

Chiral Magnetic Effect and Anomalous Hall Effect in Antiferromagnetic Insulators with Spin-Orbit Coupling

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We search for dynamical magnetoelectric phenomena in three-dimensional correlated systems with spin-orbit coupling. We focus on the antiferromagnetic insulator phases where the dynamical axion field is realized by the fluctuation of the antiferromagnetic order parameter. It is shown that the dynamical chiral magnetic effect, an alternating current generation by magnetic fields, emerges due to such time dependences of the order parameter as antiferromagnetic resonance. It is also shown that the anomalous Hall effect arises due to such spatial variations of the order parameter as antiferromagnetic domain walls. Our study indicates that spin excitations in antiferromagnetic insulators with spin-orbit coupling can result in nontrivial charge responses. Moreover, observing the chiral magnetic effect and anomalous Hall effect in our system is equivalent to detecting the dynamical axion field in condensed matter.

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Introduction.—Antiferromagnets have attracted much attention from the viewpoints of both purely scientific and applied research. Theoretically, it is known that antiferromagnetic (AF) phases are often favored in systems with strong on-site repulsive interactions. In the vicinity of AF phases, the emergence of exotic phases and phenomena such as high-temperature superconductivity and spin liquids is well acknowledged [1–3]. On the other hand, antiferromagnets have recently been studied intensively in the field of spintronics [4–7], as a possible new class of materials that are alternatives to ferromagnets. These studies suggest that the staggered magnetization can play essential roles, although antiferromagnets had not been considered suitable for practical use due to the lack of net magnetization, unlike ferromagnets.

Recent intensive and extensive studies have revealed the importance of spin-orbit coupling (SOC) in condensed matter. Especially, the discovery that strong SOC is essential to realize topologically nontrivial phases opened a new direction in modern physics [8–10]. Since topological invariants are determined from electronic band structures, studies of topological phases started as a single-particle problem. Subsequently, many-body effects in topological phases and spin-orbit coupled systems have become an attractive subject [11,12], and the emergence of novel phases such as the topological Mott insulator [13] (or more generally the fractionalized topological insulators [14]) and the Weyl semimetal [15] has been predicted. As for novel phenomena in spin-orbit coupled and correlated systems, for example, the axionic polariton, a total reflection phenomenon of light, has been suggested [16].

In this Letter, we study electromagnetic responses of antiferromagnetic insulator (AFI) phases in three-dimensional (3D) correlated systems with SOC. We explore dynamical magnetoelectric phenomena where the

staggered magnetization plays essential roles. We show that, in the presence of SOC, spin excitations in AFIs can result in nontrivial charge responses, as a consequence of the realization of the dynamical axion field. First, we show the emergence of the chiral magnetic effect (CME), an electric current generation by magnetic fields [17], in the AFI phase. The CME was originally proposed in gapless Dirac fermion systems [17], and its possibility has been discussed in Weyl semimetals [18–23]. In contrast to preceding works, we propose the dynamical realization of the CME in gapped systems. We also show the occurrence of the anomalous Hall effect (AHE) in the AFI phase. It is known that the AHE occurs usually in ferromagnetic metals [24], while the AHE arising from nontrivial spin textures has been studied in frustrated or noncollinear antiferromagnets [25,26]. We propose that spatial variations of the staggered magnetization lead to the AHE.

Realization of the dynamical axion field and its consequences.—Let us consider 3D electron systems having both on-site interactions and SOC, such as 5d transition metal oxides [12,13,27]. We focus on systems that become magnetically ordered Mott insulators when on-site interactions are strong, but which are topological band insulators when on-site interactions are weak. Once magnetic order is formed, the mean-field approximation of the interaction term can capture the essential physics of the system. In this work, we particularly consider AFIs whose mean-field lattice Hamiltonian is given by $\mathcal{H}(\mathbf{k}) = \epsilon_0(\mathbf{k})\mathbf{I} + \sum_{\mu=1}^5 R_{\mu}(\mathbf{k})\alpha_{\mu}$. Here, $\mathbf{k} = (k_1, k_2, k_3)$ is a wave vector in the Brillouin zone, \mathbf{I} is the 4×4 identity matrix, and the 4×4 matrices α_{μ} satisfy the Clifford algebra $\{\alpha_{\mu}, \alpha_{\nu}\} = 2\delta_{\mu\nu}$ with $\alpha_5 = \alpha_1\alpha_2\alpha_3\alpha_4$. The Hamiltonian of this form can be realized, for example, in the AFI phases of the Bi_2Se_3 family doped with magnetic impurities such as Fe [16] and transition metal oxides with the corundum structure such as

α -Fe₂O₃ [28]. In this case, we can derive 3D massive Dirac Hamiltonians of the form

$$\mathcal{H}_{\text{eff}}(\mathbf{q}) = q_1\alpha_1 + q_2\alpha_2 + q_3\alpha_3 + M_0\alpha_4 + M_{5f}\alpha_5 \quad (1)$$

around some momentum points X_f , where $\mathbf{q} = \mathbf{k} - X_f$. The subscript f indicates the valley degrees of freedom. The kinetic term $\sum_{\mu=1}^3 q_{\mu}\alpha_{\mu}$ is spin dependent as a consequence of SOC. $M_0\alpha_4$ is a mass term with time-reversal and parity (spatial inversion) symmetries induced by SOC, and $M_{5f}\alpha_5$ is a mass term with broken time-reversal and parity symmetries induced by the mean-field AF order parameter. We require that the system be a topological insulator when $M_0 > 0$ and $M_{5f} = 0$.

In what follows, we consider the consequences arising from the existence of the $M_{5f}\alpha_5$ mass term. The effective action of the system in the presence of an external electromagnetic potential A_{μ} is written as

$$S_{\text{eff}} = \int dt d^3r \sum_f \bar{\psi}_f(\mathbf{r}, t) [i\gamma^{\mu} D_{\mu} - M'_f e^{i\theta_f \gamma^5}] \psi_f(\mathbf{r}, t), \quad (2)$$

where t is the real time, $\psi_f(\mathbf{r}, t)$ is a four-component spinor, $\bar{\psi}_f = \psi_f^{\dagger} \gamma^0$, $D_{\mu} = \partial_{\mu} + ieA_{\mu}$, $M'_f = \sqrt{(M_0)^2 + (M_{5f})^2}$, $\cos \theta_f = M_0/M'_f$, and $\sin \theta_f = -M_{5f}/M'_f$, and we have used the fact that $\alpha_4 = \gamma^0$, $\alpha_5 = -i\gamma^0\gamma^5$, and $\alpha_j = \gamma^0\gamma^j$ ($j = 1, 2, 3$). By applying Fujikawa's method [29] to the action (2), the θ term is obtained as [30]

$$S_{\theta} = \int dt d^3r \frac{e^2}{2\pi h} \theta \mathbf{E} \cdot \mathbf{B}, \quad (3)$$

where $\theta = (\pi/2)[1 + \text{sgn}(M_0)] - \sum_f \tan^{-1}(M_{5f}/M_0)$, and \mathbf{E} (\mathbf{B}) is an external electric (magnetic) field. From this action, we obtain the magnetoelectric responses expressed by $\mathbf{P} = (e^2/2\pi h)\theta\mathbf{B}$ and $\mathbf{M} = (e^2/2\pi h)\theta\mathbf{E}$ with \mathbf{P} the electric polarization and \mathbf{M} the magnetization. In 3D time-reversal invariant topological (normal) insulators, $\theta = \pi$ ($\theta = 0$) [36]. However, the value of θ can be arbitrary when the time-reversal and parity symmetries of the system are broken [37–39]. Furthermore, when the value of θ depends on space and time, it can be said that the dynamical axion field is realized in condensed matter [16]. Some consequences of the realization have been studied so far [16,40].

Notice that, when the dynamical axion field is realized, the θ term can be rewritten in the Chern-Simons form as

$$S_{\theta} = - \int dt d^3r \frac{e^2}{4\pi h} \epsilon^{\mu\nu\rho\lambda} [\partial_{\mu}\theta(\mathbf{r}, t)] A_{\nu} \partial_{\rho} A_{\lambda}. \quad (4)$$

Then the induced four-current density j^{ν} can be obtained from the variation of the above action with respect to the four-potential A_{ν} : $j^{\nu} = \delta S_{\theta} / \delta A_{\nu} = -(e^2/2\pi h) [\partial_{\mu}\theta(\mathbf{r}, t)] \times \epsilon^{\mu\nu\rho\lambda} \partial_{\rho} A_{\lambda}$. The induced current density is given by [41,42]

$$\mathbf{j}(\mathbf{r}, t) = \frac{e^2}{2\pi h} [\dot{\theta}(\mathbf{r}, t)\mathbf{B} + \nabla\theta(\mathbf{r}, t) \times \mathbf{E}], \quad (5)$$

where $\dot{\theta} = \partial\theta(\mathbf{r}, t)/\partial t$. The magnetic-field-induced term is the CME [17]. The electric-field-induced term is the AHE, since it is perpendicular to the electric field. The induced current of the form (5) has been also studied in Weyl semimetals [18–21], where the chemical potential difference between the band touching points and the separation of the points in momentum space are required for the CME and AHE, respectively. However, the existence of the CME in Weyl semimetals is still being discussed theoretically [18–23]. Note that the situation in this Letter is completely different, since the system we consider is gapped; i.e., the above conditions required in the case of Weyl semimetals are not needed in our case.

Theoretical model.—To study the induced current (5) more concretely, let us consider a 3D lattice model with SOC and electron correlations. The model we adopt is the Fu-Kane-Mele-Hubbard model on a diamond lattice at half filling, whose Hamiltonian is given by [43–45]

$$H = \sum_{\langle i,j \rangle, \sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + i \frac{4\lambda}{a^2} \sum_{\langle\langle i,j \rangle\rangle} c_{i\sigma}^{\dagger} \boldsymbol{\sigma} \cdot (\mathbf{d}_{ij}^1 \times \mathbf{d}_{ij}^2) c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (6)$$

where $c_{i\sigma}^{\dagger}$ is an electron creation operator at a site i with spin σ (\uparrow, \downarrow), $n_{i\sigma} = c_{i\sigma}^{\dagger} c_{i\sigma}$, and a is the lattice constant of the fcc lattice. The first through third terms represent the nearest-neighbor hopping, the next-nearest-neighbor SOC, and the on-site electron-electron interaction, respectively. \mathbf{d}_{ij}^1 and \mathbf{d}_{ij}^2 are the two vectors that connect two sites i and j on the same sublattice. Namely, they are given by two of the four nearest-neighbor bond vectors. $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices for the spin degrees of freedom. We introduce a lattice distortion such that $t_{ij} = t + \delta t_1$ for the [111] direction and $t_{ij} = t$ for the other three directions, which induces a band gap of $2|M_0|$ ($M_0 \equiv \delta t_1$) in the noninteracting spectrum.

We perform the mean-field approximation to the interaction term as $H_U = U \sum_i n_{i\uparrow} n_{i\downarrow} \approx U \sum_i \{ \langle n_{i\downarrow} \rangle n_{i\uparrow} + \langle n_{i\uparrow} \rangle n_{i\downarrow} - \langle n_{i\uparrow} \rangle \langle n_{i\downarrow} \rangle - \langle c_{i\uparrow}^{\dagger} c_{i\downarrow} \rangle c_{i\downarrow}^{\dagger} c_{i\uparrow} - \langle c_{i\downarrow}^{\dagger} c_{i\uparrow} \rangle c_{i\uparrow}^{\dagger} c_{i\downarrow} + \langle c_{i\uparrow}^{\dagger} c_{i\downarrow} \rangle \langle c_{i\downarrow}^{\dagger} c_{i\uparrow} \rangle \}$. SOC breaks the spin SU(2) symmetry and the orientations of the spins are coupled to the lattice structure. Hence, we should parametrize the AF ordering between the two sublattices A and B in terms of the spherical coordinate (n, θ, φ) :

$$\langle \mathbf{S}_{i'A} \rangle = -\langle \mathbf{S}_{i'B} \rangle = (n \sin \theta \cos \varphi, n \sin \theta \sin \varphi, n \cos \theta) \equiv n_1 \mathbf{e}_x + n_2 \mathbf{e}_y + n_3 \mathbf{e}_z (\equiv \mathbf{n}), \quad (7)$$

where $\langle \mathbf{S}_{i'\mu} \rangle = \frac{1}{2} \langle c_{i'\mu\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} c_{i'\mu\beta} \rangle$ ($\mu = A, B$) with i' denoting the i' 'th unit cell. In the following we consider the ground state given by $(n_0, \theta_0, \varphi_0)$. The low-energy effective Hamiltonian of the AFI phase is written in the form (1): $H_{\text{eff}} = \sum_{\mathbf{q}} \sum_{f=1,2,3} \psi_{f\mathbf{q}}^{\dagger} \mathcal{H}_f(\mathbf{q}) \psi_{f\mathbf{q}}$, where $\psi_{f\mathbf{q}}$ is a four-component

spinor [45]. Therefore, the value of θ in the AFI phase of the Fu-Kane-Mele-Hubbard model is given by [45]

$$\theta = \frac{\pi}{2} [1 + \text{sgn}(M_0)] - \sum_{f=1,2,3} \tan^{-1}(Un_f/M_0). \quad (8)$$

From this equation, we see that the dynamical axion field is realized by the fluctuation of the AF order parameter n_f , i.e., by the spin excitations [16,28].

Dynamical chiral magnetic effect.—First, we focus on the magnetic-field-induced term in Eq. (5), i.e., the CME in the AFI phase:

$$\mathbf{j}_{\text{CME}}(\mathbf{r}, t) = -\frac{e^2}{2\pi h} \sum_f \frac{UM_0}{M_0^2 + (Un_f)^2} \dot{n}_f(\mathbf{r}, t) \mathbf{B}. \quad (9)$$

Let us consider a case where a microwave (i.e., ac magnetic field) is irradiated and a static magnetic field $\mathbf{B} = B\mathbf{e}_{n_0}$ is applied along the easy axis of the AF order. Here, $\mathbf{e}_{n_0} = \mathbf{n}_0/|\mathbf{n}_0|$ is the unit vector parallel to the easy axis. The dynamics of the sublattice magnetizations $\langle \mathbf{S}_{iA} \rangle = \mathbf{m}_A$ and $\langle \mathbf{S}_{iB} \rangle = \mathbf{m}_B$ can be described by [46,47]

$$\begin{aligned} \dot{\mathbf{m}}_A &= \mathbf{m}_A \times \{-\omega_J \mathbf{m}_B + [g\mu_B \mathbf{B} + \omega_A(\mathbf{m}_A \cdot \mathbf{e}_{n_0})] \mathbf{e}_{n_0}\}, \\ \dot{\mathbf{m}}_B &= \mathbf{m}_B \times \{-\omega_J \mathbf{m}_A + [g\mu_B \mathbf{B} + \omega_A(\mathbf{m}_B \cdot \mathbf{e}_{n_0})] \mathbf{e}_{n_0}\}, \end{aligned} \quad (10)$$

where ω_J and ω_A are the exchange field and anisotropy field, respectively. We write \mathbf{m}_A and \mathbf{m}_B as $\mathbf{m}_A = n_0 \mathbf{e}_{n_0} + \delta \mathbf{m}_{A\perp} e^{i\omega t}$ and $\mathbf{m}_B = -n_0 \mathbf{e}_{n_0} + \delta \mathbf{m}_{B\perp} e^{i\omega t}$. Up to linear order in $\delta \mathbf{m}_{A(B)\perp}$ (i.e., $|\delta \mathbf{m}_{A(B)\perp}| \ll 1$), we obtain the resonance frequencies as $\omega = \omega_{\pm} = g\mu_B B \pm \sqrt{(2\omega_J + \omega_A)\omega_A}$. In the resonance state, where all the spins are precessing around the easy axis with the same frequency ω_+ (or ω_-), the AF order parameter is described as

$$\mathbf{n}_{\pm}(t) \equiv [\mathbf{m}_A(t) - \mathbf{m}_B(t)]/2 \approx n_0 \mathbf{e}_{n_0} + \delta \mathbf{n}_{\pm} e^{i\omega_{\pm} t}. \quad (11)$$

Here, we have neglected the difference between the angles $\theta_A = \tan^{-1}(|\delta \mathbf{m}_{A\perp}|/n_0)$ and $\theta_B = \tan^{-1}(|\delta \mathbf{m}_{B\perp}|/n_0)$. The ratio θ_A/θ_B is obtained as $\theta_A/\theta_B \approx (1 + \sqrt{\omega_A/\omega_J})^2$ [46]. Typically, the ratio $\omega_A/\omega_J (\approx K/J)$, with K and J being the strength of the anisotropy and exchange coupling, respectively, is of the order of 10^{-2} to 10^{-3} [48]. Therefore, we see that $\mathbf{m}_A \approx -\mathbf{m}_B$ and thus $\dot{\mathbf{m}}_A \approx -\dot{\mathbf{m}}_B$. An illustration of the dynamics of \mathbf{m}_A and \mathbf{m}_B in the AF resonance state is shown in Fig. 1(a).

From the relation that $\mathbf{n} = n_1 \mathbf{e}_x + n_2 \mathbf{e}_y + n_3 \mathbf{e}_z$, we have $n_1 = n_0 \sin \theta_0 \cos \varphi_0 + \delta n \cos \omega t \cos \theta_0 \cos \varphi_0 - \delta n \sin \omega t \sin \varphi_0$, $n_2 = n_0 \sin \theta_0 \sin \varphi_0 + \delta n \cos \omega t \cos \theta_0 \sin \varphi_0 + \delta n \sin \omega t \cos \varphi_0$, and $n_3 = n_0 \cos \theta_0 - \delta n \cos \omega t \sin \theta_0$. Substituting these quantities into Eq. (9), we obtain the analytical expression for $\mathbf{j}_{\text{CME}}(t)$. Especially in the vicinity of the phase boundary where $Un_f/M_0 \ll 1$ [49], Eq. (9) is simplified as

$$\mathbf{j}_{\text{CME}}(t) = \frac{e^2}{2\pi h} \frac{UD_1}{M_0} \mathbf{B} \sum_{a=\pm} \omega_a \delta n_a \sin(\omega_a t + \alpha), \quad (12)$$

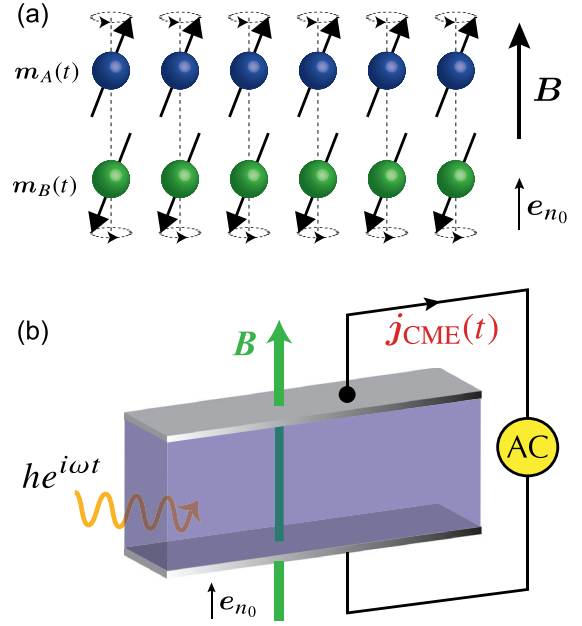


FIG. 1. (a) Schematic figure of the dynamics of \mathbf{m}_A and \mathbf{m}_B in the AF resonance state. (b) A possible experimental setup to observe the CME in our system. A static magnetic field \mathbf{B} is applied along the easy axis of the AF order, $\mathbf{e}_{n_0} = \mathbf{n}_0/|\mathbf{n}_0|$. When the external microwave (i.e., ac magnetic field) frequency ω is equal to the resonance frequencies ω_{\pm} , the induced alternating current $\mathbf{j}_{\text{CME}}(t)$ will be observed.

where $D_1 = \sqrt{p^2 + q^2}$ and $\tan \alpha = q/p$ with $p = (\cos \varphi_0 + \sin \varphi_0) \cos \theta_0 - \sin \theta_0$ and $q = \sin \varphi_0 - \cos \varphi_0$. Equation (12) means that an alternating current is induced by the AF resonance. The schematic figure of a possible experimental setup to observe the CME in our system is shown in Fig. 1(b). δn_{\pm} is a function of the external microwave frequency ω with a Lorentzian structure; i.e., $\delta n_{\pm}(\omega) \sim a/[(\omega - \omega_{\pm})^2 + a^2]$ with a being a constant. Therefore, two peaks will appear in the intensity $|\mathbf{j}_{\text{CME}}(\omega)|$.

Here, let us estimate the maximum value of the CME (12): $|\mathbf{j}_{\text{CME}}|_{\text{max}} = (e^2/2\pi h)(U|D_1|/|M_0|)B\omega_{\pm} \delta n_{\pm}$. Substituting the possible values $Un_0/|M_0| \sim 0.1$ [50], $|D_1| \sim 1$, $\delta n_{\pm}/n_0 \sim 0.02$, and $\omega_{\pm} \sim 500$ GHz at $B \sim 1$ T [51,52], we have $|\mathbf{j}_{\text{CME}}|_{\text{max}} \sim 1 \times 10^4$ A/m². This value is experimentally observable. It should be noted that the current is adiabatically induced in the gapped phase as in the case of the quantum Hall effect or the topological charge pumping effect [53]. Hence, unlike the conventional transport regime which causes Joule heat, there is no energy dissipation.

The CME was originally proposed in the ground states of massless Dirac fermion systems as a direct current generation by static magnetic fields due to the presence of the chemical potential difference between band touching points [17]. If such a static CME exists in realistic materials, there will be substantial potentials for its applications, since the current is dissipationless. However, the existence of the CME remains a theoretically controversial subject in Weyl

semimetals [18–23]. As discussed in Ref. [19], the possibility of the static CME would be ruled out in crystalline solids (i.e., lattice systems), regardless of the presence or absence of energy gaps. In contrast to preceding works, our study proposes that the CME occurs dynamically in insulating systems, which requires time dependences of the AF order parameter n_f caused by external forces such as the AF resonance state.

Anomalous Hall effect.—Next, we focus on the electric-field-induced term in Eq. (5), i.e., the AHE in the AFI phase:

$$\mathbf{j}_{\text{AHE}}(\mathbf{r}, t) = -\frac{e^2}{2\pi h} \sum_f \frac{UM_0}{M_0^2 + (Un_f)^2} \nabla n_f(\mathbf{r}, t) \times \mathbf{E}. \quad (13)$$

In order to obtain a concrete expression for this current, let us move on to a new Cartesian coordinate (X, Y, Z) . We consider a one-dimensional AF texture of length L along the Z direction, an orientational domain wall [54,55]. As shown in Fig. 2(a), the AF order parameter $\mathbf{n}(\mathbf{r}) = [\mathbf{m}_A(\mathbf{r}) - \mathbf{m}_B(\mathbf{r})]/2$ at the two edges has a relative angle δ . Namely, we have $\theta(Z=0) = \theta_0$ and $\theta(Z=L) = \theta_0 + \delta$ in the original spherical coordinate. A static electric field \mathbf{E} is applied perpendicular to the AF order as $\mathbf{E} = E_Y \mathbf{e}_Y$. For simplicity, we assume that the system lies near the phase boundary where $Un_f/M_0 \ll 1$ [49]. Noting that only the X component j_{AHE}^X survives, we see that Eq. (13) is simplified to be $j_{\text{AHE}}^X(Z) = (e^2/2\pi h)(U/M_0)E_Y \sum_f \partial n_f(Z)/\partial Z$. The total current in the X direction is given by

$$J_{\text{AHE}}^X = \int_0^L dZ j_{\text{AHE}}^X(Z) = \frac{e^2}{2\pi h} \frac{UD_2}{M_0} E_Y. \quad (14)$$

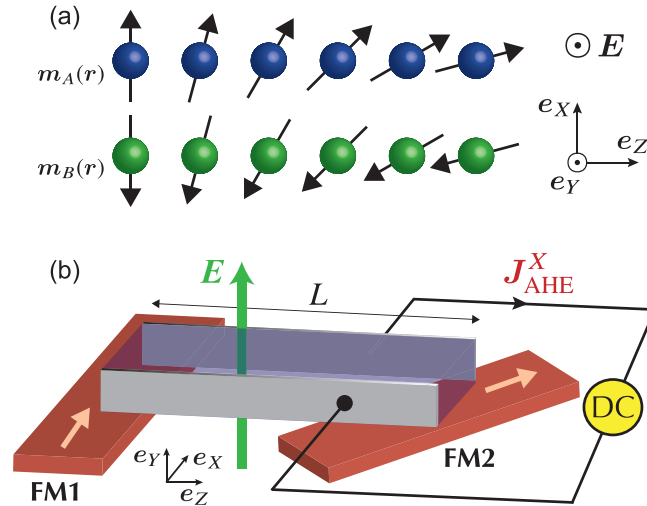


FIG. 2. (a) Schematic figure of a one-dimensional AF texture, an orientational domain wall of length L . The AF order parameter $\mathbf{n}(\mathbf{r}) = [\mathbf{m}_A(\mathbf{r}) - \mathbf{m}_B(\mathbf{r})]/2$ at the two edges has a relative angle δ . (b) A possible experimental setup to observe the AHE in our system. A static electric field \mathbf{E} is applied perpendicular to the AF order. There is a relative angle δ in the magnetization directions of ferromagnet 1 (FM1) and ferromagnet 2 (FM2).

where $D_2 = \sum_f \int_{\theta=\theta_0}^{\theta=\theta_0+\delta} dn_f = \sum_f [n_f(\theta_0 + \delta) - n_f(\theta_0)] = n_0 \{\sqrt{2} \sin[\varphi_0 + (\pi/4)] [\sin(\theta_0 + \delta) - \sin\theta_0] + \cos(\theta_0 + \delta) - \cos\theta_0\}$. The Hall conductivity is estimated as $\sigma_{XY} = (e^2/2\pi h)(U|D_2|/|M_0|) \sim 1 \times 10^{-2} e^2/h$, since $Un_0/|M_0| \sim 0.1$ [50] and $|D_2|/n_0 \sim 1$. The schematic figure of a possible experimental setup to observe the AHE in our system is shown in Fig. 2(b). Two ferromagnets with a relative angle δ in the magnetization directions are attached to the AFI [55]. In experiments, the δ dependence of the Hall conductivity will be direct evidence for the observation of the axion field. Note that, in contrast to preceding works on the AHE in antiferromagnets [25,26], the AHE studied here does not occur in uniform ground states. Namely, spatial variations of the AF order parameter n_f need to be realized by external forces.

Discussions and summary.—Let us discuss briefly the realization of our predictions in realistic correlated systems with SOC. It has been suggested that the dynamical axion field can be realized by spin excitations in the AFI phases of the Bi_2Se_3 family doped with magnetic impurities [16] and transition metal oxides with the corundum structure such as $\alpha\text{-Fe}_2\text{O}_3$ [28]. In the same manner as above, we can derive similar expressions for the CME and AHE in these systems. What about the possibility in other systems? First of all, the time-reversal and inversion symmetries of the system must be broken to induce the deviation of θ from 0 or π . Theoretically, the value of θ can be calculated numerically in any insulating systems [37–39]. The point is that the emergence of the CME and AHE depends on whether θ is a function of physical quantities such as the AF order parameter, as in our case. If θ is a function of a physical quantity, then the fluctuation of the physical quantity realizes the dynamical axion field. It should be noted that, even if the value of θ is zero in the ground states, the realization of dynamical axion fields is possible.

In summary, we have studied theoretically 3D AFIs with SOC, focusing on a role of the staggered magnetization. We have revealed that, in the presence of SOC, spin excitations in AFIs can result in nontrivial charge responses. It is shown that the dynamical CME, an alternating current generation by magnetic fields, emerges due to the time dependences of the AF order parameter. It is also shown that the AHE arises due to the spatial variations of the order parameter. These two phenomena are the consequences of the realization of the dynamical axion field in the AFI phase. The magnetic-field-induced and electric-field-induced currents in this study are understood as a polarization current in the bulk and a magnetization current in the bulk, respectively, which can flow in insulators. Observing these phenomena is equivalent to detecting the dynamical axion field in condensed matter. In other words, we propose a new way to detect the dynamical axion field.

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