

Genuinely Multipartite Entangled Quantum States with Fully Local Hidden Variable Models and Hidden Multipartite Nonlocality

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The relation between entanglement and nonlocality is discussed in the case of multipartite quantum systems. We show that, for any number of parties, there exist genuinely multipartite entangled states that admit a fully local hidden variable model, i.e., where all parties are separated. Hence, although these states exhibit the strongest form of multipartite entanglement, they cannot lead to Bell inequality violation considering general nonsequential local measurements. Then, we show that the nonlocality of these states can nevertheless be activated using sequences of local measurements, thus revealing genuine multipartite hidden nonlocality.

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The relation between quantum entanglement and nonlocality has been studied extensively in recent years; see, e.g., Refs. [1,2]. While both notions turn out to be equivalent for pure states [3,4], the case of a mixed state is still not understood. This is nevertheless desirable given the importance of entanglement and nonlocality from the point of view of the foundations of quantum theory and for quantum information processing [1].

This research was initiated by Werner [5], who presented a class of bipartite entangled states admitting a local hidden variable (LHV) model. This proved that the correlations obtained by performing arbitrary local projective measurements on such states can be perfectly simulated by a LHV model, hence using only classical resources. This was later extended to general nonsequential measurements, i.e., positive operator valued measures (POVMs) [6]. Since such states cannot lead to Bell inequality violation [7], they are referred to as “local” entangled states [8].

It turns out, however, that certain local entangled states can nevertheless lead to nonlocality when a sequence of local measurements is performed [9]. That is, the use of local filters can help to reveal (or activate) the nonlocality of the entangled state. This phenomenon, termed “hidden nonlocality,” occurs even for entangled states admitting a LHV model for POVMs [10]. Other works showed that the nonlocality of local entangled states can be activated by performing joint measurements on several copies of the state [11–13], or by placing many copies of the state in a quantum network [14,15].

Whereas the above questions have been intensively discussed for bipartite states, the relation between entanglement and nonlocality for multipartite systems is almost unexplored thus far. Here, one should nevertheless expect interesting and novel phenomena, due to the rich structure of multipartite entanglement. In particular, there is a hierarchy of different forms of entanglement in multipartite systems, the strongest of which is genuine

multipartite entanglement (GME). Similarly, the notion of genuine multipartite nonlocality (GMNL) has been discussed [16–18], which represents the strongest form of nonlocality for multipartite systems. A first natural question is then whether there exist GME states, the correlations of which can be simulated by a LHV model. This was first discussed by Tóth and Acín [19], who presented a GME state of 3 qubits admitting a LHV model, but could not extend their construction to more parties. More recently, Augusiak *et al.* [20] showed the existence of GME states of any number of parties that cannot lead to GMNL. Specifically, the authors discussed a class of GME states of N parties, and constructed a LHV model in which the parties are separated into two groups. However, this model is essentially bipartite, as the N parties cannot be completely separated. Beyond these few exploratory works, nothing is known. to the best of our knowledge.

Here we report progress in understanding the relation between GME and nonlocality. First, we present a general technique for constructing multipartite entangled states admitting a fully LHV model, i.e., where all parties are separated. This allows us to show that there exist GME states of an arbitrary number of systems, which admit a fully LHV model for arbitrary POVM measurements. Moreover, we show that the nonlocality of these states can be activated using sequential measurements. Notably, the use of local filters allows us to obtain GMNL. To summarize, there exist multipartite states, entangled in the strongest possible sense, that do not exhibit even the weakest form of nonlocality when considering nonsequential measurements. However, when using sequences of measurements, the strongest form of multipartite nonlocality can be obtained. We conclude with a series of open questions.

Genuine multipartite entanglement.—Consider N parties sharing a multipartite quantum state ρ acting on $\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$, where \mathcal{H}_i is the local Hilbert space of party i . Denote by $(b, \bar{b}) \in \mathcal{B}$ a bipartition of the N parties.

If ρ can be decomposed as a mixture of states that are each separable on some bipartition of the Hilbert space, then we have

$$\rho = \sum_{(b,\bar{b}) \in \mathcal{B}} p_b \left(\sum_j q_j^b |\Phi_j\rangle\langle\Phi_j|_b \otimes |\Phi_j\rangle\langle\Phi_j|_{\bar{b}} \right), \quad (1)$$

with $\sum_b p_b = \sum_j q_j^b = 1$, and $|\Phi_j\rangle\langle\Phi_j|_b$ acts on the Hilbert space specified by the partition b (and similarly for $|\Phi_j\rangle\langle\Phi_j|_{\bar{b}}$). If ρ does not admit such a decomposition, then it is GME. Such states can thus not be created via local operations and classical communication (LOCC) using only biseparable states.

Determining whether a given state is GME is challenging, as one must search over all possible decompositions [Eq. (1)]. However, there are sufficient conditions for an N -qubit state to be GME [21–23] (see also Ref. [24]). Write the state ρ in the canonical basis $|0, 0, \dots, 0\rangle, |0, 0, \dots, 1\rangle, \dots, |1, 1, \dots, 1\rangle$ as

$$\rho = \begin{pmatrix} c_1 & & & & & & & z_1 \\ & c_2 & & & & & & z_2 \\ & & \ddots & & & & & \ddots \\ & & & c_n & z_n & & & \\ & & & z_n^* & d_n & & & \\ & & & & & \ddots & & \\ & & & & & & d_2 & \\ z_2^* & & & & & & & d_1 \\ z_1^* & & & & & & & \end{pmatrix} \quad (2)$$

(we only write the elements of interest), where $n = 2^{N-1}$. Then ρ is GME if

$$C(\rho) = 2 \max_i \{|z_i| - w_i\} > 0, \quad (3)$$

where $w_i = \sum_{j \neq i}^n \sqrt{c_j d_j}$. Below, we will use this condition to ensure that a state is GME. Note that the value of $C(\rho)$ can also be used to quantify GME [25], an aspect that, however, will not be discussed here.

Nonlocality.—Consider again the state ρ , where now each party can make measurements labeled x_i obtaining outcomes a_i , specified by the measurement operators $M_{a_i|x_i}$, with $M_{a_i|x_i} \geq 0$ and $\sum_{a_i} M_{a_i|x_i} = \mathbb{1}$. The probability to see the outputs $\mathbf{a} = (a_1, \dots, a_N)$ given the inputs $\mathbf{x} = (x_1, \dots, x_N)$ is given by

$$p(\mathbf{a}|\mathbf{x}) = \text{Tr}[\rho(\otimes_{i=1}^N M_{a_i|x_i})]. \quad (4)$$

The state ρ is called (fully) local if, for all possible measurement operators $M_{a_i|x_i}$, the statistics $p(\mathbf{a}|\mathbf{x})$ can be reproduced by a LHV model:

$$p(\mathbf{a}|\mathbf{x}) = \int_{\lambda} q_{\lambda} p_{\lambda}(a_1|x_1) p_{\lambda}(a_2|x_2) \cdots p_{\lambda}(a_N|x_N) d\lambda, \quad (5)$$

where q_{λ} is a probability density over the shared variable λ and $p_{\lambda}(a_i|x_i)$ are probability distributions, called local

response functions. Likewise, if Eq. (5) cannot be satisfied, then the state is said to be nonlocal, as witnessed by the violation of (some) Bell inequality.

One may also consider a weaker notion of locality, whereby the correlations are not demanded to be local with respect to all parties [as in Eq. (5)], but instead to be (mixtures of) correlations that are each local across some bipartition. Again denoting by $(b, \bar{b}) \in \mathcal{B}$ a bipartition of the parties, these correlations take the form

$$p(\mathbf{a}|\mathbf{x}) = \sum_{(b,\bar{b}) \in \mathcal{B}} p_b \int_{\lambda} q_{\lambda}^b p_{\lambda}(\mathbf{a}_b|\mathbf{x}_b) p_{\lambda}(\mathbf{a}_{\bar{b}}|\mathbf{x}_{\bar{b}}) d\lambda, \quad (6)$$

where $\mathbf{a}_b, \mathbf{x}_b$ denote the inputs and outputs for the bipartition b . Note that Eq. (5) implies Eq. (6), but not necessarily the converse. Correlations that cannot be written in the above form are called genuinely multipartite nonlocal and represent the strongest form of multipartite nonlocality [16]. Here, for simplicity, we put no restrictions on the probability distributions $p_{\lambda}(\mathbf{a}_b|\mathbf{x}_b)$, $p_{\lambda}(\mathbf{a}_{\bar{b}}|\mathbf{x}_{\bar{b}})$ other than positivity and normalization (for example, they may be signaling); note that more sophisticated definitions of GMNL were proposed [17,18]. The N -party Greenberger–Horne–Zeilinger (GHZ) state, $|\text{GHZ}\rangle = (|0\rangle^{\otimes N} + |1\rangle^{\otimes N})/\sqrt{2}$, is known to produce correlations that are GMNL, as proven by the violation of the Svetlichny inequalities [16,27,28].

GME and nonlocality.—The link between GME and nonlocality is almost unexplored thus far. For $N = 3$, Tóth and Acín constructed a genuine tripartite entangled state admitting a fully LHV model [i.e., of the form Eq. (5)] for arbitrary local projective measurements [19]. Recently, Augusiak *et al.* [20] presented GME states of N qubits which cannot lead to GMNL. More precisely, they constructed a LHV model for some bipartition of N qubits, i.e., of the form Eq. (6). However, it is still unknown if there exist GME states that admit LHV models that are fully local, i.e., that satisfy Eq. (5), for any possible measurements. This is what we show in the next section.

Method.—Our main tool is a simple method to construct entangled N -party states which admit a LHV model. Specifically, we start by considering a bipartite entangled state ρ which is “unsteerable,” that is, which cannot be used to demonstrate steering. Formally, this means that ρ admits as so-called local hidden state (LHS) model [29]; hence, its correlations can be decomposed as

$$\begin{aligned} p(ab|xy) &= \text{Tr}[\rho M_{a|x} \otimes M_{b|y}] \\ &= \int q_{\lambda} p_{\lambda}(a|x) \text{Tr}[\sigma_{\lambda} M_{b|y}] d\lambda, \end{aligned} \quad (7)$$

where σ_{λ} is the local hidden state, distributed with density q_{λ} , and $M_{b|y}$ denotes Bob’s measurement operator. Clearly, an unsteerable state is local (with $p(b|y, \lambda) = \text{Tr}[\sigma_{\lambda} M_{b|y}]$), while the opposite may not hold in general.

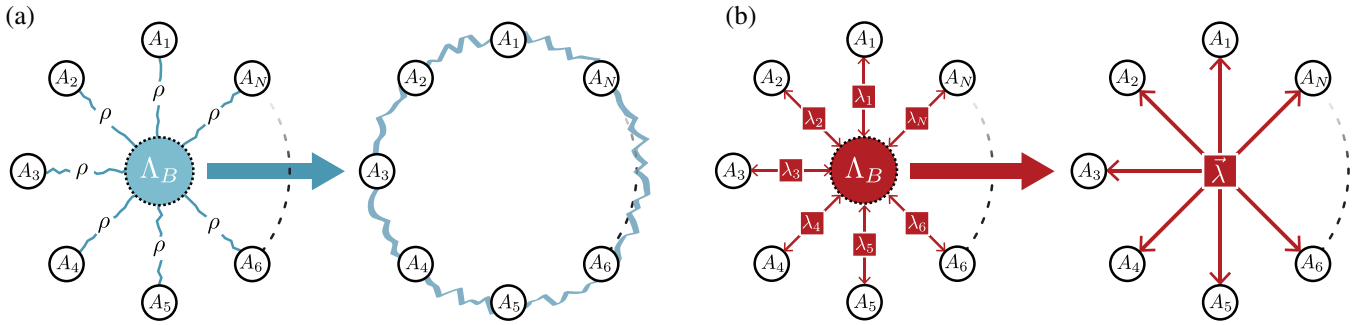


FIG. 1. Construction of multipartite states admitting a fully local model. (a) Construction of the state. First, place N copies of a bipartite state ρ in a star-shaped network. Then, apply a map Λ_B at the central node (i.e., on parties $B_1 \dots B_N$), and trace out these parties. We thus obtain an N -partite state, $\rho_{A_1 \dots A_N}$ (represented by the blue wiggly line), shared by parties $A_1 \dots A_N$. (b) LHV model. If ρ admits a LHS model, one can simulate the correlations of the star-shaped network for $\rho^{\otimes N}$, whereby the central node receives the hidden states σ_{λ_i} independently from each source and the parties A_i receive hidden variables λ_i . One may now correlate the individual λ_i 's by having the map Λ_B act on the hidden states; i.e., we can define a new distribution over $\vec{\lambda} = (\lambda_1, \dots, \lambda_N)$ that depends on $\text{Tr}[\Lambda_B(\otimes_i \sigma_{\lambda_i})]$. If each party A_i uses the same response function as in the LHS model for ρ , then the resulting statistics on parties $A_1 \dots A_N$ simulate exactly the state $\rho_{A_1 \dots A_N}$.

Next, we combine several copies of ρ in a star-shaped network (see Fig. 1). This allows one to construct a multipartite entangled state admitting a fully local model. Specifically, we have the following.

Lemma 1.—Let ρ be a quantum state acting on $\mathcal{H}_{A_1} \otimes \mathcal{H}_{B_1}$. The state $\rho^{\otimes N}$ therefore acts on $\mathcal{H}_{A_1} \otimes \dots \otimes \mathcal{H}_{A_N} \otimes \mathcal{H}_{B_1} \otimes \dots \otimes \mathcal{H}_{B_N} = \mathcal{H}_A \otimes \mathcal{H}_B$. Furthermore, let Λ_B be a completely positive linear map acting on \mathcal{H}_B . If ρ is unsteerable from A_1 to B_1 , i.e., admits a decomposition [Eq. (7)], then the N -party state,

$$\rho_{A_1 \dots A_N} = \frac{\text{Tr}_B[\mathbb{1}_A \otimes \Lambda_B(\rho^{\otimes N})]}{\text{Tr}[\mathbb{1}_A \otimes \Lambda_B(\rho^{\otimes N})]}, \quad (8)$$

admits a local hidden variable model, of the form Eq. (5), on the N -partition $A_1/A_2/\dots/A_{N-1}/A_N$.

The intuition behind the above lemma is given in Fig. 1. A complete proof is given in Appendix A in Supplemental Material [30].

Note that we have not specified the class of local measurements for which the LHV model is valid in the above lemma. If ρ has a LHS model for projective measurements, then $\rho_{A_1 \dots A_N}$ will have a LHV model for projective measurements, and similarly for POVMs. Note also that one can generalize slightly the result of Lemma 1 (see Appendix A in Supplemental Material [30]). Specifically, one can use different unsteerable states in each arm of the star-shaped network rather than the same state N times, and one can choose not to perform the trace over B and keep the center party.

GME states with fully local model.—We now use Lemma 1 to construct N -qubit states which admit a fully local model. We then prove these states to be GME for all N . Specifically, consider the class of two-qubit states,

$$\rho_{\alpha, \theta} = \alpha |\psi_\theta\rangle\langle\psi_\theta| + (1 - \alpha) \rho_A^\theta \otimes \frac{\mathbb{1}}{2}, \quad (9)$$

where $0 \leq \alpha \leq 1$, $0 \leq \theta \leq \pi/4$, $|\psi_\theta\rangle = \cos \theta |00\rangle + \sin \theta |11\rangle$, and $\rho_A^\theta = \text{Tr}_B |\psi_\theta\rangle\langle\psi_\theta|$. These states are entangled for all $\theta \in]0, \pi/4[$, if $\alpha > 1/3$. Furthermore, they are unsteerable from Alice to Bob for arbitrary projective measurements if the relation

$$\cos^2(2\theta) \geq \frac{2\alpha - 1}{(2 - \alpha)\alpha^3} \quad (10)$$

holds [31]. Hence, for any $0 \leq \alpha < 1$, one may find a corresponding $\theta > 0$ such that $\rho_{\alpha, \theta}$ is unsteerable. We now define the completely positive linear map,

$$\Lambda_B(\sigma) = F_B \sigma F_B^\dagger, \quad F_B = |0\rangle[|0, 0, \dots, 0\rangle + |1, 1, \dots, 1\rangle],$$

which projects the systems of $B_1 \dots B_N$ onto an N -qubit GHZ state. We may now define the N -party state $\rho_{A_1 \dots A_N}$ by using $\rho_{\alpha, \theta}$ and Λ_B in Eq. (8). In Appendix B in Supplemental Material [30] we show that the concurrence of this state for a fixed N , α , θ is given by

$$C(\rho_{A_1 \dots A_N}) = \frac{2 \sin^N(2\theta) \left(\alpha^N + \left[\frac{1+\alpha}{2} \right]^N + \left[\frac{1-\alpha}{2} \right]^N - 1 \right)}{[1 + \alpha \cos 2\theta]^N + [1 - \alpha \cos 2\theta]^N}. \quad (11)$$

It follows that for any N , one can find parameters α , θ such that (i) condition (10) is satisfied (ensuring that $\rho_{\alpha, \theta}$ has a LHS model) and (ii) $C(\rho_{A_1 \dots A_N}) > 0$, proving that $\rho_{A_1 \dots A_N}$ is GME. To give a specific example, take $\alpha = 1 - 1/N^2$ and $\theta > 0$, such that Eq. (10) is saturated. One sees that the denominator of Eq. (11) and $\sin^N 2\theta$ are both positive. We therefore need

$$\alpha^N + \left[\frac{1+\alpha}{2}\right]^N + \left[\frac{1-\alpha}{2}\right]^N > 1 \quad (12)$$

to be positive for all $N \geq 2$. For the case $N = 2$, one has $\alpha = 3/4$ and we find $43/32 > 1$. For $N > 2$, upon substituting $\alpha = 1 - 1/N^2$ the left-hand side becomes

$$\begin{aligned} & \left[1 - \frac{1}{N^2}\right]^N + \left[1 - \frac{1}{2N^2}\right]^N + \left[\frac{1}{2N^2}\right]^N \\ & > 2 \left[1 - \frac{1}{N^2}\right]^N > 2 \left[1 - \frac{1}{N}\right] > 1, \end{aligned} \quad (13)$$

where for the first inequality we use the fact that $[1 - 1/N^2]^N < [1 - 1/2N^2]^N$ and $[1/2N^2]^N > 0$, and the second inequality follows from Bernoulli's inequality.

Extension to general measurements.—A natural question is now to find a GME state with a fully local model, considering general POVMs. While the states $\rho_{\alpha,\theta}$ are not known to admit a LHS model for POVMs, we can nevertheless proceed differently. Starting from $\rho_{A_1 \dots A_N}$, we can in fact construct another state, ρ_{GME} , which is both GME and local for POVM measurements.

Specifically, define $\rho_{A_1 \dots A_k} = \text{Tr}_{A_{k+1} \dots A_N}[\rho_{A_1 \dots A_N}]$ and denote by $\mathfrak{U}[\rho]$ the unnormalized and symmetrized version of ρ . Then the state

$$\rho_{\text{GME}} = \frac{1}{2^N} \left[\rho_{A_1 \dots A_N} + \sum_{j=0}^{N-1} \mathfrak{U}[\rho_{A_1 \dots A_j} \otimes |2\rangle\langle 2|^{\otimes N-j}] \right] \quad (14)$$

admits a fully local model, for arbitrary local POVMs. Note that $|2\rangle\langle 2|$ denotes the projector onto a subspace orthogonal to the qubit subspace. The above follows from a straightforward extension of Protocol 2 of Ref. [10] to the case of N parties.

To conclude, we have to show that the state is GME. Note that if each party makes a local projection on the qubit subspace $|0\rangle\langle 0| + |1\rangle\langle 1|$, then the resulting (renormalized) state is $\rho_{A_1 \dots A_N}$, which is GME. Since one cannot create GME using stochastic local operations, it follows that ρ_{GME} is GME.

Hidden genuine multipartite nonlocality.—We showed that GME states can admit a fully LHV model for arbitrary nonsequential measurements. A natural question now is whether these states have hidden nonlocality [9], that is, whether nonlocality could be revealed via sequences of measurements. A sufficient condition for the existence of hidden nonlocality is the possibility of transforming the initial state using local stochastic operations, i.e., local filters, to another state that violates some Bell inequality (see, e.g., Ref. [32]). Below, we will see that the states ρ_{GME} have genuine multipartite hidden nonlocality. Furthermore, the activation of nonlocality is maximal, in the sense that the filtered state exhibits GMNL, despite the initial state being fully local.

Consider N parties sharing ρ_{GME} . Let each party perform a local filtering operation given by

$$G_\epsilon = \epsilon|0\rangle\langle 0| + |1\rangle\langle 1|, \quad (15)$$

hence transforming ρ_{GME} to the state

$$\rho_\epsilon = \frac{G_\epsilon^{\otimes N} \rho_{\text{GME}} G_\epsilon^{\otimes N}}{\text{Tr}[G_\epsilon^{\otimes N} \rho_{\text{GME}} G_\epsilon^{\otimes N}]}. \quad (16)$$

In Appendix C of Supplemental Material [30] we prove that for $\epsilon = \tan \theta$ [where θ is the parameter in Eq. (9)] the filtered state is essentially a pure N -party GHZ state $[|0\rangle^{\otimes N} + |1\rangle^{\otimes N}]/\sqrt{2}$. Specifically, the fidelity between the two states is given by

$$\begin{aligned} \mathcal{F}(\rho_\epsilon, |\text{GHZ}\rangle\langle \text{GHZ}|) &= \langle \text{GHZ} | \rho_\epsilon | \text{GHZ} \rangle \\ &= \frac{1}{2} \left[\alpha^N + \left(\frac{1+\alpha}{2}\right)^N + \left(\frac{1-\alpha}{2}\right)^N \right], \end{aligned} \quad (17)$$

which tends to 1 when α is sufficiently close to 1. Since the GHZ state is known to exhibit GMNL for any N , in particular, via violation of the Svetlichny inequalities [27,28] (which are robust to noise), it follows that ρ_ϵ can also be made GMNL.

Conclusion.—We showed that GME states can admit a fully LHV model, for any number of parties. Thus, while exhibiting the strongest form of multipartite entanglement (GME), these states can never lead to any Bell inequality violation, considering general nonsequential measurements. This can be viewed as a maximal inequivalence between multipartite entanglement and nonlocality. Interestingly, this gap can disappear when sequential measurements are considered, and the strongest form of nonlocality can be activated, thus highlighting the relevance of sequential measurements in multipartite nonlocality.

In the future, it would be interesting to investigate the above questions in quantitative terms. For instance, could one find examples of highly entangled GME states admitting a LHV model? In order to do so, one should choose a specific measure of GME [24] (as there exist no unique measure).

Also, the method we presented for constructing multipartite local entangled states could be further explored. Firstly, one could start from different bipartite unsteerable states; see, e.g., Refs. [33,34]. Secondly, by keeping the central node in the network, one can construct multipartite LHS models where one of the parties has a quantum response function, and hence may prove useful in the study of multipartite steering [35].

Finally, one could ask if there exist GME states admitting LHV models for sequential measurements, although this question is in fact still open even in the bipartite case.

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