS, respectively. These glass and superglass phases should be observable in experiments with Rydberg-dressed alkali atoms loaded into optical lattices.

The relevant Hamiltonian for hard-core bosons confined to a 2D triangular lattice reads

$$\mathcal{H} = -t\sum_{\{i,j\}}(b_i^\dagger b_j + b_j^\dagger b_i) + V\sum_{i<j;r_{ij}\le r_c} n_i n_j. \qquad (1)$$

Here, b_i ($b_i^\dagger$) are hard-core bosonic annihilation (creation) operators at site i, $n_i = b_i^\dagger b_i$, r_{ij} is the distance between sites i and j, and t is the tunneling rate on a lattice of

spacing a. In the following, t and a are used as units of energy and length, respectively. In classical physics, the *soft-shoulder* potential of Eq. (1) is of interest for soft-matter models of, e.g., colloids [34–36]. In quantum physics, this potential can be engineered in clouds of cold Rydberg atoms, where both the strength V and the range r_c of the interaction can be tuned by weakly admixing the Rydberg level to the ground state [32,33,37–44]. The additional onsite hard-core constraint can be enforced using, e.g., Feshbach resonances.

The quantum phases of Eq. (1) with $r_c = 1$ (i.e., nearest-neighbor interactions) are well known [45–49]: for densities $\rho < 1/3$ ($\rho > 2/3$), $\rho = 1/3$ ($\rho = 2/3$), and $\rho > 1/3$ ($\rho < 2/3$) the low-energy phase is a superfluid (SF), a gapped lattice S, or a gapless SS, respectively. The latter is an exotic state of matter where density correlations (here with $\sqrt{3} \times \sqrt{3}$ ordering) coexist with a finite superfluid fraction ρ_s, which is a result of doping the solid with interstitials (vacancies). The SS phase is generally robust against perturbations to the Hamiltonian (1), and may be observed experimentally, e.g., with cold quantum gases trapped in optical lattices and interacting via dipolar interactions [50–52].

In this work, we are interested in Eq. (1) with $r_c > 1$. For $r_c > 1$ the interactions belong to a large class of potentials that support the formation of self-assembled clusters of particles for sufficiently large densities $r_c\sqrt{\rho} > 1$ [35,36]. Such a phenomenon is essentially independent of the details of the interactions, as long as the latter display a negative Fourier component [34]. In the classical regime (i.e., $t = 0$) cluster formation has been shown to lead to frustration, which is manifested in an exponential growth of the ground state degeneracy as a function of the system size [53]. In the quantum regime (i.e., $t > 0$) this leads to several novel exotic phenomena at equilibrium: anomalous Luttinger-liquid behavior [53] and emergent supersymmetry in 1D lattice geometry [54] as well as free-space supersolidity in two dimensions [32,33,55]. The latter occurs, for appropriate values of interaction strength, at any density fulfilling the clusterization condition $r_c\sqrt{\rho} > 1$ [55]. In the following we consider, as a way of example, the simplest cluster forming potential with $r_c = 2$ and incommensurate particle densities consistent with such a condition. Our main focus is the demonstration of a G and SG emerging when a crystal and a SS are driven out of equilibrium via a temperature quench, respectively. Glassy phases for different ρ, r_c, and quench protocols are discussed in the Supplemental Material [56].

We study the Hamiltonian in Eq. (1) by means of path integral Quantum Monte Carlo simulations based on the worm algorithm [68]. This technique is numerically exact for bosonic systems and allows for accurate estimates of the superfluid fraction $\rho_s = \langle (W_x^2 + W_y^2)/(6\beta\rho) \rangle$ and the static structure factor $S(\mathbf{k})/N = \sum_{i,j} \exp[-i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)]\langle n_i n_j \rangle / N^2$ (N is the lattice size). The latter measure superfluidity and diagonal crystalline orders, respectively. Here, $\beta = 1/(k_B T)$, with k_B Boltzmann constant (in the following set to 1), W_i is the winding number in the ith direction, $\mathbf{k}$ is a lattice wave vector, and $\langle \cdots \rangle$ stands for statistical average. In addition, we compute the renormalized Edwards-Anderson order parameter $\tilde{q}_{\mathrm{EA}} = q_{\mathrm{EA}}/q_{\mathrm{EA}}^{\mathrm{max}}$, which, in the absence of crystalline order, is the well-accepted observable to identify glassy behavior on a lattice [28,69]. Here, $q_{\mathrm{EA}} = \sum_{i=1}^{N} \langle n_i - \rho \rangle^2$ and $q_{\mathrm{EA}}^{\mathrm{max}} = N\rho(1-\rho)$ is its maximum value obtained for a classical situation with no particle delocalization. We perform large-scale simulations with up to $N = 2304$ lattice sites and temperatures as low as $T/t = 1/12$. For each N and T, numerical values for the observables above are obtained by averaging over a minimum of 32 and a maximum of 100 different realizations of the quench.

Figure 1 [panel (a)] shows example results for the superfluid fraction ρ_s and the renormalized Edwards-Anderson parameter $\tilde{q}_{\mathrm{EA}}$ as a function of the interaction strength V/t for $N = 900$ and $T/t = 1/12$. Within the interesting range of interaction ($5.0 \lesssim V/t \lesssim 6.0$), ρ_s is found to decrease monotonically with increasing V/t from approximately 0.25 to about 0.05. In the same parameter range, $\tilde{q}_{\mathrm{EA}}$ increases up to values of the order of ~0.2. We note that in this regime the system does not feature

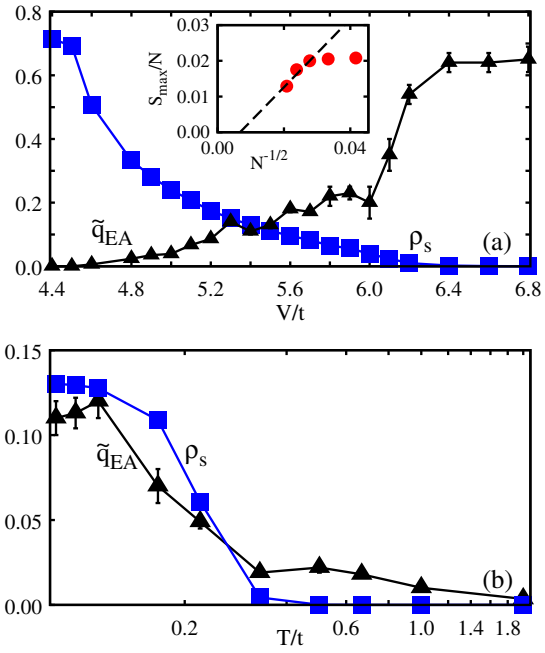

FIG. 1. (a) Superfluid fraction ρ_s, and renormalized Edwards-Anderson parameter $\tilde{q}_{\mathrm{EA}}$ as a function of V/t, for $T/t = 1/12$. (b) ρ_s and $\tilde{q}_{\mathrm{EA}}$ as a function of T/t, for $V/t = 5.4$. In both panels the density is $\rho = 13/36$ and the lattice size is $N = 900$. Solid lines are guides to the eye. Inset: maximum value of the structure factor S_{max}/N as a function of $1/\sqrt{N}$ for $\rho = 13/36$, $V/t = 5.4$, and $T/t = 1/12$; the dashed line is a linear fit for the three largest system sizes.

crystalline order; i.e., the computed structure factor $S(\mathbf{k})/N$ vanishes for any nontrivial wave vector $\mathbf{k} \neq 0$ in the thermodynamic limit, as proven in the inset, where we show the scaling with $N^{-1/2}$ of $S_{\max}/N$, the average of the largest peaks of the structure factor over several quench realizations. In the same limit the superfluid fraction stays finite. These data demonstrate one of the main results of this work, namely, the existence, in an extended region of parameters, of a SG, corresponding to an inhomogeneous noncrystalline superfluid. The dependence on T/t of both $\tilde{q}_{\mathrm{EA}}$ and ρ_s is shown in Fig. 1 [panel (b)]. For the specific value of the interaction strength in the panel superglassiness is realized below $T/t \simeq 0.2$.

For weak interactions, the SG phase quantum melts into a regular homogeneous superfluid with $\rho_s > 0$ and $\tilde{q}_{\mathrm{EA}} \simeq S(\mathbf{k}) = 0$. For the parameters of Fig. 1(a) this is obtained by decreasing the interaction strength below $V/t \simeq 4.8$. On the other hand, sufficiently large interaction strengths are found to inhibit superfluidity and turn the SG into an insulating G. The latter is characterized by a finite value of $\tilde{q}_{\mathrm{EA}}$ and $\rho_s \simeq S(\mathbf{k}) = 0$ [i.e., $V/t \gtrsim 6.2$ in the figure]. Within this glass phase quantum effects are largely suppressed. While classical glasses are well known to appear in disordered spin models, as well as in certain polydispersed systems of particles [69], here we demonstrate that (classical) glassy physics may emerge in a simple and rather general model of immediate experimental interest for bosons on a regular lattice.

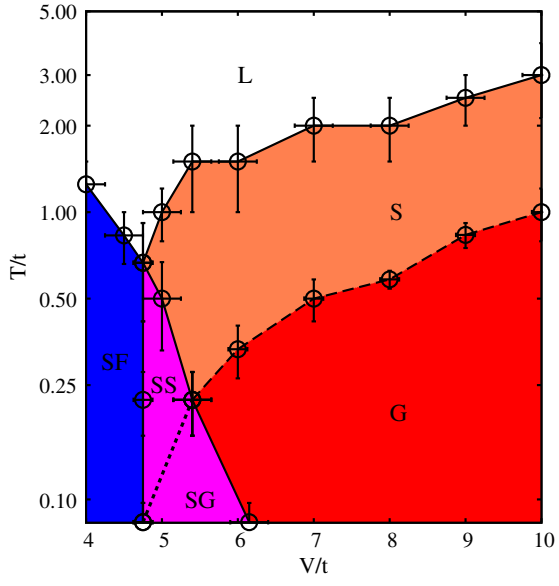

FIG. 2. Phase diagram of model Eq. (1) with $r_c = 2$ as a function of temperature T/t and interaction strength V/t, for particle density $\rho = 13/36$. Equilibrium phases: normal liquid, superfluid, stripe-crystal, and supersolid. A temperature quench to final values of T/t below the dashed line leads to a glass. The existence of a superglass is demonstrated below the dotted line using the same quench protocol.

The computed phase diagram of Eq. (1) is shown in Fig. 2 for a choice of particle density $\rho = 13/36$ as a function of T/t and V/t. At high temperatures we find a normal liquid (L) phase independently of the values of V/t, as expected. For sufficiently small interaction strength $V/t \lesssim 4.8$, this normal phase turns into a homogeneous superfluid by decreasing T/t, via a phase transition which is consistent with the Berezinskii-Kosterlitz-Thouless scenario. On the other hand, for large enough V/t and following a quench to low T the system displays a marked insulating glassy behavior with $\tilde{q}_{\mathrm{EA}} \neq 0$, $S(\mathbf{k}) = \rho_s = 0$ [Fig. 3(a) and full symbols in the inset]. The interplay between glassy physics and superfluidity is mostly evident for values of T/t below the dotted lines in Fig. 2, resulting in the SG scenario discussed above.

Interestingly, we find that fluctuations can restore crystalline order for sufficiently large T. This is shown for intermediate temperatures in Fig. 2, where a S (SS) phase intervenes between the low-temperature G (SG) and the high-temperature normal L. Here, the crystal is a floating

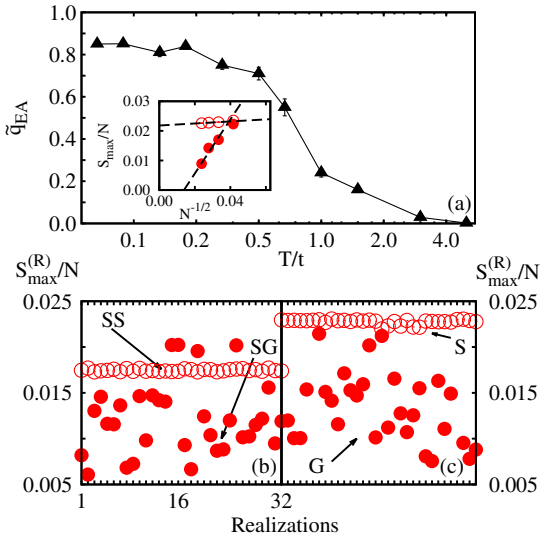

FIG. 3. (a) Normalized Edwards-Anderson order parameter $\tilde{q}_{\mathrm{EA}}$ for model Eq. (1) as a function of temperature T. The value of the interaction strength is $V/t = 10$. Data are for a lattice with size $N = 900$ and density $\rho = 13/36$. The solid line is a guide to the eye. Inset: Size dependence of $S_{\max}/N$ for $V/t = 10$. Values of the temperature are $T/t = 1/12$ and $T/t = 3/2$ for the closed and open symbols, respectively. Dashed lines are fits to our numerical data. (b)–(c) Maximum value of the structure factor $S^{(R)}_{\max}/N$ obtained in a given realization of a quench, plotted as a function of the number of different quench realizations. The latter only differ in the (random) initial condition and in the thermalization seed. The corresponding phases in the thermodynamic limit are indicated. Fluctuations in the values of $S^{(R)}_{\max}/N$ indicate glassy behavior. The parameters are $N = 1764$, $T/t = 1/2.5$, $V/t = 5$ (SS), $N = 2304$, $T/t = 1/9$, $V/t = 5.4$ (SG), $N = 1296$, $T/t = 1/0.7$, $V/t = 10$ (S), $N = 1296$, $T/t = 1/12$, $V/t = 10$ (G).

stripe solid, with finite diagonal long range order in the thermodynamic limit. Examples for the finite size scaling of the maximum value of the structure factor $S_{\max}/N$ in the S and G phases are shown in the inset of Fig. 3(a) (open and closed symbols, respectively). While in the S phase $S_{\max}/N$ is essentially independent of the system sizes investigated in this work, in the G phase $S_{\max}/N$ vanishes in the thermodynamic limit. In both phases $\rho_s \simeq 0$.

The difference between the glassy and crystalline phases is shown in Fig. 3(c), where we plot the maximum value of the structure factor $S^{(R)}_{\max}/N$ for each individual realization of a temperature quench at a given V/t and final T/t. In the crystalline phases, $S^{(R)}_{\max}/N$ is essentially identical in all realizations and $S_{\max}/N$ remains finite in the thermodynamic limit. However, within the glassy phases, $S^{(R)}_{\max}/N$ can fluctuate widely and in average decreases to zero with the system size. As shown in Fig. 3(b) the dependence of $S^{(R)}_{\max}/N$ on the realization for the SG and the SS phase is similar to that for the G and the S ones, respectively.

Further insight into the phases of Hamiltonian (1) is given by the analysis of the averaged site-density maps in Fig. 4. Specifically, we show results for a portion of the system and for a choice of T/t and V/t such that the system is a SF [panel (a)], a SG [panel (b)], and a G [panels (c)–(e)]. For comparison, panel (f) shows a cluster-type crystalline phase [i.e., $S(\mathbf{k}) \neq 0$] stabilizable at a density $\rho = 1/3$, for $V/t = 10$, and $T/t = 1$ [70]. In the homogeneous SF the average occupation number at each site equals the density ρ of the system, as expected. The resulting value of $\tilde{q}_{\mathrm{EA}}$ is thus negligible. Conversely, when V/t is large [panel (c)] the spatial density is highly inhomogeneous: particles form self-assembled clusters characterized by different numbers of constituents and spatial orientations, as well as by varying intercluster distances. These features lead, in the thermodynamic limit, to the absence of diagonal long range order, similar to an (emergent) polydispersity. Noticeably, the occupation number of lattice sites between clusters is here substantially suppressed, signaling particle localization. The resulting glass phase is insulating, similarly to, e.g., a regular Bose glass obtained by externally induced disorder [17].

Figure 4(b) shows that cluster formation (i.e., inhomogeneity) persists even at intermediate values of V/t, leading to a nonzero value of $\tilde{q}_{\mathrm{EA}}$ in the absence of crystalline order. The occupation of intercluster lattice sites is here enhanced with respect to panel (c). Such an enhancement is due to the presence of quantum fluctuations and exchanges of identical particles, responsible for the finite value of ρ_s and thus of superglassiness.

While here we have mainly focused on the minimal model Hamiltonian with $r_c = 2$ and a given density $\rho = 13/36$, we have verified that glassy phases also occur for larger r_c and densities ρ satisfying the clusterization condition, as well as when the details of the cluster forming potential are changed (see Fig. 4 and the Supplemental Material [56]). In particular, glassy phases are obtained for soft-core van der Waals type interactions relevant to experiments with Rydberg dressed atoms [42,43] (see Fig. 4 and the Supplemental Material [56]). In view of possible experiments, where V/t can be easily varied, we have verified that (see the Supplemental Material [56]) (i) quenches in V/t for a fixed low T from a superfluid lead to the same SG and G phases described above, and, in addition, (ii) the SG and G phases are robust against density variations at the percent level. In particular, the G phase is found at all densities explored, up to $\rho \simeq 0.4$, for sufficiently large V/t. We emphasize that while the finite lifetime of a Rydberg-dressed gas can be a limitation to the observability of phases in thermal equilibrium, glassy phases are realized out of equilibrium and their relevant properties are essentially those emerging in the first stages of the experiment.

The glass and superglass phases discussed here are the low-T quenched counterparts of the equilibrium solid and supersolid found for intermediate T. These latter equilibrium phases are analogous to those found in, e.g., Ref. [55]

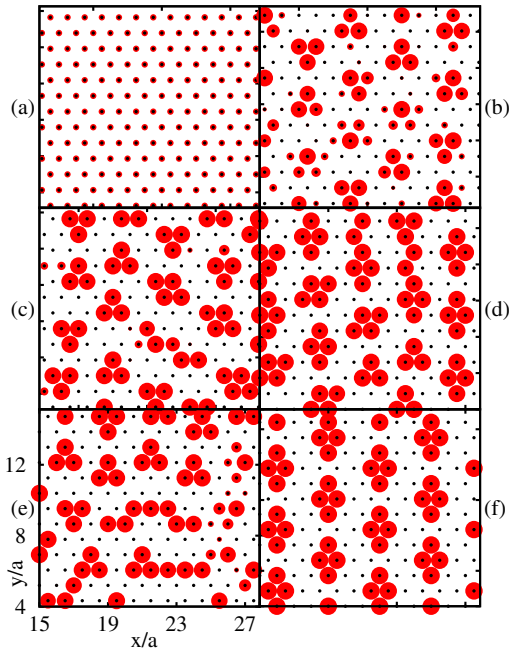

FIG. 4. Averaged site density for a portion of the system. Black circles depict the lattice sites. Density values are proportional to the size of red circles. Panel (a) shows an homogeneous superfluid phase ($T/t = 1/9$ and $V/t = 4$); panels (b) and (c), refer to a SG ($T/t = 1/12$, $V/t = 5.4$), and to a normal G ($T/t = 1/12$, $V/t = 10$), respectively. In panels (a)–(c) the density is $\rho = 13/36$. Panels (d) and (e) show the glassy density map obtained for $\rho = 0.401$ ($V/t = 10$, $T/t = 1/12$), and for the same density of panels (a)–(c), using the van der Waals purely repulsive soft potential with $V_{\mathrm{ryd}}/t = 30$, $T/t = 1/3$ (see details in the Supplemental Material [56]), respectively. Panel (f) shows a crystalline structure stabilizable at $\rho = 1/3$, $V/t = 10$, and $T/t = 1$.

in continuous space. While recent results point to the existence of out-of-equilibrium glassy-type phenomena in the classical regime for cluster-forming systems [71] (see also Ref. [72] for a discussion on the dynamics of the equilibrium phases), the existence of glassy dynamics in the quantum regime and in continuous space remains an open question.

In conclusion, we have demonstrated that glassy phases can be realized for a broad class of simple bosonic frustration-free Hamiltonians of the extended Hubbard-type. For intermediate interaction strength the interplay between quantum fluctuations, statistics, and glassy physics gives rise to an exotic SG scenario, where glassiness coexists with superfluidity, in contrast to a conventional Bose glass. In our model, frustration arises from the self-assembling of clusters, which is a direct consequence of the (isotropic) interparticle interaction potential at high enough density. The physics described in this work should be directly relevant for experiments with ultracold Rydberg-dressed atoms confined to optical lattices [32,41,73]. We hope that our work will provide new insights for unveiling other general mechanisms to glassy physics, and in general to frustration-induced phenomena both in the classical and quantum regime. Interesting extensions might include the search for exotic phenomena beyond the SG, such as frustration-induced Bose metals [74,75] and emergent gauge fields [76].

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