Cosmic Explosions, Life in the Universe, and the Cosmological Constant

Tsvi Piran,[1,*] Raul Jimenez,[2,3,†] Antonio J. Cuesta,[4,‡] Fergus Simpson,[4,§] and Licia Verde[2,5,∥]
[1]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[2]*ICREA & ICC, University of Barcelona, Marti i Franques 1, Barcelona 08028, Spain*
[3]*Institute for Applied Computational Science, Harvard University, Cambridge, Massachusetts 02138, USA*
[4]*ICC, University of Barcelona, Marti i Franques 1, Barcelona 08028, Spain*
[5]*Institute of Theoretical Astrophysics, University of Oslo, Oslo 0315, Norway*

(Received 10 August 2015; revised manuscript received 27 January 2016; published 23 February 2016)

Gamma-ray bursts (GRBs) are copious sources of gamma rays whose interaction with a planetary atmosphere can pose a threat to complex life. Using recent determinations of their rate and probability of causing massive extinction, we explore what types of universes are most likely to harbor advanced forms of life. We use cosmological N-body simulations to determine at what time and for what value of the cosmological constant (Λ) the chances of life being unaffected by cosmic explosions are maximized. Life survival to GRBs favors Lambda-dominated universes. Within a cold dark matter model with a cosmological constant, the likelihood of life survival to GRBs is governed by the value of Λ and the age of the Universe. We find that we seem to live in a favorable point in this parameter space that minimizes the exposure to cosmic explosions, yet maximizes the number of main sequence (hydrogen-burning) stars around which advanced life forms can exist.

DOI: 10.1103/PhysRevLett.116.081301

Why the value of the cosmological constant Λ is neither zero nor of the order of the Planck density (the Planck mass to the fourth power, $M_{\rm Pl}^4$) remains one of the deepest mysteries of nature. References [1,2] have argued that in order for observers (and, thus, cosmic structure) to exist, the value of Λ could not be larger than $10^{-120}M_{\rm Pl}^4$. This was the first indication that the value of Λ could not be arbitrary and that requiring the existence of observers bounded Λ from above. Subsequent works (e.g., Refs. [3–7]) have firmed up this argument. However, to this date, no argument has been given to provide a lower bound to Λ; in particular, it is not clear why Λ does not simply vanish (a lower bound to Λ based on the stability of atoms [8] applies only in the distant future for spatially flat universes). Interestingly, the necessity to avoid massive life extinction events by gamma-ray bursts (GRBs) can shed a new light on this issue.

GRBs are potentially catastrophic events for biological organisms. (In what follows, when we refer to life, we consider biological organisms which are sufficiently complex to act as observers. Our considerations will be for Earth-like planets where the UV protection provided by the atmosphere is due to an ozone layer.) In particular, copious flux of γ-ray photons with energies above 10–100 keV could destroy the ozone layer of a habitable Earth-like planet, exposing living organisms to damaging UV radiation and compromising its habitability. This has led to the suggestion [9–11] that galactic GRBs have been responsible for some mass extinction events on Earth. Yet the rate or energy of nearby GRBs were not sufficient to avoid the emergence of observers. However, such GRBs take place more frequently at the inner parts of the Milky Way and may cause a serious problem for development of life there [12]. On Earth and, in general, in the outskirts of large galaxies, the most luminous GRBs, the ones around the knee of the luminosity function (see Fig. 3 in Ref. [12]) pose the greatest threat for the development of complex organisms (a small fraction of mass extinction from short GRBs [13] will not change our results, as it would just add a baseline that equally penalizes all values of Λ), as they could cause catastrophic damage even if located in a sufficiently nearby satellite galaxy.

The rate of GRBs within a given galaxy depends on the metallicity: most GRBs take place where metallicity < 0.3 solar and the stellar mass of the host galaxy is above $5\times10^7 M_\odot$ [14,15]. Such low-metallicity environments are rare within the Milky Way and the ~50 kpc region around it. Small-mass, low-metallicity, Magellanic Cloud (SMC and LMC)-type galaxies are the typical host of GRBs and, thus, the most likely location for potentially damaging nearby GRBs. We explore next the rate of such catastrophic extragalactic events.

The observed global GRB rate is 10^9 GRB/Gpc^3/Gyr [16]. (We use here the rate of GRBs beamed towards a given observer. The overall rate is larger by a factor corresponding to the beaming. However, this is not relevant for this work.) GRB hosts have stellar masses between 5×10^7 and $10^{10}M_\odot$ [14,15]. Integrating the stellar mass function of Ref. [17], in this mass range we find a stellar density of $10^{16}M_\odot/{\rm Gpc}^3$, yielding a rate of 10^{-7} GRB$/M_\odot$/Gyr. This rate depends only on stellar physics and, thus, is independent of cosmology. Integrating (out to 200 kpc from the center) the dark matter profile from cold dark matter (CDM) simulations [18], the dark matter mass in satellites is 20% of the total halo mass (2×10^{12}) of the Milky Way. Since

$\sim$1% of this mass is in stars [19], we obtain a stellar mass of 4×10^9 in the satellites (this number is in excellent agreement with direct integration of the observed stellar mass in Milky Way satellites [20]). Thus, we expect 400 GRBs/Gyr in satellites. Using the observed GRB luminosity function [16] assuming an effective duration of 10 s, we expect $280(70\%) > 10^{52}$ erg, $72(18\%) > 10^{53.5}$ erg, $4\,(1\%) > 10^{54.5}$, and 0.5–1 (0.15%) with energy of $\sim 10^{55}$ erg. Reference [12] has shown that most likely there has been one GRB during the last Gyr with a fluence on Earth of 100 kJ/m^2; this fluence is the value found by Refs. [21,22] for massive life extinction to take place. This event is believed to have caused the Ordovician extinction [23], which wiped out 85% of all species present on Earth at the time. Following Refs. [21,22], we take a fluence of 100 kJ/m^2 to be the threshold: higher fluence would have catastrophic consequences for having observers. (The amount of ozone depletion and DNA damage scales slowly with fluence: they are reduced by factors of 2 and 2.5, respectively, by reducing fluence from 100 to 10 kJ/m^2 [24]). Then the equivalent damaging distance from the center of the Galaxy for a planet at 10 kpc from the center is 17 kpc for $10^{53.5}$ erg, 27 kpc for $10^{54.5}$ erg, and 50 kpc for 10^{55} erg. This implies, conservatively, that a region of 20 kpc (from the center of the host halo) should be devoid of GRB-hosting subhalos for harboring planets suitable to support observers. If there are no satellites in this region, then there will be no damaging GRBs (of fluence 100 kJ/m^2) with a rate much higher than that on Earth. (About 8% of the satellites are within a radius of 20 kpc for a $\Lambda = 0$ cosmology. Regardless of the cosmology, almost all satellites are within a radius of 50–100 kpc.) Since there is a nonzero probability of a $\geq 10^{55}$ erg GRB/Gyr in the satellites, which yields a lethal 100 kJ/m^2 fluence at a distance of 50 kpc, we also discuss this case.

The existence of many nearby satellites will have an additional effect. Many of these satellites infall into the main galaxy, bringing low-metallicity material and triggering further star formation that will increase GRB production. We do not consider this enhanced rate in this Letter, but it clearly makes development of intelligent life in cosmologies with numerous nearby satellites even more difficult than considered here.

Any inference of cosmological parameters ought to take into account the selection effects which have led us to observe the Universe from this particular vantage point [25]. Of particular interest is the value of Λ, which can govern the growth of regions around large galaxies devoid of LMC-type satellites. The accelerated expansion induced by a cosmological constant slows the growth of cosmic structures and increases the mean intergalaxy separation. This reduces the number of nearby satellites likely to host catastrophic GRBs. Below we quantify this effect.

Using N-body simulations, we search for halos with dark matter masses $10^{11.5} < M/M_\odot < 10^{12}$ that have no satellites of dark matter mass $> 2 \times 10^8 M_\odot$ within a radius of 20 (50) kpc (proper); we refer to these halos as isolated. In particular, we use the Millennium-II [26] publicly available dark-matter-only simulation [27] through the Millennium database portal created by the Virgo Consortium [28]. We search for these isolated halos over all 37 available snapshots from $z = 2$ to $z = 0$ to determine the redshift evolution. We determine the number of subhalos $N_{\rm SH}$ inside the search radius for each host halo (identified with a Friends-of-Friends ID number), which can be either empty ($N_{\rm SH} = 0$, isolated host halo) or not empty ($N_{\rm SH} > 0$, nonisolated host).

The Millenium-II simulation is performed for a flat ΛCDM cosmology with parameters (unless otherwise stated, cosmological parameters are at $z = 0$): $\Omega_m = 0.25$, $\Omega_b = 0.045$, $\Omega_\Lambda = 0.75$, $h = 0.73$, $\sigma_8 = 0.9$, $n_s = 1$. The size of the box is $L = 100 h^{-1}$ Mpc (comoving) on a side. The spatial resolution is $1 h^{-1}$ kpc, so the search radius for subhalos is not affected by resolution effects. The mass resolution of the simulation is $6.89 \times 10^6 h^{-1} M_\odot$, so, effectively, our threshold on the mass of the satellite halo is verified by *all* subhalos resolved in the simulation, regardless of their mass. The host halo is required to have 30 000 to 100 000 particles, which corresponds to the halo mass bin stated above.

No public N-body simulations with the required mass resolution exist for models other than the "vanilla" ΛCDM. Fortunately, numerical simulations of one cosmology can be remapped into a different one accurately both for the dark matter field and the corresponding halos [29,30]. Using this algorithm, we have remapped the results of the Millennium-II ΛCDM simulation to cosmologies with other values of Λ. Note that by keeping the geometry fixed ($\Omega_m + \Omega_\Lambda = 1$) and the early Universe quantities (such as the physical matter density $\Omega_m h^2$, the amplitude of primordial perturbations, and the baryon fraction) fixed, the current value of Ω_Λ specifies the cosmology.

It is challenging to keep track of changes of all other cosmological parameters, and it is clear that other changes could mimic some of the effects that we emphasize here; for example, a significant amount of massive neutrinos would also suppress the growth of structure. However, it has been shown [31] that even modest modifications to a range of cosmological parameters lead to adverse consequences for the abundance of life. As such, we focus on changing only the value of the cosmological constant, while keeping the conditions of the early Universe fixed.

Figure 1 shows the number of isolated halos per comoving 1000 Mpc^3 for four relevant flat cosmologies spanning the range $0 \leq \rho_\Lambda / M_{\rm Pl}^4 \leq 2.7 \times 10^{-123}$: (1) $\Omega_m = 0.15$, $\Omega_\Lambda = 0.85$, $\rho_\Lambda / M_{\rm Pl}^4 = 2.7 \times 10^{-123}$, (2) (fiducial ΛCDM) $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $\rho_\Lambda / M_{\rm Pl}^4 = 1.4 \times 10^{-123}$, (3) $\Omega_m = 0.5$, $\Omega_\Lambda = 0.5$, $\rho_\Lambda / M_{\rm Pl}^4 = 4.5 \times 10^{-124}$, and (4) (Einstein–de Sitter) $\Omega_m = 1$, $\Omega_\Lambda = 0$, $\rho_\Lambda = 0$. In the Λ-dominated models, only fairly recently, on a cosmological time scale, the number of GRB-protected halos

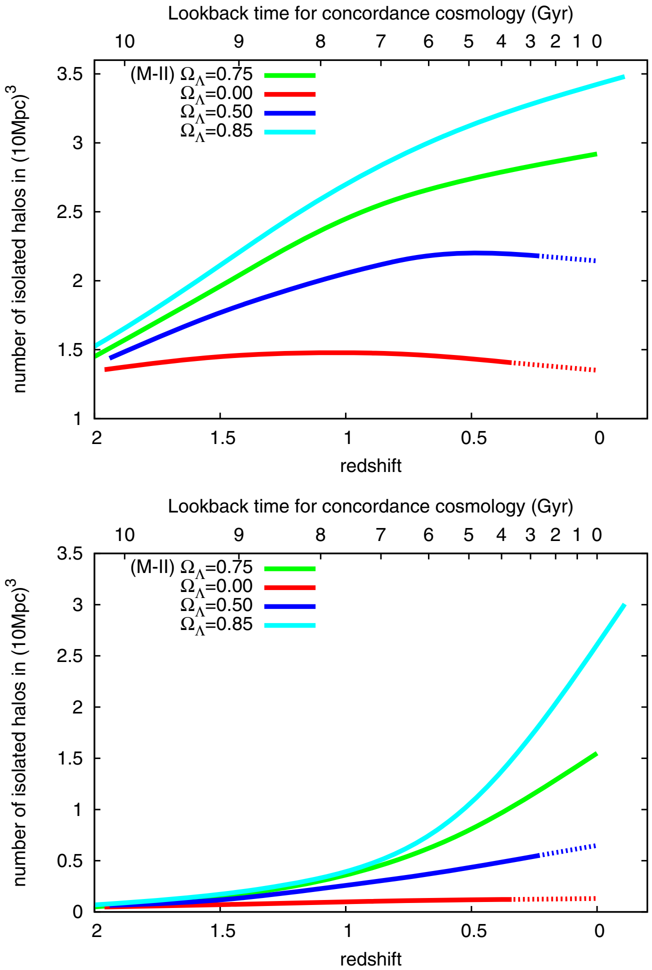

FIG. 1. Number of isolated halos as a function of redshift that are protected from the damaging effect on life of GRBs; we show four cosmologies and two isolated radii of satellites: 20 kpc (upper panel) and 50 kpc (lower panel). To guide the eye, dashed lines show extrapolations to $z = 0$ due to limitations in remapping the MILLENNIUM-II simulation, as this would have needed it to be run into the future. The number of life-protected regions is significantly larger in the Λ-dominated cosmologies. This is due to the effect of Λ at late times at clearing and smoothing virialized regions. Also remarkable is the fact that only below $z \approx 1$, the number of isolated halos grows significantly. This is about 7 Gyr ago, not too dissimilar from the age of Earth.

grew significantly. In fact, a large number (for a Milky Way (MW)-like halo in the concordance ΛCDM model, 1/3 of the halos fulfill the 50 kp isolation criterion, so our MW is not a very special halo) of isolated halos appear only below $z \approx 1.5$. This corresponds to a lookback time of about 7 Gyr, not dissimilar from the age of Earth. As structure formation proceeds faster in the $\Lambda = 0$ universe, the amount of substructure grows faster and earlier: the number of isolated halos is much smaller than in the case of a Λ-dominated universe.

In order for a particular galaxy to harbor life, it must reside within a habitable region of the parameter space. The halo mass should be in the range $10^{11.5} < M/M_\odot < 10^{12}$ as to give rise to a large galaxy like the Milky Way so that it has significant outer regions in which the GRB rate is low but there are sufficient numbers of stars. Additionally, the galaxy should undergo sufficient chemical evolution so that its average metallicity is relatively large thereby reducing the GRB rate. Furthermore, the halo should be young enough that stars with mass $M_{\rm min}$ have not left the main sequence. For $M > 0.5 M_\odot$ stars, this corresponds to ($\lesssim$50 Gyr), for $M_{\rm min} > 0.7 M_\odot$, this corresponds to $\lesssim$20 Gyr. For example, for the model with $\Omega_\Lambda = 0.5$, the age of the Universe is ~20 Gyr; by the time Ω_Λ takes over the expansion, the Universe is already too old and out of the habitable epoch for all main sequence stars above $M_{\rm min} = 0.7 M_\odot$. For $M_{\rm min} = 0.5 M_\odot$, we estimate this to correspond instead to $\Omega_\Lambda = 0.2$ ($\rho_\Lambda / M_{\rm Pl}^4 = 1.2 \times 10^{-124}$).

One can imagine waiting for a long time for the GRB rate to be sufficiently low. The current decrease of the GRB rate with time [16] at $z < 1$ is much flatter than the decrease in the star formation rate (see Ref. [16], Fig. 9); extrapolating this rate implies that the star formation will be exhausted sooner than GRBs. By $z \sim = -0.4$, when flat universes with a cosmological constant that is 0.3–0.6 of that in the vanilla model become similar to the present Universe with $\Omega_\Lambda = 0.75$, the GRB rate will be down by a factor of 2, while the star formation rate will be down by a factor of 10. Since the decrease in the star formation rate is the dominant effect, for simplicity, in what follows, we assume that the GRB rate remains constant. This approximation does not change qualitatively our argument.

In the range of ρ_Λ that we have explored, and for $M_{\rm min} = 0.7 M_\odot$, we find that the number density of habitable and isolated halos can be approximated by (for 50 kpc radius $\eta = 0.1$ and $\kappa = 1.1 \times 10^{123}$ for ρ_Λ in $M_{\rm PL}^4$; for 20 kpc radius, we find that a broken power law is a slightly better fit: $\kappa_1 = 0.72 \times 10^{123}$, $\eta_1 = 0.52$ and $\kappa_2 = 0.24 \times 10^{123}$, $\eta_2 = 0.72$, respectively, at low and high values of ρ_Λ)

$$I(\rho_\Lambda) \sim \kappa \rho_\Lambda + \eta. \tag{1}$$

The coefficients of the fit may change for different values of $M_{\rm min}$ or different radii, but the qualitative behavior remains with a sharp decrease for $\Omega_\Lambda < 0.2$ ($\rho_\Lambda < 1.2 \times 10^{-124}$). For larger values of Λ, this relation must flatten since virtually all MW-size halos are isolated for $\Omega_\Lambda > 0.85$. We also note that, always in the range of ρ_Λ explored, the number of MW-sized halos in the same volume is roughly constant, implying that $I(\rho_\Lambda)$ is roughly proportional to the fraction of isolated halos (for larger values of Λ when this fraction reaches unity, it is assumed to become a constant). If we assume that, for the values of ρ_Λ we considered so far, $I(\rho_\Lambda)$ is proportional to the probability $p(I|\rho_\Lambda)$, to infer the posterior probability for having a cosmological constant $p(\Lambda|I) \propto p(\Lambda) p(I|\Lambda)$, we must define a prior distribution $p(\Lambda)$. Previous studies take a flat prior on ρ_Λ based on the premise that its value may arise from the cancellation of much larger terms. However, given the large uncertainty regarding the nature of dark energy, this might not be the case. A more conservative approach is to select an uninformative prior such that $p(\Lambda) \propto 1/\rho_\Lambda$ (see, also, Ref. [6]). In this case, one shall not consider negative values of the cosmological constant, although they are likely to generate even more inhospitable consequences.

The other mechanism by which Λ influences the habitability of the Universe is the suppression of galaxy formation. This was studied in detail in Ref. [31], who used the fraction of baryons $F(\mu)$ residing in halos of mass $\mu \equiv \xi^2 M$, where ξ is the matter density per photon, as a proxy for the habitability of the Universe. They present the following prescription for the late-time solution:

$$F(\mu) = \mathrm{erfc}\left[\frac{A\rho_\Lambda^{1/3}}{\xi^{4/3} Q s(\mu) G_\infty}\right]. \qquad (2)$$

Here, A and G_∞ can be considered fixed quantities, $A = 5.59$ and $G_\infty = 1.43$; we take $s = 28$ and $\xi^4 Q^3 = 10^{-124}$ following Ref. [31] (see, e.g., Ref. [31], Fig. 7.). It is clear from Eq. (2) and, e.g., Fig. 7 of Ref. [31], that this imposes a sharp suppression for $\rho_\Lambda \gtrsim 10^{-122} M_{\mathrm{Pl}}^4$.

In summary,

$$p(\Lambda|I) \propto p(\Lambda) F(\Lambda) I(\Lambda), \qquad (3)$$

where $F(\Lambda)$ gives the number of suitable MW-size halos and describes the sharp upper cutoff imposed by Eq. (2), and $I(\Lambda)$ imposes a much slower suppression towards 0. In Fig. 2, we sketch the probability distribution function for ρ_Λ normalized to the Einstein–de Sitter case using the computed abundance of isolated Milky-Way-size halos from Fig. 1. The low-end suppression is described by Eq. (1). For high values of ρ_Λ, the suppression discussed in Refs. [1,2,31], effectively due to the requirement of cosmic structure to form, becomes relevant. It is not surprising then that our Universe has a value of $\Lambda \approx 1$ ($\rho_\Lambda \sim 10^{-123} M_{\mathrm{Pl}}^4$). It is instructive to compare to Fig. 7 of Ref. [31] and notice that the allowed region for ρ_Λ is now greatly reduced.

Since, on average, there is *one* isolated region in a patch of 10 Mpc radius (which, incidentally, is the mean intergalaxy cluster separation today), to ensure (at $\sim 3\sigma$) that there is at least one habitable galaxy in the observable universe, the horizon size should be at least 40 Mpc, i.e., not more than ~ 100 times smaller than the current horizon size. Given our specifications for galactic habitability, both in terms of the required separations between galaxies, and the minimum age of the Universe which permits the formation of planets, a large universe is necessary for life to emerge. A loitering model with a finely tuned cosmological constant could satisfy the age requirement but does not provide adequate intergalactic spacing.

If our location in the Universe, and potentially the multiverse, is preferentially selected by the absence of nearby GRB-hosting small halos, then we will expect to find that most Milky-Way-sized galaxies beyond the local group possess a heightened number of LMC-like satellites. To some extent, this has already been observed [32]. The reduced number of observed Milky Way satellites compared to predictions derived from simulations is often attributed to the inefficiency of star formation within low-mass halos. Yet this puzzle may also be partially resolved by our proposed selection effect.

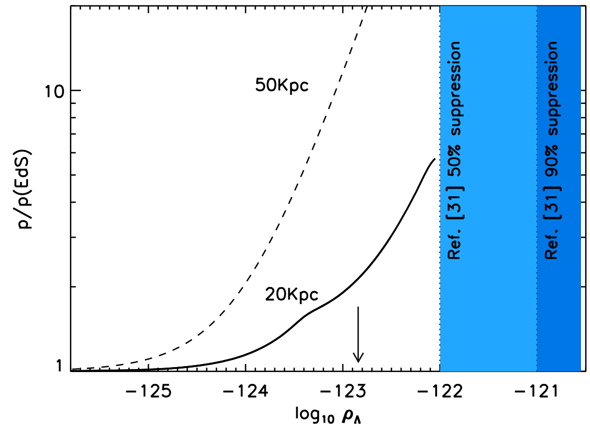

FIG. 2. The probability distribution function for ρ_Λ normalized to the probability for the Einstein–de Sitter (EdS, $\Lambda = 0$) case. The solid (dashed) line corresponds to exclusion regions of 20 (50) kpc and result from this work. For high Λ values, the exponential suppression of Ref. [31] takes over. This is indicated by the shaded regions. The arrow shows the value of ρ_Λ for the concordance ΛCDM model. Our result suppresses the probability of low values of Λ previously allowed and, in fact, favored, by the argument in Refs. [1,2].

Another interesting implication is that if the amplitude of fluctuations Q is increased, then halos of a given mass form earlier, but they do so in a very crowded environment. In previous investigations, Q is one of the few cosmological parameters which could be enlarged by an order of magnitude without any clearly adverse effects (e.g., Fig. 12 in Ref. [31]). Within the context of GRB-limited habitability, there is likely to be much less freedom in this parameter.

In summary, we have shown that Λ plays a crucial role at creating habitable regions for galaxies in a habitable epoch. These considerations may be used to disfavor very low values for Λ. Negative values of Λ will yield even more satellites, and, hence, these arguments strongly disfavor such values.

Millennium II data can be obtained by following the link in Ref. [27].

We thank R. Angulo for help with interpreting the simulation outputs. This research was supported by the ISF I-Core Center of Excellence and by an Israel-China grant (T. P.), by Mineco Grants No. AYA2014-58747-P (R. J. and L. V.) and No. FP7-IDEAS-Phys.LSS 240117 (L. V., A. J. C., and F. S.), and the Spanish MINECO under Project No. MDM-2014-0369 of ICCUB (Unidad de Excelencia 'María de Maeztu').

*tsvi.piran@mail.huji.ac.il
†raul.jimenez@icc.ub.edu
‡ajcuesta@icc.ub.edu
§fergus2@icc.ub.edu
‖liciaverde@icc.ub.edu

[1] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Clarendon Press, Oxford, 1986).
[2] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
[3] G. Efstathiou, Mon. Not. R. Astron. Soc. **274**, L73 (1995).
[4] M. Tegmark and M. J. Rees, Astrophys. J. **499**, 526 (1998).
[5] J. Garriga, M. Livio, and A. Vilenkin, Phys. Rev. D **61**, 023503 (1999).
[6] J. Garriga and A. Vilenkin, Phys. Rev. D **61**, 083502 (2000).
[7] J. A. Peacock, Mon. Not. R. Astron. Soc. **379**, 1067 (2007).
[8] J. D. Barrow, H. B. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 123501 (2002).
[9] S. E. Thorsett, Astrophys. J. **444**, L53 (1995).
[10] J. Scalo and J. C. Wheeler, Astrophys. J. **566**, 723 (2002).
[11] A. L. Melott, B. S. Lieberman, C. M. Laird, L. D. Martin, M. V. Medvedev, B. C. Thomas, J. K. Cannizzo, N. Gehrels, and C. H. Jackman, Int. J. Astrobiol. **3**, 55 (2004).
[12] T. Piran and R. Jimenez, Phys. Rev. Lett. **113**, 231102 (2014).
[13] A. L. Melott, arXiv:astro-ph/0604440.
[14] S. Savaglio, K. Glazebrook, and D. L. Borgne, Astrophys. J. **691**, 182 (2009).
[15] R. Jimenez and T. Piran, Astrophys. J. **773**, 126 (2013).
[16] D. Wanderman and T. Piran, Mon. Not. R. Astron. Soc. **406**, 1944 (2010).
[17] B. Panter, R. Jimenez, A. F. Heavens, and S. Charlot, Mon. Not. R. Astron. Soc. **378**, 1550 (2007).
[18] V. Springel, J. Wang, M. Vogelsberger, A. Ludlow, A. Jenkins, A. Helmi, J. F. Navarro, C. S. Frenk, and S. D. M. White, Mon. Not. R. Astron. Soc. **391**, 1685 (2008).
[19] M. Fukugita and P. J. E. Peebles, Astrophys. J. **616**, 643 (2004).
[20] P. A. James and C. F. Ivory, Mon. Not. R. Astron. Soc. **411**, 495 (2011).
[21] B. C. Thomas, A. L. Melott, C. H. Jackman, C. M. Laird, M. V. Medvedev, R. S. Stolarski, N. Gehrels, J. K. Cannizzo, D. P. Hogan, and L. M. Ejzak, Astrophys. J. **634**, 509 (2005).
[22] A. L. Melott and B. C. Thomas, Astrobiology **11**, 343 (2011).
[23] A. L. Melott and B. C. Thomas, Paleobiology **35**, 311 (2009).
[24] B. C. Thomas and A. L. Melott, New J. Phys. **8**, 120 (2006).
[25] F. Simpson, Mon. Not. R. Astron. Soc. **456**, L59 (2016).
[26] M. Boylan-Kolchin, V. Springel, S. D. M. White, A. Jenkins, and G. Lemson, Mon. Not. R. Astron. Soc. **398**, 1150 (2009).
[27] See http://www.mpa-garching.mpg.de/galform/millennium-II/.
[28] G. Lemson (Virgo Consortium Collaboration), arXiv: astro-ph/0608019.
[29] R. E. Angulo and S. D. M. White, Mon. Not. R. Astron. Soc. **405**, 143 (2010).
[30] A. Mead and J. Peacock, Mon. Not. R. Astron. Soc. **445**, 3453 (2014).
[31] M. Tegmark, A. Aguirre, M. J. Rees, and F. Wilczek, Phys. Rev. D **73**, 023505 (2006).
[32] P. Côt, M. J. West, and R. O. Marzke, Astrophys. J. **567**, 853 (2002).