Detecting the Disruption of Dark-Matter Halos with Stellar Streams

Jo Bovy

Department of Astronomy and Astrophysics, University of Toronto, 50 St. George Street, Toronto, Ontario M5S 3H4, Canada (Received 29 November 2015; revised manuscript received 27 January 2016; published 22 March 2016)

Narrow stellar streams in the Milky Way halo are uniquely sensitive to dark-matter subhalos, but many of these subhalos may be tidally disrupted. I calculate the interaction between stellar and dark-matter streams using analytical and N-body calculations, showing that disrupting objects can be detected as lowconcentration subhalos. Through this effect, we can constrain the lumpiness of the halo as well as the orbit and present position of individual dark-matter streams. This will have profound implications for the formation of halos and for direct- and indirect-detection dark-matter searches.

DOI: [10.1103/PhysRevLett.116.121301](http://dx.doi.org/10.1103/PhysRevLett.116.121301)

Introduction.—One of the key predictions of the colddark-matter paradigm is that the extended dark-matter halos of galaxies contain a large amount of small-scale structure in the form of subhalos [\[1\]](#page-3-1). At the high-mass end of the subhalo spectrum, this structure is visible in the form of dwarf galaxies [\[2\].](#page-3-2) But if dark matter is truly cold, the mass spectrum should extend well below the halo mass scale where baryons can condense and form stars and below the scales constrained by the Ly- α forest ($M \gtrsim 3 \times 10^8 M_{\odot}$) [\[3\]](#page-3-3). An observational determination of the subhalo mass spectrum to well below $10^{9}M_{\odot}$ would provide one of the most important astrophysical constraints on the nature of dark matter.

In our own Milky Way galaxy, one of the most promising methods for detecting low-mass dark-matter subhalos is through their effects on cold stellar streams in the halo [\[4](#page-3-4)–6]. Cold stellar streams form when a globular cluster in the halo gets tidally disrupted; mass loss at pericentric passages is deposited into orbits with slightly higher and lower orbital energies, leading to narrow leading and trailing arms [\[7\]](#page-3-5). Many examples of such streams are now known from wide-area photometric surveys [\[8,9\].](#page-3-6) The kinematical coldness of tidal streams makes them sensitive to the influence of subhalos with masses $\lesssim 10^8 M_{\odot}$. Dynamical modeling of the smooth stream itself [\[10,11\]](#page-3-7) and of the impact of subhalos [\[12,13\]](#page-3-8) has been shown to be able to detect and characterize subhalos with masses down to $10^7 M_{\odot}$ with Gaia and the Large Synoptic Survey Telescope (LSST) [\[14\]](#page-3-9).

While many of the dark-matter subhalos are expected to survive as separate entities, some of them, especially the more massive ones within a few tens of kpc from the Galactic center, may be in the process of being tidally disrupted in a similar manner as the globular clusters [\[15\]](#page-3-10). If this is the case, a null detection of the expected cold-darkmatter mass spectrum at $M \lesssim 10^9 M_{\odot}$ could be misconstrued as evidence against cold dark matter. On the more positive side, if a significant fraction of the dark matter in the solar neighborhood is coherent in velocity space, annual modulation in dark-matter direct-detection experiments may be enhanced [\[16,17\]](#page-3-11) and dark matter in models with high minimum scattering thresholds would be easier to detect [\[18,19\].](#page-4-0) A determination of the fraction of darkmatter subhalos that are in the process of tidal disruption is therefore crucial for future astrophysical and directdetection experiments into the nature of dark matter.

In this Letter, I compute the impact of dark-matter streams—formed from subhalos that are in the process of being tidally disrupted—on stellar streams. The kinematical coldness of stellar streams makes them excellent probes of this scenario. The interaction between a darkmatter and a stellar stream is more extended than that between a surviving subhalo and a stellar stream, and therefore a larger part of the stream is affected by the interaction. This implies that dark-matter streams will most easily be detected in conventional analyses (i.e., which assume a surviving subhalo) as detections with subhalo parameters that imply anomalously low concentrations. The extended interaction, however, also causes the impulse approximation—which is typically accurate for subhalostream interactions—to break down, and I demonstrate that large velocity kicks can occur, even for very extended dark-matter tidal tails, for which the cross section is high. The breakdown of the impulse approximation also opens up the possibility that we can infer the orbit and current location of individual, entirely dark subhalos.

Impulse approximation.—The interaction between a stellar stream and a surviving dark-matter subhalo is typically well described using the impulse approximation [\[13,20\]](#page-3-12). In this approximation, both the subhalo and the stream are approximated as moving on a straight line at the time of closest approach; the interaction is modeled as an instantaneous velocity kick along the stellar stream. We can calculate the interaction between a dark-matter and a stellar stream in a similar manner, approximating the dark-matter stream as a set of Plummer spheres [\[21\]](#page-4-1) for which the interaction with the stellar stream can be computed analytically. Using the same setup as in Ref. [\[12\]](#page-3-8), where the stellar stream moves along the y axis with velocity v_v and

the dark matter moves with velocity $(-w_1 \sin \alpha, w_y)$; $w_1 \cos \alpha$) through the point of closest approach at $(b \cos \alpha, 0, b \sin \alpha)$, the velocity kicks along the stream are

$$
\Delta v_y = -\int dt \frac{dGM}{dt} \frac{w_{\perp}^2 \tilde{y}(t)}{w\{[b^2 + r_s^2(t)]w^2 + w_{\perp}^2 \tilde{y}(t)^2\}}, \quad (1)
$$

$$
\int dGM \, bw^2 \cos[\sin(\alpha \pm \tilde{y}(t))w_{\perp}w_{\parallel} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp} \sin[\cos(\alpha \pm \tilde{y}(t))w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}w_{\perp}
$$

$$
\Delta v_{x/z} = 2 \int dt \frac{dGM}{dt} \frac{bw^2 \cos[\sin(\alpha \pm \tilde{y}(t))w \perp w_{\parallel} \sin[\cos(\alpha \pm \tilde{y}(t))]w^2 + w_{\perp}^2 \tilde{y}(t)^2]}{w\{[b^2 + r_s^2(t)]w^2 + w_{\perp}^2 \tilde{y}(t)^2\}},
$$
\n(2)

where $\tilde{y}(t) = y - v_y t$.

In these expressions, $w_{\parallel} = v_y - w_y$, $w = \sqrt{w_{\perp}^2 + w_{\parallel}^2}$,

and dGM/dt is the mass of the Plummer sphere with scale radius $r_s(t)$ that passes through the point of closest approach at time t ; choose the sin and cos in brackets and the minus sign for Δv_z . This expression assumes that the velocity kicks arising from different parts of the darkmatter stream add linearly, which is a good assumption for the small kicks from $M \lesssim 10^9 M_{\odot}$ subhalos.

As discussed in Ref. [\[12\],](#page-3-8) the overall amplitude of the kick of a single Plummer sphere is primarily set by the mass. The extent Δy over which the kicks are significant is set by the scale radius r_s and the impact parameter b. It is clear from the expressions in Eq. [\(1\)](#page-1-0) that if the dark-matter stream has a length L that is short compared to Δy in the sense that $v_y L / \sqrt{w_\perp^2 + w_y^2} < \Delta y$, then the interaction is similar to that with a single subhalo. Because the two stream velocities are generically similar, this is only the case for dark-matter streams that are not much longer than r_s , i.e., very early in the disruption process. For longer dark-matter streams, the interaction will be softened as the various parts of the leading and trailing tails of the dark-matter stream typically produce kicks in opposite directions. The net effect is to reduce the amplitude of the velocity kick below that from a surviving subhalo, while simultaneously acting over a larger part of the stellar stream. This is illustrated in Fig. [1](#page-1-1). It is straightforward to generalize the impulse approximation in the previous paragraph to take into account the curved nature of the stellar stream (cf. Ref. [\[13\]\)](#page-3-12). This can be efficiently done by moving the stream along an orbit to compute the velocities of the stream segments over the time interval of the interaction.

N-body simulations.—To investigate the interaction between two streams in further detail, I use N-body simulations to compute the full, nonlinear interaction. The setup of these simulations is as in Ref. [\[13\].](#page-3-12) A mock stellar stream is generated by evolving a King cluster [\[22\]](#page-4-2) of $10^5 M_{\odot}$ with $W_0 = 5$ and a core radius of 13 pc represented with 10^5 particles for 10.125 kpc/(km s⁻¹) $(\approx 10 \text{ Gyr})$ in a logarithmic host potential with a circular velocity of 220 km s^{-1} and a potential flattening of 0.9. This mock stellar stream sustains a direct hit by dark-matter

FIG. 1. Interaction between a stream on a circular orbit at 10 kpc in the x-y plane moving at 220 km s⁻¹ anticlockwise with a dark-matter stream with a total mass of $10^8 M_{\odot}$ moving at $(0, 132, 176)$ km s⁻¹, making a closest approach at 625 pc from $(x, y) = (10, 0)$ kpc (the setup of Fig. 1 in Ref. [\[13\]\)](#page-3-12). The interaction is computed in the impulse approximation of Eq. [\(1\),](#page-1-0) assuming uniform dGM/dt for different stream lengths and $r_s(t) = 625$ pc. The top panel shows the velocity kick in the x direction and the bottom panel that in the y direction. The curve labeled as "halo, no arms" has all of the mass in a single Plummer sphere; that labeled as "halo $+2$ kpc arms" has half of the mass in a stream and half in a single Plummer halo. An interaction with a stream rather than a surviving subhalo has a lower amplitude, but affects a much larger part of the stellar stream.

streams at $(X, Y, Z) = (-13.5, 2.84, -1.84)$ kpc moving at $(v_x, v_y, v_z) = (6.82, 132.77, 149.42)$ km s⁻¹, generated by evolving a Plummer sphere with $M = 10⁸M_o$ and $r_s = 625 \text{ pc (using } 10^5 \text{ particles}) \text{ for } 125 \text{ pc/(km s⁻¹)}$, 250 pc/ (kms^{-1}) , and 500 pc/ (kms^{-1}) . The dark-matter and stellar streams are evolved together for $250 \text{ pc}/(\text{km s}^{-1})$ (≈250 Myr) starting 125 pc/ (km s^{-1}) before the direct-impact time (defined as the time at which the progenitor dark-matter subhalo would have directly hit the stellar stream), to be able to study the interaction in a clean manner. All N-body simulations are run using gyrfalcON and NEMO [\[23,24\]](#page-4-3). These N-body simulations are demonstrated in Fig. [2.](#page-2-0)

I compute the velocity kicks in the N-body simulations by backwards-orbit integration using galpy [\[25\]](#page-4-4) of the stellar-stream particles after the interaction in the host potential and comparing the velocity with that of the particles in a simulation of the stellar stream with the same initial conditions, but without the dark-mater subhalo.

FIG. 2. N-body simulations of the interaction between a dark-matter (DM) and globular-cluster (GC) stellar stream. The stellar stream is shown at the point of closest approach between the stream and the dark-matter progenitor. The dark-matter is displayed 125 pc/(km s⁻¹) (≈125 Myr) before and 125 pc/(km s⁻¹) after the interaction, which is the time interval over which the DM and GC streams are evolved together. The orbit of the dark-matter progenitor during this time is given in red. Three different dark-matter streams are generated by letting the dark matter disrupt for different amounts of time. In the simulation on the left, the DM stream is only starting to form, in the middle panel a long DM stream is in the process of forming, and on the right the DM subhalo is fully disrupted, but still forms a coherent stream.

These velocity kicks are displayed in Fig. [3](#page-3-13) as a function of angle along the stream. Compared to the kicks from an interaction with a surviving subhalo on the same orbit as the dark-matter stream, which peak at \approx (0.4, 0.4, 1.8) km s⁻¹ in (v_x, v_y, v_z) and are approximately zero by $|\theta_{\parallel}| = 1$, it is clear that the kicks are smaller and act over a more extended part of the stream, in agreement with the considerations based on the impulse approximation above. Interestingly, the kicks in v_x are larger than that for the surviving-subhalo interaction, with a similar amplitude for streams of different lengths.

To understand the dynamics in the N-body simulation further, I estimate the amount of stream mass passing through the impact point as a function of time, by analyzing the dark-matter stream in action-angle coordinates (cf. Ref. [\[10\]](#page-3-7)). I then compute the kicks using the impulse approximation above, accounting for the movement of the stellar stream during the interaction. The motions of the particles in the most diffuse stream are consistent with being test particles in the host potential, but for the dark-matter streams that are still in the process of tidal disruption I add a small contribution from a single Plummer sphere to represent the remnant subhalo. The resulting kicks are displayed as dashed lines in Fig. [3.](#page-3-13) While the impulse approximation works well for v_y and v_z for the shorter two streams, it fails for the most diffuse stream and for all streams for v_x . The solid lines show kicks computed by representing each darkmatter stream with a random subsample of 300 particles, modeled as Plummer spheres with $r_s = 10$ pc and computing the kicks from each of these 300 interactions independently using orbit integration in the host plus Plummer potential. For the two shortest streams, I again add small contributions from a subhalo remnant. It is clear that this approximation to the kicks matches the full N-body kicks in all dimensions well, even at large offsets from the impact point. This demonstrates that the impulse approximation breaks down because the orbital motion of the dark-matter stream is important, rather than due to the nonlinear contributions from different parts of the stream.

Dark-matter subhalos are more realistically represented as Navarro-Frenk-White (NFW) spheres [\[26\]](#page-4-5) rather than Plummer spheres. To determine whether the effects discussed above are different for NFW halos, I have repeated the simulations above, but modeling the dark-matter halos as NFW halos with $M = 10^8 M_{\odot}$, $r_s = 900$ pc, and a tidal truncation radius of 2 kpc (chosen to be similar to subhalos in the Via Lactea-2 simulation [\[27\]](#page-4-6) in the mass and radial range considered here). The particle data for this NFW halo are sampled using the method of Ref. [\[28\]](#page-4-7). These NFW dark-matter halos disrupt and form tidal tails of almost the same length and width as those in the Plummer simulation above, and the effect on the globular-cluster stream is qualitatively the same.

Discussion.—Stellar streams within tens of kpc from the Galactic center typically encounter a few subhalos with masses of $10^8 M_{\odot}$ to $10^9 M_{\odot}$ [\[20\]](#page-4-8). Many of these may be in the process of tidal disruption and give rise to velocity kicks along the stellar streams similar to those in Figs. [1](#page-1-1) and [3](#page-3-13). These kicks affect a larger part of the stream and are slightly lower in amplitude. In standard analyses of the impact of subhalos on stellar streams [\[13,14\]](#page-3-12), both of these effects will lead to inferred (M, r_s) with anomalously low concentrations compared to the cold-dark-matter prediction. This will be the telltale sign that the stellar stream has been hit by a dark-matter stream rather than a surviving subhalo. From the N-body simulations above, diffuse

FIG. 3. Velocity kicks computed from the three N-body simulations shown in Fig. [2](#page-2-0)—"disruption start" from the left-hand panel, "disrupting" from the middle panel, and "fully disrupted" from the right-hand panel—represented as dots compared to two approximations. The kicks are shown as a function of the parallel angle coordinate θ_{\parallel} along the stream with respect to the impact point in action-angle coordinates [\[10\]](#page-3-7); the range in θ_{\parallel} shown spans almost the entire trailing arm of the stellar stream. The breakdown of the impulse approximation for the diffuse stream and for all streams in v_x demonstrates that the full orbital path of the stream is responsible for the observed kicks.

streams can give substantial kicks for at least ≈ 0.5 Gyr, so the probability of catching a dark-matter halo in the act of disrupting is high.

Analyses of the kinematics of stellar streams (cf. Ref. [\[14\]](#page-3-9)) can therefore determine the prevalence of dark-matter streams in the Milky Way halo. Many additional stellar streams within tens of kpc are expected to be found soon using data from Gaia [\[29\],](#page-4-9) and we will therefore soon have plenty of potential targets for a dark-matter-stream search. Such a measurement would have profound implications for dark-matter direct-detection experiments [\[16\]](#page-3-11) and would provide an important constraint on the formation of halos in the hierarchical cosmological framework.

The N-body simulations above demonstrate that stellar streams are uniquely sensitive to the full orbital path of dark-matter streams. This is unlike the case of subhalo– stream interactions, which are typically well modeled using the impulse approximation. In this approximation, the velocity kicks remain the same when the mass of the perturber and the relative fly-by velocity are changed by the same factor [\[14\].](#page-3-9) Computing the kicks for a simulation like the "disrupting" case in Fig. [3](#page-3-13), but with the mass and relative fly-by velocity scaled down by half, I find velocity kicks that are different in amplitude and width by 50%, while these can be measured to \approx 10% from Gaia and LSST data (cf. Ref. [\[14\]\)](#page-3-9), and the full orbit of the dark-matter stream could thus be precisely constrained. If a likely dark-matter stream from a recently disrupted subhalo is detected as discussed above, detailed observations of the kinematics along the stellar stream may therefore reveal the full orbital path and present position of a dark subhalo. Such an object would be a tantalizing target for indirect dark-matter detection experiments.

It is my pleasure to thank Neal Dalal for conversations that inspired this work and the anonymous referees for helpful comments that improved this Letter. This research received financial support from the Natural Sciences and Engineering Research Council of Canada.

[*](#page-0-0) bovy@astro.utoronto.ca

- [1] V. Springel, J. Wang, M. Vogelsberger, A. Ludlow, A. Jenkins, A. Helmi, J. F. Navarro, C. S. Frenk, and S. D. M. White, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1111/j.1365-2966.2008.14066.x) 391, 1685 (2008).
- [2] V. Belokurov, [New Astron. Rev.](http://dx.doi.org/10.1016/j.newar.2013.07.001) 57, 100 (2013).
- [3] M. Viel, G.D. Becker, J.S. Bolton, and M.G. Haehnelt, Phys. Rev. D 88[, 043502 \(2013\)](http://dx.doi.org/10.1103/PhysRevD.88.043502).
- [4] K. V. Johnston, D. N. Spergel, and C. Haydn, [Astrophys. J.](http://dx.doi.org/10.1086/339791) 570[, 656 \(2002\)](http://dx.doi.org/10.1086/339791).
- [5] R. A. Ibata, G. F. Lewis, M. J. Irwin, and T. Quinn, [Mon.](http://dx.doi.org/10.1046/j.1365-8711.2002.05358.x) [Not. R. Astron. Soc.](http://dx.doi.org/10.1046/j.1365-8711.2002.05358.x) 332, 915 (2002).
- [6] R. G. Carlberg, [Astrophys. J.](http://dx.doi.org/10.1088/0004-637X/748/1/20) 748, 20 (2012).
- [7] K. V. Johnston, [Astrophys. J.](http://dx.doi.org/10.1086/305273) 495, 297 (1998).
- [8] M. Odenkirchen et al., Astrophys. J. 548[, L165 \(2001\).](http://dx.doi.org/10.1086/319095)
- [9] C. J. Grillmair and O. Dionatos, [Astrophys. J.](http://dx.doi.org/10.1086/505111) 643, L17 [\(2006\).](http://dx.doi.org/10.1086/505111)
- [10] J. Bovy, [Astrophys. J.](http://dx.doi.org/10.1088/0004-637X/795/1/95) **795**, 95 (2014).
- [11] J. L. Sanders, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1093/mnras/stu1159) 443, 423 (2014).
- [12] D. Erkal and V. Belokurov, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1093/mnras/stv655) 450, [1136 \(2015\)](http://dx.doi.org/10.1093/mnras/stv655).
- [13] J. L. Sanders, J. Bovy, and D. Erkal, [Mon. Not. R. Astron.](http://dx.doi.org/10.1093/mnras/stw232) [Soc., doi:10.1093/mnras/stw232 \(2016\).](http://dx.doi.org/10.1093/mnras/stw232)
- [14] D. Erkal and V. Belokurov, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1093/mnras/stv2122) 454, [3542 \(2015\)](http://dx.doi.org/10.1093/mnras/stv2122).
- [15] M. Zemp, J. Diemand, M. Kuhlen, P. Madau, B. Moore, D. Potter, J. Stadel, and L. Widrow, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1111/j.1365-2966.2008.14361.x) 394[, 641 \(2009\)](http://dx.doi.org/10.1111/j.1365-2966.2008.14361.x).
- [16] M. Kuhlen, N. Weiner, J. Diemand, P. Madau, B. Moore, D. Potter, J. Stadel, and M. Zemp, [J. Cosmol. Astropart.](http://dx.doi.org/10.1088/1475-7516/2010/02/030) [Phys. 2 \(2010\) 030.](http://dx.doi.org/10.1088/1475-7516/2010/02/030)
- [17] M. Lisanti and D. N. Spergel, [Phys. Dark Univ.](http://dx.doi.org/10.1016/j.dark.2012.10.007) 1, 155 [\(2012\).](http://dx.doi.org/10.1016/j.dark.2012.10.007)
- [18] D. Smith and N. Weiner, Phys. Rev. D 64[, 043502 \(2001\).](http://dx.doi.org/10.1103/PhysRevD.64.043502)
- [19] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, [Phys.](http://dx.doi.org/10.1103/PhysRevD.69.037302) Rev. D 69[, 037302 \(2004\)](http://dx.doi.org/10.1103/PhysRevD.69.037302).
- [20] J. H. Yoon, K. V. Johnston, and D. W. Hogg, [Astrophys. J.](http://dx.doi.org/10.1088/0004-637X/731/1/58) 731[, 58 \(2011\).](http://dx.doi.org/10.1088/0004-637X/731/1/58)
- [21] H. C. Plummer, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1093/mnras/71.5.460) 71, 460 (1911).
- [22] I. R. King, Astron. J. **71**[, 64 \(1966\).](http://dx.doi.org/10.1086/109857)
- [23] W. Dehnen, [J. Comput. Phys.](http://dx.doi.org/10.1006/jcph.2002.7026) **179**, 27 (2002).
- [24] P. Teuben, in Astronomical Data Analysis Software and Systems IV, Astronomical Society of the Pacific Conference

Series Vol. 77, edited by R. A. Shaw, H. E. Payne, and J. J. E. Hayes (ASP, San Francisco, 1995), p. 398.

- [25] J. Bovy, [Astrophys. J. Suppl. Ser.](http://dx.doi.org/10.1088/0067-0049/216/2/29) 216, 29 (2015).
- [26] J. F. Navarro, C. S. Frenk, and S. D. M. White, [Astrophys. J.](http://dx.doi.org/10.1086/304888) 490[, 493 \(1997\)](http://dx.doi.org/10.1086/304888).
- [27] J. Diemand, M. Kuhlen, P. Madau, M. Zemp, B. Moore, D. Potter, and J. Stadel, [Nature \(London\)](http://dx.doi.org/10.1038/nature07153) 454, 735 (2008).
- [28] P.J. McMillan and W. Dehnen, [Mon. Not. R. Astron. Soc.](http://dx.doi.org/10.1111/j.1365-2966.2007.11753.x) 378[, 541 \(2007\)](http://dx.doi.org/10.1111/j.1365-2966.2007.11753.x).
- [29] M. A. C. Perryman, K. S. de Boer, G. Gilmore, E. Høg, M. G. Lattanzi, L. Lindegren, X. Luri, F. Mignard, O. Pace, and P. T. de Zeeuw, [Astron. Astrophys.](http://dx.doi.org/10.1051/0004-6361:20010085) 369, 339 (2001).