Quantum Electrodynamics in $d = 3$ from the ϵ Expansion

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We study quantum electrodynamics in $d = 3$ coupled to N_f flavors of fermions. The theory flows to an IR fixed point for N_f larger than some critical number N_f^c. For $N_f \leq N_f^c$, chiral-symmetry breaking is believed to take place. In analogy with the Wilson-Fisher description of the critical $O(N)$ models in $d = 3$, we make use of the existence of a fixed point in $d = 4 - 2\epsilon$ to study the three-dimensional conformal theory. We compute, in perturbation theory, the IR dimensions of fermion bilinear and quadrilinear operators. For small N_f, a quadrilinear operator can become relevant in the IR and destabilize the fixed point. Therefore, the epsilon expansion can be used to estimate N_f^c. An interesting novelty compared to the $O(N)$ models is that the theory in $d = 3$ has an enhanced symmetry due to the structure of 3D spinors. We identify the operators in $d = 4 - 2\epsilon$ that correspond to the additional conserved currents at $d = 3$ and compute their infrared dimensions.

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Introduction.—We consider an $\mathbb{R}$ gauge theory in $d = 2 + 1$ dimensions, coupled to $2N_f$ complex two-component massless fermions of unit charge, ψ^i ($i = 1, \ldots, 2N_f$). This theory has an $SU(2N_f)$ global symmetry [1]. When N_f is sufficiently large, the theory flows to a stable interacting fixed point with an $SU(2N_f)$ global symmetry. (It is stable in the sense that there are no relevant operators preserving all the symmetries). However, the IR behavior is different if the number of fermions is smaller than a critical value $N_f \leq N_f^c$, leading to spontaneous symmetry breaking according to the pattern [2]

$$SU(2N_f) \to SU(N_f) \times SU(N_f) \times U(1). \tag{1}$$

This symmetry breaking pattern can be triggered by the condensation of the parity-even operator

$$\sum_{a=1}^{N_f} (\bar{\psi}_a \psi^a - \bar{\psi}_{a+N_f} \psi^{a+N_f}). \tag{2}$$

Various estimates of the critical number N_f^c exist in the literature.

In condensed matter physics this theory has been advocated as an effective description of various strongly correlated materials. Quantum electrodynamics in $d = 3$ (QED_3) can arise as the continuum limit of spin systems with various values of N_f, e.g., $N_f = 2, 4$.[3–5] The theory with $N_f = 2$ also has applications in high-temperature superconductivity [6–8].

A method that has been employed to study QED_3 is the large-N_f expansion [9–14]. At large N_f, the theory simplifies and a systematic expansion in $1/N_f$ can be carried out. For an alternative to large N_f that uses the functional renormalization group approach see Ref. [15].

Here, we study QED_3 using the epsilon expansion. Clearly, since the theory is IR free in $d = 4$ and since the gauge coupling has positive mass dimension for $d < 4$, there is an IR fixed point at $d = 4 - 2\epsilon$ with $\epsilon > 0$. The fixed point is generated analogously to the Wilson-Fisher fixed point [16].

In the development of the epsilon expansion for QED one encounters some new technical difficulties that do not arise for $O(N)$ models. Perhaps one reason that (to our knowledge) it has not been considered before is that spinor representations of the Poincaré group do not behave very simply as a function of the number of dimensions (unlike tensor representations) [17]. Hence, it may not be obvious how to analytically continue to d. However, there appears to be no fundamental obstruction to studying QED with $d \leq 4$. We will keep the spinor structure that exists in $d = 4$ also in lower dimension. In lower integer dimension, the representation is reducible and can be interpreted in terms of the existing spinor structures in $d = 3$ and $d = 2$.

The fact that spinor representations are smaller in $d = 3$ than in $d = 4$ enhances the symmetry of the theory. The theory in $d = 3$ enjoys an $SU(2N_f)$ global symmetry, while the theory in $d = 4$ (around which we expand) only an $SU(N_f) \times SU(N_f)$ symmetry. We find that in $d = 4 - 2\epsilon$ certain antisymmetric tensor operators, bilinear in the fermions, are naturally interpreted as continuations of the enhanced currents of the three-dimensional theory. This suggests that the epsilon expansion provides the necessary elements to correctly describe the theory in $d = 3$.

Here, we only perform leading-order computations in the epsilon expansion of QED. Going to higher orders will be necessary to acquire more confidence about the accuracy of the method, and to estimate the uncertainties [21]. We consider bilinear and quadrilinear operators in the fermions. We shall see that a certain quadrilinear operator invariant under $SU(2N_f)$ and parity can become relevant in the IR for low values of N_f, and may destabilize the fixed point. At leading order in ϵ, evaluating the dimension naively at $\epsilon = 1/2$ without any resummation leads to $N_f^c = 2$. This is consistent both with the F theorem [22] and with lattice data [23–26]. (A different estimate that uses input from the $d = 2$ flavored Schwinger model gives $N_f^c = 4$). We also estimate the dimensions of these bilinear and quadrilinear operators at the fixed point for $N_f > N_f^c$. For comments on the theory with compact gauge group, see the Supplemental Material [27].

Generalities of the epsilon expansion.—To illustrate the procedure of the epsilon expansion, consider the two-point function of an operator $\mathcal{O}$ in $d = 4$, expanded in perturbation theory in a classically marginal coupling g

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = p^{2\Delta-4} \sum_{0\leq m\leq n, n=0}^{\infty} c_{nm} g^n \left(\log \frac{\Lambda^2}{p^2}\right)^m, \quad (3)$$

where Δ is the dimension of $\mathcal{O}$ in $d = 4$ at $g = 0$, and Λ is an UV cutoff. Introducing the renormalized operator $\mathcal{O}^{\rm ren} = Z\mathcal{O}$, we can cancel the Λ dependence of the correlator by allowing the coupling g and the normalization Z to evolve according to $\frac{dg}{d\log\Lambda} \equiv \beta(g)$, $\frac{d\log Z}{d\log\Lambda} \equiv \gamma(g)$, such that the Callan-Symanzik equation holds

$$\left(\frac{\partial}{\partial \log\Lambda} + \beta(g)\frac{\partial}{\partial g} + 2\gamma(g)\right)\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = 0. \quad (4)$$

The terms in Eq. (3) with coefficients c_{nn}, $n \geq 1$, are the leading logs. It follows from Eq. (4) that they are all fixed in terms of the coefficients β_1 and γ_1 in the leading order expansion of β and γ

$$\beta(g) = \beta_1 g^2 + O(g^3), \qquad \gamma(g) = \gamma_1 g + O(g^2). \quad (5)$$

One can then resum the leading logs to obtain

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle \simeq p^{2\Delta-4}\left(1 + \frac{1}{2}\beta_1 g \log\frac{\Lambda^2}{p^2}\right)^{-(2\gamma_1/\beta_1)}. \quad (6)$$

For $d = 4 - 2\epsilon$ we assume that g acquires a positive mass dimension $c\epsilon$ (where c is some positive number). The analogue perturbative expansion of the two-point function in $d = 4 - 2\epsilon$ is

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = p^{2\Delta-d}\sum_{n=0}^{\infty} c_n \left(\frac{g}{p^{c\epsilon}}\right)^n. \quad (7)$$

Requiring that Eq. (7) approaches Eq. (3) in the limit $\epsilon \to 0$, we find the matching condition

$$c_n = \sum_{m=0}^{n} c_{nm} m! \left(\frac{2}{c\epsilon}\right)^m + O(\epsilon). \quad (8)$$

The leading contribution to the two-point function, Eq. (7), in the limit $\epsilon \ll 1$ comes from the terms containing c_{nn}, which we can thus resum similarly to Eq. (6)

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle \simeq p^{2\Delta-d}\left(1 + \frac{\beta_1}{c\epsilon}\frac{g}{p^{c\epsilon}}\right)^{-(2\gamma_1/\beta_1)} \underset{p\to 0}{\approx} p^{2\Delta-d} p^{2\gamma_1(c\epsilon/\beta_1)}. \quad (9)$$

In the IR limit $p \to 0$ a new scaling law emerges. The contribution to the IR dimension of the operator at first order in ϵ is thus

$$\Delta_{\rm IR} = \Delta + \gamma_1 \frac{c\epsilon}{\beta_1} + O(\epsilon^2). \quad (10)$$

The crossover to the IR scaling in Eq. (9) happens when

$$1 \ll \frac{\beta_1}{c\epsilon}\frac{g}{p^{c\epsilon}} \Rightarrow p \ll \left(\frac{\beta_1}{c\epsilon}\right)^{(1/c\epsilon)} g^{(1/c\epsilon)}. \quad (11)$$

We see here the physical consequence of introducing the small parameter ϵ: the crossover towards the IR happens at a scale that is enhanced by the parametrically large factor $(\beta_1/c\epsilon)^{(1/c\epsilon)}$ with respect to the naive scale $g^{(1/c\epsilon)}$. As a result, the IR fixed point is parametrically close to the one in the UV.

Indeed, the IR fixed point corresponds to the zero of the β function for the dimensionless combination $\hat{g} = g\Lambda^{-c\epsilon}$

$$\frac{d\hat{g}}{d\log\Lambda} \equiv \beta(\hat{g}) = -c\epsilon\hat{g} + \beta_1\hat{g}^2 + O(\hat{g}^3), \quad \beta(\hat{g}_*) = 0 \Rightarrow \hat{g}_* = \frac{c\epsilon}{\beta_1} + O(\epsilon^2). \quad (12)$$

Comparison with Eq. (10) shows explicitly that, at leading order in ϵ, the difference $\Delta_{\rm IR} - \Delta$ is the anomalous dimension γ evaluated at the fixed point [28,29]. Extrapolating the results to $\epsilon = \frac{1}{2}$, we obtain an estimate for the observables of the IR theory in three dimensions.

Wilson-Fisher fixed point in QED.—The Lagrangian for QED in $d = 4$ is

$$\mathcal{L} = -\frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} + i\sum_{a=1}^{N_f}\bar{\Psi}_a\gamma^\mu D_\mu \Psi^a. \quad (13)$$

We use the usual four-dimensional Dirac notation for the spinors. Their decomposition in terms of two-component fermions is

$$\Psi^a = \begin{pmatrix} \psi^a \\ i\sigma_2 \psi^{a+N_f} \end{pmatrix}, \qquad a = 1, \ldots, N_f. \quad (14)$$

In dimension d we take the Clifford algebra to be $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$, with $\eta^{\mu\nu}\eta_{\mu\nu} = d$. To leading nontrivial order, the beta function in $d = 4 - 2\epsilon$ is given by (here $\hat{e} = e\Lambda^{-\epsilon}$)

$$\beta(\hat{e}) = -\epsilon\hat{e} + \frac{N_f}{12\pi^2}\hat{e}^3 + O(\hat{e}^5). \quad (15)$$

The value of the coupling at the Wilson-Fisher fixed point is $\hat{e}_*^2 = 12\pi^2\epsilon/N_f$. The theory is therefore weakly coupled when we are close to $d = 4$ or when the number of flavors is large.

A comment on γ_5 is in order. A consistent definition of γ_5 in noninteger dimension is due to 't Hooft and Veltman [30–32]. According to this prescription, γ_5 anticommutes only with the γ^μ's of the four-dimensional subspace, and commutes with all others. This implies an explicit breaking of axial symmetries in $d = 4 - 2\epsilon$, and reproduces the chiral anomaly for the singlet axial current $\sum_a \bar{\Psi}_a \gamma^\mu \gamma_5 \Psi^a$ in the limit $\epsilon \to 0$ [33,34]. For the leading-order calculations that we present here, this prescription is in practice equivalent to a naive continuation of γ_5 as totally anticommuting. However, the difference from the naive continuation becomes relevant at higher orders.

QED in $d = 4$ has an $SU(N_f) \times SU(N_f)$ global symmetry with associated conserved currents

$$(J_\mu)_a^b = \bar{\Psi}_a \gamma_\mu \Psi^b - \frac{1}{N_f} \delta_a^b \sum_c \bar{\Psi}_c \gamma_\mu \Psi^c,$$
$$(J_\mu^5)_a^b = \bar{\Psi}_a \gamma_\mu \gamma_5 \Psi^b - \frac{1}{N_f} \delta_a^b \sum_c \bar{\Psi}_c \gamma_\mu \gamma_5 \Psi^c. \qquad (16)$$

Their anomalous dimension at one loop vanishes and, therefore, at leading order the IR dimension is the same as the classical one, i.e., $d - 1$. This is the correct scaling dimension for conserved currents. For the vector current this argument is valid at all orders in perturbation theory, because they are conserved for any d. On the other hand, the axial currents J_μ^5 are explicitly broken for noninteger d, [33] and this can affect the IR dimension at higher orders. Nevertheless, we do expect them to be conserved in $d = 3$, because the nonconservation is given by an operator that vanishes both in $d = 4$ and $d = 3$.

So far, we have argued that the epsilon expansion predicts the existence of currents associated with the global symmetry $SU(N_f) \times SU(N_f)$ in the IR Conformal Field Theory (CFT) for $d = 3$. However, QED_3 has an enhanced $SU(2N_f)$ symmetry. For $N_f \geq N_f^c$, the full $SU(2N_f)$ is realized linearly at the IR fixed point. This entails the existence of $2N_f^2 + 1$ additional conserved operators of spin 1 with protected dimension $\Delta = 2$. It is natural to ask whether these operators are visible also in the theory continued to noninteger dimension.

One of the additional currents is the singlet axial current $J_\mu^s = \sum_a \bar{\Psi}_a \gamma_\mu \gamma_5 \Psi^a$. Indeed, the continuation of the anomaly operator $F \wedge F$ vanishes for $d = 3$. As for the remaining $2N_f^2$ currents, we note that in $d = 4 - 2\epsilon$ we can define the following antisymmetric tensor operators

$$(K_{\mu\nu})_a^b = \bar{\Psi}_a \gamma_{\mu\nu} \Psi^b, \qquad \bar{\Psi}_a \gamma_{\mu\nu} \gamma_5 \Psi^b. \qquad (17)$$

They carry the correct flavor and Lorentz quantum numbers to be identified with the additional currents, because in $d = 3$ we can use the totally antisymmetric tensor $\epsilon_{\mu\nu\rho}$ and dualize them to spin 1 operators.

We are led to the expectation that the IR dimension of J_μ^s and $K_{\mu\nu}$ should evaluate to 2 for $\epsilon = 1/2$. The one-loop computation gives

$$\Delta_{\rm IR}(J^s) = 3 - 2\epsilon + O(\epsilon^2),$$
$$\Delta_{\rm IR}(K) = 3 - 2\epsilon + \frac{3\epsilon}{2N_f} + O(\epsilon^2). \qquad (18)$$

The anomalous dimension of J^s only starts at two-loop order [34,35]. As we will show in the next section, we can estimate that the IR critical point exists only for $N_f \geq 3$. Plugging $N_f = 3$ and $\epsilon = 1/2$ into Eq. (18) we find $\Delta_{\rm IR}^{\text{1-loop}}(K) = 2.25$, which agrees with the expectation within a 10% margin. The precision improves for larger values of N_f. We view this as a hint that the continuation to noninteger dimensions correctly captures the properties of the 3D CFT that we ultimately want to study. A preliminary check of higher orders in ϵ shows that the agreement *improves*. This will be discussed in Ref. [21].

Note that we can also study the anomalous dimension of the operator $F_{\mu\nu}$. The Bianchi identity is obeyed for all d, and one can verify that $\Delta_{\rm IR}(F) = 2$ holds to all orders in ϵ.

Quadrilinear and bilinear operators.—In the three-dimensional theory there are two parity-even quadrilinear scalar operators that are invariant under the full $SU(2N_f)$,

$$\mathcal{O}_1 = \left(\sum_i \bar{\psi}_i \sigma^\mu \psi^i\right)^2 \quad \text{and} \quad \mathcal{O}_2 = \left(\sum_i \bar{\psi}_i \psi^i\right)^2. \qquad (19)$$

The operator $\mathcal{O}_1$ can be easily continued to $d = 4 - 2\epsilon$. In Dirac notation we can rewrite it as $\mathcal{O}_1 = (\sum_a \bar{\Psi}_a \gamma_\mu \Psi^a)^2$. To continue the operator $\mathcal{O}_2$, we use the fact that in $d = 3$ the antisymmetrization of three γ matrices is proportional to the identity. Therefore, we can rewrite it as $\mathcal{O}_2 = 6(\sum_a \bar{\Psi}_a \gamma_{[\mu} \gamma_\nu \gamma_{\rho]} \Psi^a)^2$, which is a well-defined expression also for $d = 4 - 2\epsilon$. Note that in $d = 4$ this operator can be identified with the square of the axial current $(\sum_a \bar{\Psi}_a \gamma_\mu \gamma_5 \Psi^a)^2$.

To obtain their IR dimension we compute the mixing between these two operators. Typical diagrams at one loop are shown in Fig. 1. To obtain the correct mixing at one loop, it is necessary to also take into account the one-loop mixing with the operator $\mathcal{O}_{\rm EOM} = (\sum_a \bar{\Psi}_a \gamma^\mu \Psi^a) \times [(1/e)\partial^\nu F_{\mu\nu} - \sum_b \bar{\Psi}_b \gamma_\mu \Psi^b]$ that vanishes on the equations

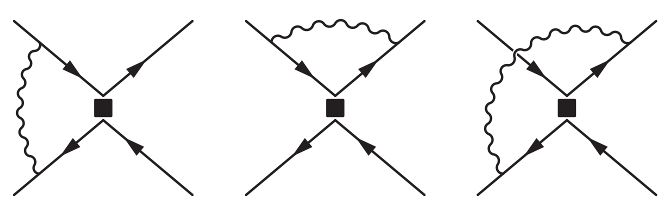

FIG. 1. Diagrams giving the mixing matrix of the quadrilinear operators at one loop.

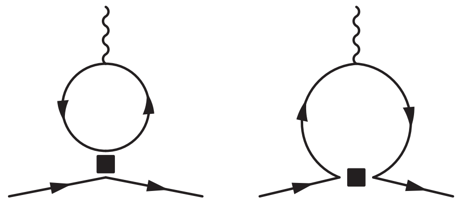

FIG. 2. Diagrams giving the mixing matrix of $\mathcal{O}_1$ and $\mathcal{O}_2$ into $\mathcal{O}_{\mathrm{EOM}}$.

of motion. This mixing is induced by the diagrams in Fig. 2.

In the basis $\{\mathcal{O}_1, \mathcal{O}_2\}$, the matrix of anomalous dimensions reads [36]

$$\gamma_{\mathcal{O}}(\hat{e}) = \frac{\hat{e}^2}{16\pi^2}\begin{pmatrix} \frac{8}{3}(2N_f+1) & 12 \\ \frac{44}{3} & 0 \end{pmatrix} + O(\hat{e}^4). \qquad (20)$$

Its eigenvalues are $\frac{\hat{e}^2}{12\pi^2}(2N_f + 1 \pm 2\sqrt{N_f^2 + N_f + 25})$.

By evaluating them for the fixed-point value $\hat{e}_*^2 = 12\pi^2\epsilon/N_f$, we find that the operator corresponding to the negative eigenvalue becomes relevant when $N_f \le (9\epsilon/2) + O(\epsilon^2)$. Thus, for $\epsilon = \frac{1}{2}$ the operator is relevant in the IR when $N_f = 1$, 2, while it remains irrelevant for any integer $N_f > 2$. Because the quadrilinear is neutral under all the global symmetries, when it becomes relevant it may be generated and trigger a flow to a new IR phase (e.g., a Goldstone phase). From this we obtain the estimate $N_f^c \le 2$ [37].

The same mechanism for the onset of chiral-symmetry breaking has been studied using the large-N_f expansion [38]. The large-N_f approximation of the anomalous dimensions is such that $\mathcal{O}_1$ and $\mathcal{O}_2$ are always irrelevant at the IR fixed point for $N_f \ge 1$. (See, however, the renormalization group study at large N_f in [39]).

Let us make a few comments on $d = 2$. There, the quadrilinear operator is marginal already at the tree-level. Therefore, the criterion above implies that in $d = 2$ the anomalous dimension must evaluate to 0 at N_f^c. This is satisfied for $N_f^c = \infty$. This value is consistent with the IR behavior of QED in $d = 2$: for every finite N_f the theory flows to an $SU(2N_f)$ Wess-Zumino-Witten (WZW) interacting CFT [40,41], namely, a σ model with coset target space deformed by a WZW term (exactly as one expects for the Lagrangian of Nambu-Goldstone bosons). Even though Nambu-Goldstone bosons do not exist in $d = 2$, it appears natural to interpret this theory as the continuation of the chirally broken phase to $d = 2$. See also Refs. [42,43]. Assuming that the anomalous dimension approaches 0 as $1/N_f$ for $N_f \to \infty$, the divergence of $N_f^c(d)$ for $d = 2$ is given by a simple pole. This suggests using the modified ansatz $N_f^c(d) = (d-2)^{-1}f(d)$ in the equation for N_f^c, which can then be solved for f perturbatively in ϵ. With this ansatz, the leading-order estimate becomes $N_f^c \le 4.5$. The difference with the previous estimate is higher-order terms in ϵ; it may be viewed as a measure of uncertainty. Improving on this requires computations beyond one loop [21].

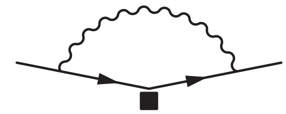

FIG. 3. The diagram giving the anomalous dimension of bilinear operators at one loop.

Further data about the fixed point can be obtained by considering bilinear scalar operators. There are two types of scalar operators in the three-dimensional theory. Operators of the first type are scalars also in $d \ne 3$, i.e.,

$$(B_1)_a^b = \bar{\Psi}_a\Psi^b, \qquad \bar{\Psi}_a\gamma^5\Psi^b. \qquad (21)$$

Operators in this class preserve at most the diagonal $SU(N_f)$ subgroup of $SU(N_f)\times SU(N_f)$, and the most symmetric ones are $\sum_a \bar{\psi}_a\psi^{a+N_f} \pm \text{c.c.}$ The one-loop computation (see Fig. 3) gives

$$\Delta_{\mathrm{IR}}(B_1) = 3 - 2\epsilon - \frac{9\epsilon}{2N_f} + O(\epsilon^2). \qquad (22)$$

The second type of scalar operators are given by rank-three antisymmetric tensors in $d = 4 - 2\epsilon$

$$(B_{2\mu\nu\rho})_a^b = \bar{\Psi}_a\gamma_{[\mu}\gamma_\nu\gamma_{\rho]}\Psi^b, \qquad \bar{\Psi}_a\gamma_{[\mu}\gamma_\nu\gamma_{\rho]}\gamma^5\Psi^b. \qquad (23)$$

They give rise to scalars in $d = 3$ because they can be contracted with the totally antisymmetric tensor $\epsilon_{\mu\nu\rho}$. The chiral condensate Eq. (2) and the parity-odd, $SU(2N_f)$-invariant bilinear $\sum_i \bar{\psi}_i\psi^i$ belong to this class of operators. Since their anomalous dimension vanishes at leading order in perturbation theory, their IR dimension to first order in ϵ is captured by just the classical contribution

$$\Delta_{\mathrm{IR}}(B_2) = 3 - 2\epsilon + O(\epsilon^2). \qquad (24)$$

The anomalous dimension starts being nonzero at two-loop order [34,44]. This implies that higher orders in ϵ in Eq. (24) will be nonzero. Nevertheless, the epsilon expansion suggests that the IR dimension of these scalar operators is perhaps close to $\Delta = 2$.

Future directions.—In this Letter we initiated a study of the critical point of QED_3 based on the epsilon expansion. Our results were based on leading-order computations. It would be very interesting to sharpen the theoretical predictions by higher-order computations [45]. The necessary preliminary step of computing the two-loop counterterms was done for generic gauge theories with fermions in Ref. [48]. Due to the asymptotic nature of the epsilon expansion, efficiently including higher-order terms requires the use of resummation techniques. (In this context, it would be interesting to understand if data from the flavored Schwinger model can be efficiently included).

The epsilon expansion can be used to compute additional observables of the IR fixed point. For instance, correlators of the stress tensor and of conserved currents [49]. Another interesting datum of the 3D theory is the universal coefficient F of the partition function on the three-sphere, which gives also the universal part of the entanglement entropy across a circular region. For this one can utilize the techniques of Refs. [52,53]. The calculation of F in QED_3 via the epsilon expansion has been recently presented in Ref. [54]. Another interesting line of investigation would be to see if the conformal bootstrap techniques shed any light on QED_3 [55].

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[1] Since we are discussing the $\mathbb{R}$ gauge theory, the topological symmetry current $j = \star F$, where F is the field strength two-form, does not have any local operators charged under it and can thus be ignored for the time being.
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