

Interparticle Attraction in 2D Complex Plasmas

Roman Kompaneets,^{1,*} Gregor E. Morfill,^{1,2} and Alexei V. Ivlev¹

¹Max-Planck-Institut für extraterrestrische Physik, Giessenbachstrasse 1, 85748 Garching, Germany

²BMSTU Centre for Plasma Science and Technology, Moscow, Russia

(Received 7 August 2015; published 22 March 2016)

Complex (dusty) plasmas allow experimental studies of various physical processes occurring in classical liquids and solids by directly observing individual microparticles. A major problem is that the interaction between microparticles is generally not molecularlike. In this Letter, we propose how to achieve a molecularlike interaction potential in laboratory 2D complex plasmas. We argue that this principal aim can be achieved by using relatively small microparticles and properly adjusting discharge parameters. If experimentally confirmed, this will make it possible to employ complex plasmas as a model system with an interaction potential resembling that of conventional liquids.

DOI: 10.1103/PhysRevLett.116.125001

A complex (or dusty) plasma is a plasma that contains charged microparticles (dust) [1–9]. In laboratory complex plasmas, the particles are typically of a few μm in diameter and charged primarily by collection of free ions and electrons from the plasma [2–6]. Such systems allow experimental studies of various physical processes occurring in liquids and solids by directly observing individual particles. This idea has inspired a great deal of experiments (see, e.g., studies of shock waves [10,11], solitons [12], crystallization and melting fronts [13], and dislocations in crystals [14]). In contrast to colloidal suspensions [15,16], which can be used for similar purposes, complex plasmas are characterized by weak damping and, therefore, allow studying various processes on their intrinsic dynamic time scale.

A major problem in the field of complex plasmas is that the interaction between microparticles is generally not molecularlike, as the pair potential does not exhibit long-range attraction. This raises questions as to what extent complex plasmas are suitable to study various fundamental processes occurring in conventional liquids, such as the liquid-vapor phase transition and critical phenomena [17]. In isotropic complex plasmas, which can be experimentally realized under microgravity conditions [18–22], the interaction potential is believed to be repulsive at distances of the order of the interparticle separation [2,4,23]. Under laboratory conditions, the interaction potential $\varphi(\mathbf{r})$ is generally substantially anisotropic and also nonreciprocal [i.e., $\varphi(\mathbf{r}) \neq \varphi(-\mathbf{r})$, $\text{actio} \neq \text{reactio}$] due to the presence of plasma flow [2,4,24]. Often, a two-dimensional (2D) complex plasma is formed in the plane perpendicular to the flow; in this case, the interactions in the monolayer are reciprocal but believed to be repulsive, too (see, e.g., the experiment of Ref. [25]).

In this Letter, we use a theoretical foundation for calculating the pair interaction potential in the presence of ion flow, developed by us before [26–29], to make an

easy-to-verify prediction as to how to achieve attraction between particles in 2D complex plasmas. We argue that this can be done in a ground-based experiment with the most common experimental setup. No external fields need to be applied (in contrast to Refs. [18,28]), as we predict that the attraction can be achieved by merely adjusting parameters such as the gas pressure, rf power, and particle size. Our theoretical approach is robust and realistic as it is kinetic and accounts for collisions, the non-Maxwellian velocity distribution of ions, and the electric field that drives the ion flow; the potential calculated using this approach has been shown to be in excellent agreement with direct measurements [27]. If our prediction is confirmed, it will make it possible to use 2D complex plasmas as a model system to study fundamental processes in 2D liquids.

Most laboratory experiments on 2D complex plasmas are performed in an rf plasma device, where charged microparticles are levitated against gravity by the (time-averaged) electric field of the (pre)sheath near the lower electrode [2,4]. This region is characterized by the presence of strong ion flow (with a substantially non-Maxwellian velocity distribution) driven by the field towards the electrode [30–33]; the field is induced in the plasma to balance the absorption of ions and electrons on the electrode (see Bohm criterion [30–33]). The presence of the ion flow is a key factor determining the plasma shielding and, hence, the interactions between microparticles. Thus, to describe the shielding of a particle levitated in the (pre)sheath (not in the plasma bulk), it is essential to employ the kinetic description for ions, incorporating the field driving the flow and an ion-neutral collision operator:

$$\mathbf{v} \cdot \nabla f + \frac{e}{m} (\mathbf{E}_{\text{sh}} - \nabla \varphi) \cdot \frac{\partial f}{\partial \mathbf{v}} = \text{St}[f], \quad (1)$$

$$\nabla \cdot \mathbf{E}_{\text{sh}} - \nabla^2 \varphi = \frac{e}{\epsilon_0} \left(\int f d\mathbf{v} - n_e + Q\delta(\mathbf{r}) \right). \quad (2)$$

Here, $f(\mathbf{r}, \mathbf{v})$ is the ion distribution function, \mathbf{E}_{sh} is the unperturbed (pre)sheath field, which is generally a function of the vertical coordinate, $\varphi(\mathbf{r})$ is the potential perturbation due to the charged particle, $\text{St}[f]$ is the ion-neutral collision operator, Q is the particle charge, e is the elementary charge (ions are assumed to be singly ionized), m is the ion mass, and ε_0 is the permittivity of free space. The electron density n_e is assumed to either have the Boltzmann response, $\delta n_e = n_e e \varphi / T_e$, where T_e is the electron temperature, or, as a particular case, to be unperturbed by the particle at all, which corresponds to the limit of infinitely large T_e . Note that we neglect ionization, as the latter is expected to have little effect on the interparticle interactions, at least at pressures we will consider [34].

The principal assumption of our present approach is the homogeneous plasma approximation, in which all unperturbed (by the particle) quantities do not depend on the vertical coordinate. This is a common assumption, and we have recently shown it to be quite accurate to describe the shielding at moderate distances, particularly in the direction perpendicular to the flow [35]. Thus, the steady state in our model is determined simply by the balance of the electric field and collisions, $(e\mathbf{E}_{\text{sh}}/m) \cdot \partial f / \partial \mathbf{v} = \text{St}[f]$. To calculate $\varphi(\mathbf{r})$, we use the linear perturbation approximation [2,4,26,27,35]; i.e., we linearize Eqs. (1) and (2) with respect to the perturbations induced by the particle.

Cold-neutral approximation.—Since in many experiments the ion flow velocity at the levitation height of the 2D crystal significantly exceeds the neutral thermal velocity, we start our analysis with the cold-neutral approximation:

$$\text{St}[f] = -\frac{v f(\mathbf{v})}{\ell} + \frac{\delta(\mathbf{v})}{\ell} \int f(\mathbf{v}') v' d\mathbf{v}', \quad (3)$$

where $\delta(\mathbf{v})$ is the delta function. The collision length ℓ is assumed to be velocity independent, which is a quite accurate approximation for superthermal flow velocities and noble gases (typical for experiments with 2D complex plasmas). The dominant collision mechanism in this case is charge transfer [36], which is characterized by a weak (logarithmic) velocity dependence of its cross section [36,37].

The corresponding form of the potential is derived in Ref. [27] and given by Eq. (6) of that paper. The inclusion of the electron response results in addition of the term

$$\tau_\ell = e E_{\text{sh}} \ell / T_e \quad (4)$$

to the numerator under the square root in the above equation. The potential essentially depends on two dimensionless numbers, τ_ℓ and

$$\zeta_\ell = \lambda / \ell, \quad (5)$$

where

$$\lambda = \sqrt{\frac{\varepsilon_0 E_{\text{sh}} \ell}{n e}}, \quad (6)$$

and n is the ion density. Let us analyze this potential in the plane perpendicular to the flow.

We start with the asymptotic expressions for small and large distances. At small distances, the potential obviously becomes the Coulomb potential, $\varphi = Q / (4\pi \varepsilon_0 r)$. It is easy to show that for $\tau_\ell = 0$, the corresponding range of distances is $r \lesssim \lambda \zeta_\ell^{1/3}$ for $\zeta_\ell \ll 1$, and $r \lesssim \lambda \zeta_\ell$ for $\zeta_\ell \gg 1$. At larger distances, the potential exhibits a power-law decay,

$$\varphi = \frac{Q \lambda^2 \sqrt{2}}{48 \pi \varepsilon_0 r^3} (60 \zeta_\ell^2 - 1) + O(r^{-4}). \quad (7)$$

However, numerical calculations show that this asymptotic behavior is reached only at very large distances (e.g., $r \sim 10^3 \lambda$) and that a finite τ_ℓ changes the power-law decay to an exponential decay. Nevertheless, Eq. (7) is helpful in that it already demonstrates the principal possibility of attraction at low collisionality (small ζ_ℓ).

Figure 1 shows the results of our numerical analysis of the potential. In the absence of the electron response ($\tau_\ell = 0$), the potential is repulsive at small distances and attractive at large distances for $\zeta_\ell < 0.13$. For $0.13 < \zeta_\ell < 0.24$, there is attraction at intermediate distances and repulsion at small and large distances, while for $\zeta_\ell > 0.24$, the potential is repulsive at all distances. In the presence of the electron response, our calculations show that the potential has an attractive well when

$$\zeta_\ell < \frac{0.067}{\tau_\ell + 0.28}. \quad (8)$$

For $\tau_\ell \leq 2$, this condition is accurate to less than 5%.

Role of a finite neutral temperature.—Before we analyze Eq. (8) in terms of experimental parameters such as the gas pressure, plasma density, and particle size, let us first address the role of a finite neutral temperature. Accurately doing so requires cumbersome velocity calculations, but, to probe into the principal effect, we simplify the problem by employing the model Bhatnagar-Gross-Krook (BGK) collision operator,

$$\text{St}[f] = -\nu f(\mathbf{v}) + \nu \Phi_M(v) \int f(\mathbf{v}') d\mathbf{v}', \quad (9)$$

where

$$\Phi_M(v) = \frac{1}{(2\pi v_T^2)^{3/2}} \exp\left(-\frac{v^2}{2v_T^2}\right) \quad (10)$$

is the normalized Maxwellian velocity distribution of neutrals, ν is the (velocity-independent) collision

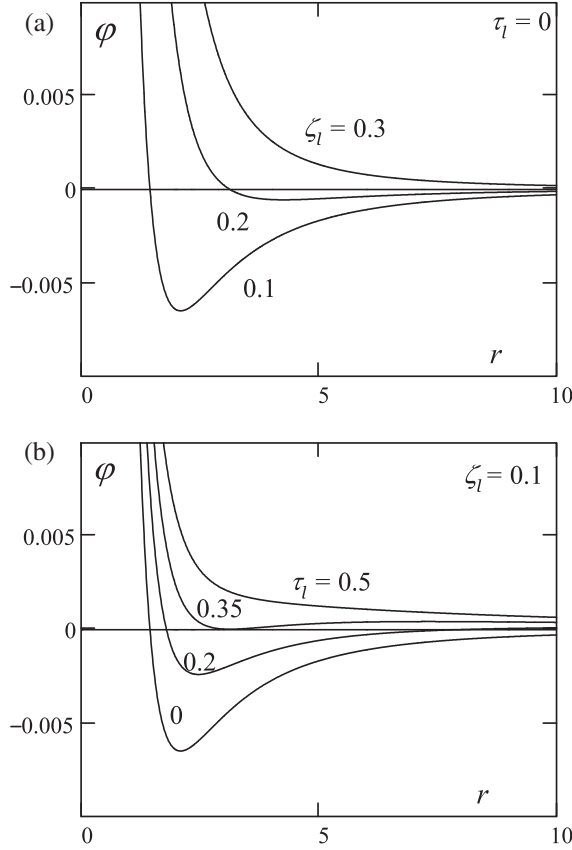


FIG. 1. Potential in the plane perpendicular to the flow, calculated in the constant-collision-length model under the cold-neutral approximation. Here, the potential and distance are normalized by $Q/(4\pi\epsilon_0\lambda)$ and λ , respectively. (a) The limit of absence of the electron response ($\tau_e = 0$) plotted for various ζ_e . (b) Effect of a finite electron response (for $\zeta_e = 0.1$ and various τ_e).

frequency, and $v_T = \sqrt{T_n/m}$ is the thermal velocity of neutrals. The corresponding potential is given in Ref. [26] and depends on three dimensionless parameters:

$$\tau_\nu = mv_\parallel^2/T_e, \quad (11)$$

$$\zeta_\nu = \nu/\omega_p, \quad (12)$$

and

$$u = v_\parallel/v_T, \quad (13)$$

where $v_\parallel = eE_{\text{sh}}/(m\nu)$ is the flow velocity and $\omega_p = \sqrt{ne^2/(\epsilon_0 m)}$ is the ion plasma frequency. In certain situations (see below), it is convenient to employ the temperature ratio

$$\tau_n = T_n/T_e \quad (14)$$

instead of using τ_ν .

In the limit of cold neutrals, $v_T \rightarrow 0$ (or $u \rightarrow \infty$), the potential (in the plane perpendicular to the flow) given by the BGK model almost matches the one given by the constant-collision-length model, provided that the parameters are properly rescaled. Figure 2 shows that while the potential curves differ considerably when τ_e and τ_ν are chosen to be the same, changing any of these parameters by about 30% results in almost perfect matching of the curves. For $\tau_\nu = 0$, the potential at large distances is

$$\phi = \frac{Qv_\parallel^2(\zeta_\nu^2 - 2)}{4\pi\epsilon_0\omega_p^2 r^3} + O(r^{-4}). \quad (15)$$

That is, a similar r^{-3} dependence is recovered as in the constant-collision-length case.

For finite τ_ν (or τ_n), the far-field potential at small flow velocities ($u \ll 1$) is

$$\begin{aligned} \phi(r) = & \frac{Q\tau_n\zeta_\nu^2 u^2}{8\pi\epsilon_0 r(1+\tau_n)^3} \\ & + \frac{Q\lambda_n^2 u^2(\zeta_\nu^2 - 2 - 2\tau_n\zeta_\nu^2 + 2\tau_n^2)}{4\pi\epsilon_0 r^3(1+\tau_n)^4} \\ & + O\left(\frac{u^4}{r^3}\right) + O\left(\frac{u^2}{r^4}\right) + O\left[\frac{1}{r} \exp\left(-\frac{r}{\lambda_n}\right)\right], \end{aligned} \quad (16)$$

where

$$\lambda_n = \sqrt{\frac{\epsilon_0 T_n}{ne^2}}, \quad (17)$$

and O refers to the limit $u \rightarrow 0$, $r \rightarrow \infty$. Equation (16) shows that at very large distances, the potential is always repulsive and Coulomb-like. However, since in

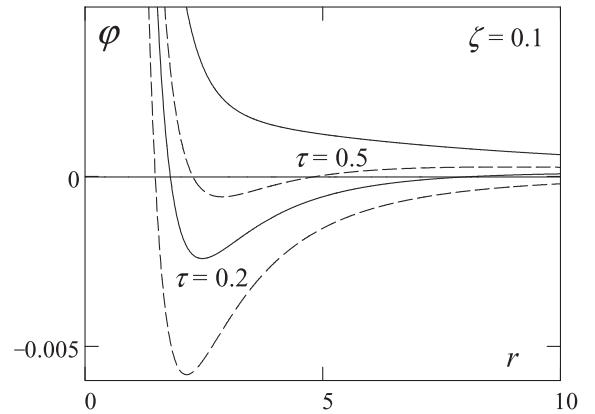


FIG. 2. Comparison of the constant-collision-length model (solid lines) with the BGK model (dashed lines), both under the cold-neutral approximation. For the constant-collision-length model ($\zeta = \zeta_e$ and $\tau = \tau_e$), the potential and distance are normalized by $Q/(4\pi\epsilon_0\lambda)$ and λ , respectively. For the BGK model ($\zeta = \zeta_\nu$ and $\tau = \tau_\nu$), the potential and distance are normalized by $Q\omega_p/(4\pi\epsilon_0 v_\parallel)$ and v_\parallel/ω_p , respectively.

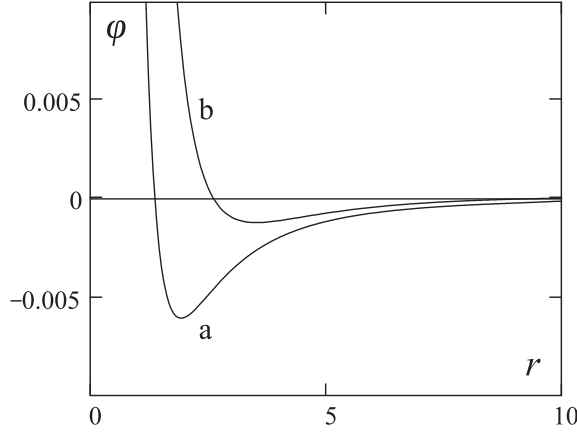


FIG. 3. Potential for the BGK model and finite realistic values of u , ζ_ν , and τ_n . The potential and distance are normalized as in Fig. 2. (a) $u = 6$, $\zeta_\nu = 0.05$, $\tau_n = 10^{-2}$. (b) $u = 4$, $\zeta_\nu = 0.2$, $\tau_n = 10^{-2}$.

experiments $\tau_n \sim 10^{-2}$, the first term is significant only at very large distances; in the second term, the small parameter τ_n plays little role, so the attraction occurs for $\zeta_\nu < \sqrt{2}$. Note that for $\tau_\nu = 0$, the potential (16) reduces exactly to Eq. (15); that is, we get the same far-field potential for small and large flow velocities.

In Fig. 3 we demonstrate that the attraction is present for various realistic combinations of u , ζ_ν , and τ_n .

Thus, since a finite neutral temperature does not suppress the attraction, it seems safe to say that all what is needed to achieve the attraction in an experiment is to satisfy the condition (8), provided that the flow velocity substantially exceeds the thermal velocity, i.e., $eE_0\ell \gg T_n$.

It is noteworthy that in the BGK model, the plasma is unstable with respect to the formation of ion kinetic waves when both $\zeta_\nu \lesssim 0.3$ and $u \gtrsim 8$; otherwise, the plasma is stable [29]. Thus, the attraction emerges before the instability sets in (i.e., before the model itself becomes physically unmeaningful).

We also note that the attraction disappears in the limit $\zeta_\nu \rightarrow 0$, as in this case, the potential well moves to $r \rightarrow \infty$ and becomes infinitely weak, which indicates that collisions are essential for the attraction. Mathematically, the limit $\zeta_\nu \rightarrow 0$ implies infinitely small \mathbf{E}_{sh} and ν but a finite ratio of these (and, thus, a finite flow velocity). In this limit, the far-field potential at small flow velocities can be analytically shown to be

$$\phi(r) = \frac{Q\lambda_n^2 u^2}{4\pi\epsilon_0 r^3} \left[\frac{2}{(1 + \tau_n)^2} - \frac{\pi}{2(1 + \tau_n)^3} \right] + \dots, \quad (18)$$

where \dots are the same O terms as in Eq. (16). The term in the square brackets is always positive, so there is always repulsion at large distances. We have numerically verified that the potential remains repulsive at all distances and

finite flow velocities as long as the limit $\zeta_\nu \rightarrow 0$ is considered.

Experimental conditions for attraction.—To convert ζ_ℓ and τ_ℓ into experimentally controllable parameters, we use the vertical force balance, $-QE_{sh} = Mg$ (neglecting the ion drag force [2,4]), where M is the particle mass. This yields

$$\zeta_\ell = \sqrt{\frac{\rho a^2 P g \sigma}{3n z T_e T_n}} \quad (19)$$

and

$$\tau_\ell = \frac{\rho a^2 g e^2 T_n}{3\epsilon_0 P z T_e^2 \sigma}, \quad (20)$$

where a is the particle radius, P is the gas pressure, ρ is the particle material density, $z = -Qe/(4\pi\epsilon_0 a T_e)$ is the normalized particle charge, usually of the order of unity [4], and σ is the ion-neutral cross section.

By analyzing the condition for attraction [Eq. (8)] together with Eqs. (19) and (20), we find it rather restrictive but quite possible to satisfy. For instance, for the parameters of the experiments of Ref. [38], namely, $P = 0.66$ Pa, $\rho = 1510$ g/cm³, $T_e = 2.5$ eV, $n \sim 2 \times 10^9$ cm⁻³, and $T_n = 300$ K, the attraction should occur when the particle diameter $2a$ is less than about $4.8 \mu\text{m}$. (This critical size corresponds to $\zeta_\ell \approx 0.035$ and $\tau_\ell \approx 1.6$; to calculate this size, we assumed $z = 3$ [2,4] and $\sigma = 6.5 \times 10^{-15}$ cm² [27].) This is quite a realistic size, as the above experiments were performed with particles of $3.4\text{--}8.8 \mu\text{m}$ in diameter. Note that the plasma density in the experiment of Ref. [38] was measured in the bulk of the discharge, not at the levitation height, but this should not affect estimates. Also note that for the above-calculated critical size, the flow velocity at the levitation height is about 10 times the thermal velocity of neutrals, so the cold-neutral approximation should indeed be applicable. We have also calculated, in the cold-neutral approximation, that the use of $2 \mu\text{m}$ particles would result in a potential well located at about ≈ 0.13 mm with the depth $\sim 300T_n$, i.e., a deep potential well close to the particle.

While this Letter focuses on the regime where the flow velocity is much larger than the thermal one, we should note that this is probably not necessarily required for the attraction. Indeed, in the opposite limit $u \ll 1$, the BGK model still yields the attraction [see Eq. (16)]. Note that in this regime, the dominant collision mechanism is elastic scattering, which is not accurately described by the BGK operator, so the attraction is not guaranteed. The attraction condition $\zeta_\nu < \sqrt{2}$ becomes

$$\sigma P \sqrt{\frac{\epsilon_0}{2n_0 e^2 T_n}} < 1, \quad (21)$$

which can be very easily satisfied. For example, for $n = 10^9$ cm⁻³ and $T = 300$ K, Eq. (21) yields $P < 240$ Pa.

However, the condition $u \lesssim 1$ represents a stronger restriction: By using $-QE_{\text{sh}} = Mg$, we rewrite it as

$$\frac{\rho a^2 g e^2}{3\epsilon_0 P T_e z \sigma} \lesssim 1. \quad (22)$$

For $2a = 3 \mu\text{m}$, $T_e = 2.5 \text{ eV}$, $\rho = 1510 \text{ g/cm}^3$, $z = 3$, and $T_n = 300 \text{ K}$, this condition yields $P \gtrsim 40 \text{ Pa}$, which is quite a realistic pressure for experiments with complex plasmas. Larger particles, however, would require a larger pressure to satisfy the condition $u \lesssim 1$.

To conclude, we have theoretically shown that it is possible to obtain a molecularlike interaction potential in 2D complex plasmas by using relatively small particles and properly adjusting discharge parameters. We hope that our results will stimulate experimental work in this direction. If experimentally confirmed, the interparticle attraction will make it possible to employ complex plasmas as a model system with an interaction potential resembling that of conventional liquids.

The work received funding from the European Research Council under the European Union's Seventh Framework Programme, European Research Council Grant No. 267499, and the Russian Scientific Foundation, Project No. 14-43-00053.

*kompaneets@mpe.mpg.de

- [1] V. E. Fortov and G. E. Morfill, *Complex and Dusty Plasmas* (CRC Press, London, 2010).
- [2] V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, *Phys. Rep.* **421**, 1 (2005).
- [3] O. Ishihara, *J. Phys. D* **40**, R121 (2007).
- [4] G. E. Morfill and A. V. Ivlev, *Rev. Mod. Phys.* **81**, 1353 (2009).
- [5] P. K. Shukla and B. Eliasson, *Rev. Mod. Phys.* **81**, 25 (2009).
- [6] M. Bonitz, C. Henning, and D. Block, *Rep. Prog. Phys.* **73**, 066501 (2010).
- [7] M. Schwabe, S. Zhdanov, C. R  th, D. B. Graves, H. M. Thomas, and G. E. Morfill, *Phys. Rev. Lett.* **112**, 115002 (2014).
- [8] A. V. Ivlev, S. K. Zhdanov, M. Lampe, and G. E. Morfill, *Phys. Rev. Lett.* **113**, 135002 (2014).
- [9] P. Hartmann, A. Z. Kov  cs, A. M. Douglass, J. C. Reyes, L. S. Matthews, and T. W. Hyde, *Phys. Rev. Lett.* **113**, 025002 (2014).
- [10] Y. Nakamura, H. Bailung, and P. K. Shukla, *Phys. Rev. Lett.* **83**, 1602 (1999).
- [11] D. Samsonov, S. K. Zhdanov, R. A. Quinn, S. I. Popel, and G. E. Morfill, *Phys. Rev. Lett.* **92**, 255004 (2004).
- [12] D. Samsonov, A. V. Ivlev, R. A. Quinn, G. Morfill, and S. Zhdanov, *Phys. Rev. Lett.* **88**, 095004 (2002).
- [13] M. Rubin-Zuzic, G. E. Morfill, A. V. Ivlev, R. Pompl, B. A. Klumov, W. Bunk, H. M. Thomas, H. Rothmel, O. Havnes, and A. Fouquet, *Nat. Phys.* **2**, 181 (2006).
- [14] V. Nosenko, G. E. Morfill, and P. Rosakis, *Phys. Rev. Lett.* **106**, 155002 (2011).
- [15] V. J. Anderson and H. N. W. Lekkerkerker, *Nature (London)* **416**, 811 (2002).
- [16] D. Frenkel, *Science* **314**, 768 (2006).
- [17] H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Clarendon, Oxford, 1971).
- [18] A. V. Ivlev, G. E. Morfill, H. M. Thomas, C. R  th, G. Joyce, P. Huber, R. Kompaneets, V. E. Fortov, A. M. Lipaev, V. I. Molotkov *et al.*, *Phys. Rev. Lett.* **100**, 095003 (2008).
- [19] S. A. Khrapak, B. A. Klumov, P. Huber, V. I. Molotkov, A. M. Lipaev, V. N. Naumkin, A. V. Ivlev, H. M. Thomas, M. Schwabe, G. E. Morfill *et al.*, *Phys. Rev. E* **85**, 066407 (2012).
- [20] M. Y. Pustynnik, M. H. Thoma, G. E. Morfill, R. Grimm, and C. Hock, *J. Plasma Phys.* **78**, 289 (2012).
- [21] J. Beckers, D. J. M. Trienekens, and G. M. W. Kroesen, *Phys. Rev. E* **88**, 055101 (2013).
- [22] S. A. Khrapak, M. H. Thoma, M. Chaudhuri, G. E. Morfill, A. V. Zobnin, A. D. Usachev, O. F. Petrov, and V. E. Fortov, *Phys. Rev. E* **87**, 063109 (2013).
- [23] M. Lampe and G. Joyce, *Phys. Plasmas* **22**, 023704 (2015).
- [24] V. A. Schweigert, I. V. Schweigert, A. Melzer, A. Homann, and A. Piel, *Phys. Rev. E* **54**, 4155 (1996).
- [25] U. Konopka, G. E. Morfill, and L. Ratke, *Phys. Rev. Lett.* **84**, 891 (2000).
- [26] A. V. Ivlev, S. K. Zhdanov, S. A. Khrapak, and G. E. Morfill, *Phys. Rev. E* **71**, 016405 (2005).
- [27] R. Kompaneets, U. Konopka, A. V. Ivlev, V. Tsytovich, and G. Morfill, *Phys. Plasmas* **14**, 052108 (2007).
- [28] R. Kompaneets, G. E. Morfill, and A. V. Ivlev, *Phys. Plasmas* **16**, 043705 (2009).
- [29] R. Kompaneets, A. V. Ivlev, S. V. Vladimirov, and G. E. Morfill, *Phys. Rev. E* **85**, 026412 (2012).
- [30] V. A. Godyak, *Phys. Lett. A* **89**, 80 (1982).
- [31] K.-U. Riemann, *J. Phys. D* **24**, 493 (1991).
- [32] R. N. Franklin, *J. Phys. D* **36**, R309 (2003).
- [33] N. Hershkowitz, *Phys. Plasmas* **12**, 055502 (2005).
- [34] M. Chaudhuri, S. A. Khrapak, R. Kompaneets, and G. E. Morfill, *IEEE Trans. Plasma Sci.* **38**, 818 (2010).
- [35] R. Kompaneets, A. V. Ivlev, V. Nosenko, and G. E. Morfill, *Phys. Rev. E* **89**, 043108 (2014).
- [36] M. A. Lieberman and A. J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing* (Wiley, New York, 1994).
- [37] B. M. Smirnov, *Physics of Ionized Gases* (Wiley, New York, 2001).
- [38] V. Nosenko, A. V. Ivlev, S. K. Zhdanov, M. Fink, and G. E. Morfill, *Phys. Plasmas* **16**, 083708 (2009).