Photonic Circuits with Time Delays and Quantum Feedback

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We study the dynamics of photonic quantum circuits consisting of nodes coupled by quantum channels. We are interested in the regime where the time delay in communication between the nodes is significant. This includes the problem of quantum feedback, where a quantum signal is fed back on a system with a time delay. We develop a matrix product state approach to solve the quantum stochastic Schrödinger equation with time delays, which accounts in an efficient way for the entanglement of nodes with the stream of emitted photons in the waveguide, and thus the non-Markovian character of the dynamics. We illustrate this approach with two paradigmatic quantum optical examples: two coherently driven distant atoms coupled to a photonic waveguide with a time delay, and a driven atom coupled to its own output field with a time delay as an instance of a quantum feedback problem.

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Introduction.—Wiring up increasingly complex quantum devices from basic modules is central in the effort to build large scale quantum circuits [1]. Quantum optical systems provide a natural framework to implement such a modular approach as a photonic quantum circuit [2,3]. Here, left- and right-propagating modes in optical fibers or waveguides provide the channels for communication between the nodes, and represent input and output ports to drive and observe the circuit. Such networks can involve quantum communication between "local" nodes, or in a distributed network between "distant" nodes, where time delays can be important. Recent advances in building small scale quantum processors with atoms and ions [4], and the development of atom-photon interfaces in circuit QED [5], or in coupling atoms to photonic nanostructures [6–8], have demonstrated—at least on a conceptual level—the basic building blocks for such a scalable photonic network [9,10].

On the theory side this raises the question of formulating a quantum theory of photonic quantum networks. Such a theory must account for the quantum many-body dynamics induced by multiple photon exchanges between the nodes, and relating the input and output quantum signals on the level of quantum states. Theoretical quantum optics has provided tools for modeling Markovian quantum networks, i.e., when time delays can be ignored [12,13]. It is the purpose of the present work to address non-Markovian aspects of the dynamics introduced by these time delays. This refers to the retarded interaction between the nodes of the network involving the exchange of (possibly many) photons, and also addresses the problem of quantum feedback [14], where the quantum signal emitted from a system is fed back with a time delay [16]. Our approach is based on solving the quantum stochastic Schrödinger equation (QSSE) [13] with time delays based on (continuous) matrix product states (MPSs) [20–24], as developed originally in a condensed matter context [25–31]. This technique allows for an efficient description of entanglement, which scales with finite time delays between the nodes of the network and the stream of photons propagating in the quantum channels, including the quantum fields and relevant observables at the output of the photonic quantum network.

Quantum optical model.—Our approach is best illustrated for the paradigmatic model [32] consisting of two distant nodes $n = 1, 2$ connected to an infinite waveguide, representing the input and output ports of our system [cf. Fig. 1(a)]. The nodes are located at positions $x_1 < x_2$, and there is a time delay $\tau = (x_2 - x_1)/v \equiv d/v$ associated with photon exchange with v the velocity of light. Our treatment considers photons in a bandwidth $\mathcal{B}$ around some mean optical frequency $\bar{\omega}$. To describe the dynamics, we

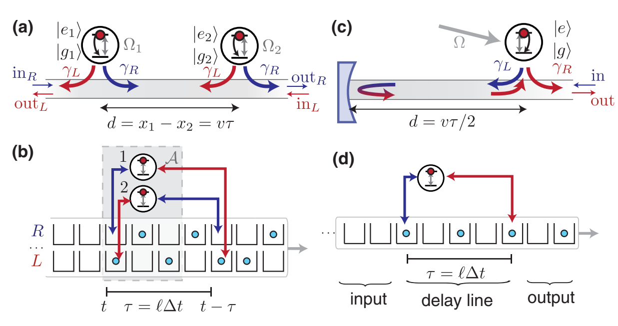

FIG. 1. Basic building blocks of a photonic quantum circuit. (a) Two distant driven atoms coupled to a one-dimensional waveguide (quantum channel). While (a) is a spatial representation of the circuit, (b) is the corresponding time interpretation according to a (discretized) QSSE. The nodes interact sequentially with photon modes defined for time bins according to the stroboscopic map (3) (see text). The time delay $\tau > 0$ corresponds to a nonlocal interaction in time. (c) Delayed quantum feedback, realized by a driven atom in front of a mirror, and (d) the time bin interpretation of the QSSE.

follow the familiar quantum optical formulation [13,33] and write a QSSE $i\hbar(d/dt)|\Psi\rangle = H(t)|\Psi\rangle$ for the total system of the nodes and waveguide. The total Hamiltonian is denoted by $H(t) = H_{\rm sys} + H_{\rm int}(t)$ with $H_{\rm sys} = \sum_n H_{\rm sys}^{(n)}$ the sum of the Hamiltonian of the nodes. The interaction term is given by

$$H_{\rm int}(t) = i\hbar((\sqrt{\gamma_R}b_R^\dagger(t) + \sqrt{\gamma_L}b_L^\dagger(t-\tau)e^{i\phi})c_1 - \text{H.c.}) + i\hbar((\sqrt{\gamma_R}b_R^\dagger(t-\tau)e^{i\phi} + \sqrt{\gamma_L}b_L^\dagger(t))c_2 - \text{H.c.}), \tag{1}$$

which describes the emission and absorption of photons by the nodes $n = 1, 2$ into the left- and right-propagating modes $i = L, R$ of the waveguide within a rotating wave approximation. Here, the operators $b_R(t)$ and $b_L(t)$ are defined by

$$b_i(t) = \frac{1}{\sqrt{2\pi}}\int_{\mathcal{B}} d\omega b_i(\omega)e^{-i(\omega-\bar{\omega})t} \quad (i = L, R)$$

with $b_i(\omega)$ $[b_i^\dagger(\omega)]$ the destruction [creation] operators of photons of frequency ω, satisfying $[b_i(\omega), b_{i'}^\dagger(\omega')] = \delta_{i,i'}\delta(\omega-\omega')$. In quantum optics they have the meaning of quantum noise operators with the white noise bosonic commutation relations $[b_i(t), b_{i'}^\dagger(t')] = \delta_{i,i'}\delta(t-t')$, which lead to the interpretation as a QSSE. The photon propagation phase is denoted by $\phi = -\bar{\omega}\tau$. The operators $c_{1/2}$ are the transition operators for the nodes in the emission of a photon. The coupling to the left- and right-propagating modes is given by the decay rates γ_L and γ_R into the radiation modes of the waveguide, respectively ($\gamma \equiv \gamma_R + \gamma_L$). We note that for a chiral coupling, as naturally realized in nanophotonics [6–8], we have $\gamma_R \neq \gamma_L$. While the assumptions behind the derivation of the QSSE are still Markovian in nature [13,34], it is the time delays, reflecting the retardation between the absorption and emission events, that introduce a non-Markovian element into the dynamics [see Fig. 1(b)].

The simplest physical realization of two nodes ($n = 1, 2$) is given by coherently driven two level systems with ground and excited states $|g_n\rangle$, $|e_n\rangle$ [6]. The corresponding system Hamiltonian is in a rotating frame $H_{\rm sys}^{(n)} = -\hbar\Delta_n|e_n\rangle\langle e_n| - (\hbar/2)(\Omega_n|g_n\rangle\langle e_n| + \text{H.c.})$ with $\Delta_n = \omega_L - \omega_{eg}$ the detuning of the driving laser from atomic resonance, Ω_n the Rabi frequency, and $c_n = |g_n\rangle\langle e_n|$. We return to this example below in some detail.

In the Markovian limit $\tau \to 0^+$, the above QSSE can be interpreted according to Stratonovich quantum stochastic calculus [13,34]. There are well established techniques to convert this QSSE to Ito calculus, and eventually to a master equation for the dynamics of the reduced state of the nodes $\rho_{\rm sys}$ tracing over the waveguide as a quantum reservoir. For vacuum inputs we obtain

$$\frac{d}{dt}\rho_{\rm sys} = -\frac{i}{\hbar}[H_{\rm sys}, \rho_{\rm sys}] + \gamma(\mathcal{D}[c_1]\rho_{\rm sys} + \mathcal{D}[c_2]\rho_{\rm sys}) - (\gamma_L e^{i\phi}[c_1, \rho_{\rm sys}c_2^\dagger] + \gamma_R e^{i\phi}[c_2, \rho_{\rm sys}c_1^\dagger] - \text{H.c.}) \tag{2}$$

with $\mathcal{D}[c]\rho \equiv c\rho c^\dagger - \frac{1}{2}\{c^\dagger c, \rho\}$ [35,36]. We note that Eq. (2) contains instantaneous dipole-dipole interactions, and collective atomic decay terms related to the 1D character of the reservoir. For the case of symmetric decay, $\gamma_L = \gamma_R$, Eq. (2) describes super- and subradiant decay processes [37]. In the limit of purely unidirectional couplings, $\gamma_L = 0$, where node 1 drives node 2, but there is no back scattering from 2 to 1, Eq. (2) reduces to the master equation for a cascaded quantum system, as first derived by Carmichael and Gardiner [35,36]. We note that for the cascaded case a finite time delay $\tau > 0$ can always be absorbed in a retarded time for node 2; i.e., the system dynamics can be described by a Markovian master equation [12,13]. This is not the case, however, when we allow for back scattering or two-way communication. Below we address this problem by solving the QSSE for $\tau > 0$, where a Markovian master equation of the type (2) does not exist.

To give a meaning to a QSSE with time delays and to prepare our MPS formulation to its solution we find it convenient to discretize time in small steps Δt, that is $t_k = k\Delta t$ with $k \in \mathbb{Z}$. We represent the time evolution as a dynamical map $|\Psi(t_{k+1})\rangle = U_k|\Psi(t_k)\rangle$. We choose a time step that is small compared to the time scale of the system evolution (including $\gamma_{L,R}\Delta t \ll 1$) but large compared to the inverse bandwidth $\mathcal{B}$ of the waveguide. Moreover, we conveniently choose Δt to be a unit fraction of the delay time $\tau = \ell\Delta t$. Thus, we have

$$|\Psi(t_{k+1})\rangle = U_k|\Psi(t_k)\rangle \equiv \exp\left(-\frac{i}{\hbar}H_{\rm sys}(t_k)\Delta t + O_{k,1} + O_{k,2}\right)|\Psi(t_k)\rangle \tag{3}$$

with

$$O_{k,1} = (\sqrt{\gamma_R}\Delta B_R^\dagger(t_k) + \sqrt{\gamma_L}\Delta B_L^\dagger(t_{k-\ell})e^{i\phi})c_1 - \text{H.c.},$$
$$O_{k,2} = (\sqrt{\gamma_R}\Delta B_R^\dagger(t_{k-\ell})e^{i\phi} + \sqrt{\gamma_L}\Delta B_L^\dagger(t_k))c_2 - \text{H.c.} \tag{4}$$

Here, we have defined quantum noise increments $\Delta B_i(t_k) = \int_{t_k}^{t_{k+1}} dt b_i(t)$. They obey (up to a normalization factor) the bosonic commutation relations $[\Delta B_i(t_k), \Delta B_{i'}^\dagger(t_{k'})] = \Delta t\delta_{i,i'}\delta_{k,k'}$ and can be interpreted as annihilation (creation) operators for photons in the time bin k. Since we have two channels (L, R) each time bin contains two such modes. The above equation states that in time step $t_k \to t_{k+1}$ the first node can emit a photon into two modes, the L mode of bin k and the R mode of bin $k-\ell$, and vice versa for the second node. Thus, we can visualize the time evolution as a conveyor belt of time bins representing the modes [cf. Fig. 1(c)]: each time step shifts this conveyor belt by one unit, and after ℓ steps the first (second) system interacts with the photons emitted by the second (first) one.

Matrix product state description.—In the following we employ a MPS representation of $|\Psi(t)\rangle$. In a condensed matter context time-dependent density matrix renormalization group techniques have been developed to integrate the many-particle Schrödinger equation in 1D systems, and ladder geometries, and a close relationship between MPSs and the output fields from photonic systems has been established [20–24]. Here, we build on these developments to integrate Eq. (3) efficiently and for long times, approaching the steady state.

We assume that the full state is initially ($t = 0$) completely uncorrelated, that is, $|\Psi(t=0)\rangle = |\psi_S\rangle \bigotimes_p |\phi_p\rangle$, where $|\psi_S\rangle$ denotes the initial state of the nodes (emitters) and $|\phi_p\rangle$ the state of the photons in time bin p. In particular this includes a waveguide initially in the vacuum state $|\phi_p\rangle = |\mathrm{vac}_p\rangle$ with $\Delta B_{L/R}(t_p)|\mathrm{vac}_p\rangle = 0$. The stroboscopic evolution until a time t_k gives a state $|\Psi(t_k)\rangle = |\phi_{\mathrm{in}}(t_k)\rangle \otimes |\psi(t_k)\rangle$, where $|\phi_{\mathrm{in}}(t_k)\rangle = \bigotimes_{p \geq k} |\phi_p\rangle$ is the remaining input state and

$$|\psi(t_k)\rangle = \sum_{i_S,\{i_p\}} \psi_{i_S,i_{k-1},i_{k-2},\cdots} |i_S, i_{k-1}, i_{k-2}\cdots\rangle \tag{5}$$

is the entangled state of the nodes and radiation field. Here, i_S and i_p label the basis states in the Hilbert space of the nodes and the time bin p, respectively:

$$|i_p\rangle \equiv |i_p^L, i_p^R\rangle = \frac{(\Delta B_L^\dagger)^{i_p^L}}{\sqrt{\Delta t^{i_p^L} i_q^L!}} \frac{(\Delta B_R^\dagger)^{i_p^R}}{\sqrt{\Delta t^{i_p^R} i_q^R!}} |\mathrm{vac}_p\rangle, \tag{6}$$

where $i_p^{L(R)} = 0, 1, 2, \ldots$ denote the number of photons in the $L(R)$ mode in time bin p. The modes $p \in (k-1, \ldots, k-\ell)$ represent the photonic state in the quantum circuit, while $p < k - \ell$ labels the modes of the output field [cf. Fig. 1(b)].

In a MPS form [26,28–30,38] we can write these amplitudes as

$$\psi_{i_S,i_{k-1},\cdots} = \mathrm{tr}\{A[S]^{i_S} A[k-1]^{i_{k-1}} A[k-2]^{i_{k-2}} \cdots\}, \tag{7}$$

where $A[p]^{i_p}$ are $D_p \times D_{p-1}$ matrices and D_p is the bond dimension for a bipartite cut between time bins p and $p+1$. We emphasize that the quantum state (5), and its MPS decomposition (7), refer to an entangled state of nodes and photons in time bins. This is in contrast to condensed matter systems where the many-body wave function refers to spatial correlations at a given time. The stroboscopic evolution of the full state from t_k to t_{k+1} via Eq. (4) involves an interaction of each node with two time-bin modes of the radiation fields, with time delays appearing as "long range interactions." While instantaneous (short range) interactions are standard to implement in the MPS formalism [28,29], time-delayed (long range) interactions can be handled by methods introduced in Refs. [39–41]. In each propagation step the system gets entangled with the time bin k such that the MPS (7) grows by one site ($A[k]$). The update of the MPS (7) in the kth time step according to the interaction of the system with the delayed feedback in time bin $k - \ell$ involves all matrices representing the field in the circuits, that is, the time bins in $[t_k - \tau, t_k]$, but does not involve the matrices for time bins $p < k - \ell$. For the technical details on updating the MPS in each time step we refer the reader to the Supplemental Material [33].

We now illustrate this method with two examples: (i) two coherently driven, distant atoms interacting via the waveguide as discussed above, and (ii) a single driven atom coupled to a waveguide terminated by a distant mirror as an illustration of a quantum feedback problem. In contrast to previous work [32,37,42–55], which includes transfer matrix and Wigner-Weisskopf type approaches applicable to a single or a few excitations propagating through the system, we are interested here in strongly driven systems with multiple photon exchange resulting in significant entanglement. The problem of delayed quantum feedback was addressed recently by Grimsmo [56] in the transient regime. This approach is based on a replication of the system Hilbert space after each delay cycle, which allows for an exact evolution for a few τ. The exponential increase of the replicated Hilbert space however inhibits a propagation to long times. Our approach is complementary to this as we will be able to follow the evolution of the circuit for long times, reaching the steady state.

Two driven distant atoms.—Figures 2(a) and 2(b) show results for the time evolution of two atoms driven through the waveguide from the left input port, which are separated by a distance corresponding to a delay $\gamma\tau = 5$, and a propagation phase $\phi = \pi/2$ [see Fig. 1(b)]. Figure 2(a) plots the atomic excitation probabilities and the mean photon number in the waveguide between the two atoms (delay line). Starting at time $t = 0$ the atoms are coupled to the waveguide. They will "not see each other" for times $0 \leq t < \tau$, and thus obey Rabi dynamics described by the single atom Bloch equations. For $t > \tau$ the atoms interact both with the coherent drive and also with the time-delayed nonclassical stream of photons emitted by the other atom. We assume a driving laser field from the left and thus the output field of atom 1 acquires the same phase as the laser, when traveling to atom 2. On the other hand the output field of atom 2 is out of phase with respect to the coherent drive by 2ϕ when it reaches atom 1. This causes destructive and constructive interference in the atomic populations for $t > \tau$ in Fig. 2(a).

In Fig. 2(b) we plot the time evolution of the entanglement of the time bins in terms of the entropy $S(\rho_{\mathcal{A}}) \equiv -\mathrm{Tr}\{\rho_{\mathcal{A}} \log_2 \rho_{\mathcal{A}}\}$ of the reduced state $\rho_{\mathcal{A}}(t) = \mathrm{Tr}_{\bar{\mathcal{A}}}\{|\Psi(t)\rangle\langle\Psi(t)|\}$. Here, $\mathcal{A}$ refers to the radiation field in time bins $[t - t_{\mathcal{A}}, t]$ [see Fig. 1(b)], while $\overline{\mathcal{A}}$ refers to the

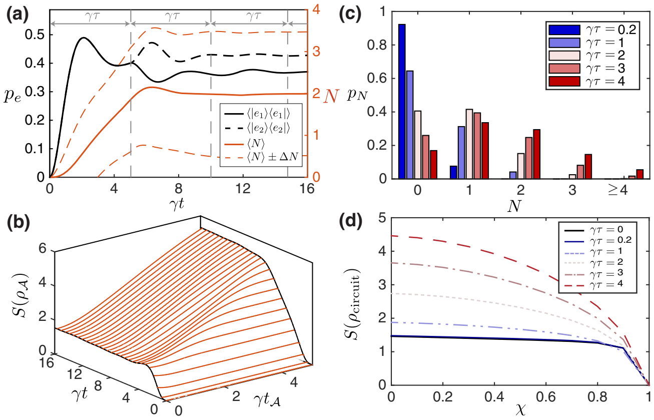

FIG. 2. Two driven two-level atoms coupled to a waveguide [cf. Fig. 1(a)]. (a) Excitation probabilities of atoms 1 and 2 (solid and dashed black lines, respectively) and the photon number in the waveguide between the atoms (red) as a function of time for $\gamma\tau = 5$ and $|\Omega| = 1.5\gamma$. Vertical dashed lines indicate multiples of the delay time. (b) Entanglement entropy $S(\rho_{\mathcal{A}})$ of the atoms and radiation field in the interval $[t - t_{\mathcal{A}}, t]$ as a function of time. The line at $t_{\mathcal{A}} = 0$ gives the entanglement of the atoms with the entire radiation field, while the line at $t_{\mathcal{A}} = \tau = 5/\gamma$ corresponds to the entanglement of the entire circuit with the output field. (c) Probabilities p_N for having N photons (on top of the coherent driving field) in the delay line in the steady state (calculated by time evolution to $t_{\max} = 200/\gamma$). (d) Entanglement entropy of the entire circuit with the output field in the steady state for asymmetric coupling $\gamma_{L/R} = \gamma(1 \pm \chi)/2$. Unless otherwise stated the parameters in (a)–(d) are $\gamma_L = \gamma_R \equiv \gamma/2$, $\gamma\tau = 5$, $\phi = \pi/2$, $\Omega_1 = \Omega_2 e^{i\phi} = 1.5\gamma$, $\Delta = 0$, $\gamma\Delta t = 0.1$, $D_{\max} = 256$.

field in the complement of $\mathcal{A}$. Note that for $t_{\mathcal{A}} = 0$ the state $\rho_{\mathcal{A}} \equiv \rho_{\text{sys}}$ corresponds to the reduced density operator of the atoms, and $S(\rho_{\text{sys}})$ quantifies the atom-photon entanglement, as the total state $|\Psi(t)\rangle$ is pure. On the other hand, for $t_{\mathcal{A}} = \tau$, the state $\rho_{\mathcal{A}} \equiv \rho_{\text{circuit}}$ includes also the radiation field in the delay line, and thus $S(\rho_{\text{circuit}})$ quantifies the entanglement of the circuit with the output field [57]. $S(\rho_{\text{circuit}})$ increases approximately linearly during the first round-trip time, but does not increase afterwards. The necessary bond dimension to represent the state, $D_{\max}$, scales thus exponentially with $\gamma\tau$, which limits the achievable time delays. However, for a fixed τ, the bond dimension does not increase with the total integration time, allowing us to reach the steady state. This can be understood by noting that each photon is emitted as a superposition state into the left- and right-moving channel, and contributes an entropy $S_1 = -(\gamma_L/\gamma)\log_2(\gamma_L/\gamma) - (\gamma_R/\gamma)\log_2(\gamma_R/\gamma)$ to the total entanglement of the circuit for a time τ after its emission, i.e., before the photon leaves the circuit [33]. $S(\rho_{\text{circuit}})$ thus scales linearly with the number of photons in the delay line [58]. In Fig. 2(c) we plot this photon number distribution in the steady state for increasing τ. Figure 2(d) shows the corresponding increase of $S(\rho_{\text{circuit}})$, and the role of chirality $\gamma_L \neq \gamma_R$. The entropy per photon S_1 is maximal for a bidirectional system

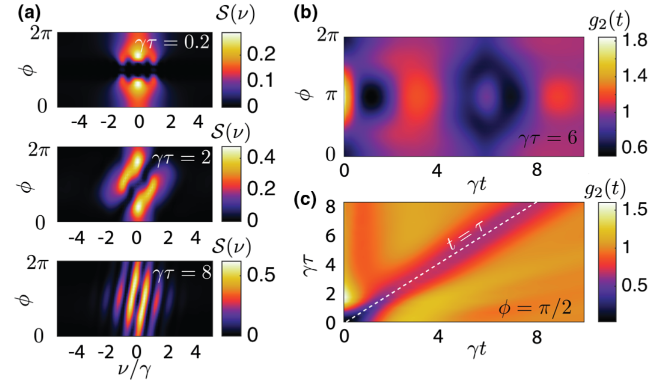

FIG. 3. Steady state properties of the output field for different delays $\gamma\tau$ and feedback phases ϕ ($\gamma_L = \gamma_R = \gamma/2$). (a) Incoherent part of the output field spectrum as a function of the feedback phase for $\gamma\tau = 0.2, 2, 4$ (from top to bottom). (b) Autocorrelation function of the output field for $\gamma\tau = 6$ as a function of ϕ and (c) for different delay times at $\phi = \pi/2$. The parameters are $\gamma\Delta t = 0.1$, $D_{\max} = 50$. Steady state obtained by evolution to $t_{\max} = 200/\gamma$, with $\Omega = \gamma$ and $\Delta = 0$.

($\gamma_L = \gamma_R$), and vanishes in the unidirectional case ($\gamma_R = 0$), such that in the cascaded limit $S(\rho_{\text{circuit}})$ becomes independent of τ. This connects our approach to the well known fact that time delays can be trivially eliminated in the cascaded limit [35,36]. The vanishing of $S(\rho_{\text{circuit}})$ in Fig. 2(d) for the unidirectional coupling $\gamma_R = 0$ indicates the existence of a dark (pure) quantum state as the steady state of the circuit. The formation as quantum dimers of two-level atoms has been discussed in the Markovian case [59,60], and these persist as dimer correlations shifted by the time delay between the two atoms even for $\tau > 0$.

Quantum feedback—atom in front of a mirror.—Finally, we illustrate our approach for the example of a driven atom in front of a mirror at distance d shown in Fig. 1(c) (see also Ref. [56]), and calculate the properties of the atomic steady state and the corresponding output field (Fig. 3). The quantum stochastic Hamiltonian is given by Ref. [33]

$$H_{\text{int}}(t) = i\hbar[(\sqrt{\gamma_L}b^\dagger(t) + \sqrt{\gamma_R}b^\dagger(t-\tau)e^{i\phi})|g\rangle\langle e| - \text{H.c.}]. \tag{8}$$

The parameters characterizing this setup are the delay time $\tau = 2d/v$ and the round-trip phase $\phi = \pi - \bar{\omega}\tau$. In the Markovian limit $\tau \to 0^+$, the system is described by the well known optical Bloch equation, with the effective decay rate $\gamma_{\text{eff}} = 2\gamma\cos^2(\phi/2)$ and effective detuning $\Delta_{\text{eff}} = \Delta - (\gamma/2)\sin(\phi)$ (for $\gamma_L = \gamma_R$) [61–64]. In this limit the output power spectrum in the steady state, $\mathcal{S}(\nu)$, shows a Mollow triplet [65] and the autocorrelaltion function $g_2(t)$ exhibits photon antibunching. With our methods we can go systematically beyond this limit and calculate these steady state quantities for long delays $\tau \gg \gamma^{-1}, \Omega^{-1}$ (Fig. 3). As depicted in Fig. 3(b) with increasing τ the incoherent part of the spectrum develops a series of peaks at $\nu = (\phi + 2\pi\mathbb{Z})/\tau$.

This reflects the coherence of photons that are emitted in a superposition of states corresponding to propagation towards and away from the mirror, resulting in correlations of time bins separated by τ. As shown in Fig. 3(c), $g_2(t)$ also reveals long time correlations $g_2(\tau) < 1$. This reduced probability of detecting two photons delayed by τ can be traced back to the antibunching of photons emitted towards and away from the mirror. Moreover, depending on the phase of the feedback ϕ, $g_2(t = 0)$ can change from the well known antibunching dip $[g_2(0) < 1]$ to a bunching peak $[g_2(0) > 1]$, where photons in the feedback line interfere with the emission of photons directly into the output port [Fig. 3(c)].

In summary, we have developed a MPS approach to describe the dynamics of photonic quantum networks with time delays, and quantum feedback. This provides a systematic framework for the simulation of nonlinear photonic quantum circuits and quantum optical devices with several input and output channels [33,66]. With increasing complexity of the circuit, and the associated scaling of computational resources (see Ref. [33]), a classical simulation will eventually be inefficient, or impossible, which is, of course, the original motivation for the development of quantum circuits and devices.

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