Driven Markovian Quantum Criticality

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We identify a new universality class in one-dimensional driven open quantum systems with a dark state. Salient features are the persistence of both the microscopic nonequilibrium conditions as well as the quantum coherence of dynamics close to criticality. This provides a nonequilibrium analogue of quantum criticality, and is sharply distinct from more generic driven systems, where both effective thermalization as well as asymptotic decoherence ensue, paralleling classical dynamical criticality. We quantify universality by computing the full set of independent critical exponents within a functional renormalization group approach.

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Introduction.—There has been a surge of activity in a broad spectrum of experimental platforms, which implement driven open quantum systems. In such systems, coherent and driven-dissipative dynamics occur on an equal footing. While such a situation is reminiscent of conventional quantum optics, the systems in point are set apart from more traditional realizations by a large, continuous number of spatial degrees of freedom, giving rise to genuine driven many-body systems. Indeed, experimental realizations range from exciton-polariton systems [1,2] over ultracold atoms [3–5], large systems of trapped ions [6,7], and photon Bose-Einstein condensates [8] to microcavity arrays [9,10]. The driven nature at the microscale leads to an intrinsic nonequilibrium (NEQ) situation even in the stationary state due to the explicit breaking of detailed balance. At this microscopic level, the dynamics of such systems is Markovian, i.e., memoryless in time. This is not in fundamental contradiction to genuine quantum effects playing a role, as has been demonstrated theoretically [11,12] and experimentally [13,14] in many-body systems, where phase coherence or entanglement ensue in the stationary state of tailored driven-dissipative evolution. For the universal critical behavior of such systems, however, despite the fact that they are "made of quantum ingredients," the Markovian character generically leads to a NEQ analogue of classical dynamical criticality: typically, as the result of dissipation, phase transitions in driven-dissipative systems are governed by an emergent effective temperature together with the loss of quantum coherence, and their bulk critical behavior is captured by equilibrium universality classes [15–24].

This sparks a natural and fundamental set of questions: Given the intrinsic quantum origin, together with the flexibility in designing such systems, to which extent can effects of quantum mechanical coherence persist asymptotically at the largest distances in the vicinity of a critical point? And if so, what are the precise parallels and differences to criticality in closed equilibrium systems at zero temperature? In other words, is there a driven analogue of quantum critical behavior?

In this work, we address these questions driving a one-dimensional open Bose gas with a strong Markovian quantum diffusion, implemented, e.g., with microcavity arrays. When diffusion dominates over the Markovian noise level induced by the environment, a novel critical regime associated to NEQ condensation can be realized, where the coherent quantum mechanical origin of the system and the NEQ driven nature persist at infrared scales. Our results establish a new driven quantum universality class in one dimension, which we characterize by computing the full set of static and dynamical critical exponents. In particular, we obtain the following key results. (i) New nonequilibrium fixed point. The fixed point (FP) associated to quantum NEQ condensation cannot be mapped to the classical FP of driven-dissipative condensation—as evidenced from novel scaling of the correlation length close to criticality, and we identify a scaling regime where it governs the fluctuation dominated renormalization group (RG) flow. Because of the fine-tuning of the Markovian noise level in addition to the mass gap, it is less stable—in a RG sense—than a classical fixed point, in analogy to the double fine-tuning of mass and temperature to zero necessary to reach an equilibrium quantum critical point [25]. (ii) Absence of decoherence. In classical equilibrium and driven critical behavior, decoherence causes the fadeout of all coherent couplings in the infrared RG flow, leading to a FP where only purely dissipative dynamics persists. In contrast, the anomalous dispersion relation of the critical modes proper of the novel NEQ fixed point, manifests the simultaneous presence of coherent and diffusive processes. The absence of decoherence is reflected in the degeneracy of the two critical exponents encoding kinetic mechanisms. (iii) Absence of asymptotic thermalization. Many driven systems exhibit effective thermal behavior at low frequencies [15–22,24]. The present system does not show this property, and this is characterized by the nonthermal character of the distribution function, as well as universally by a new independent critical exponent entering the NEQ fluctuation-dissipation relation. A hallmark of the interplay of these effects is an oscillatory behavior of the spectral density as a consequence of the (iv) RG limit-cycle behavior of the complex wave function renormalization coefficient.

Finally, notice that the noise in our system is Markovian, in contrast to previous realizations of NEQ quantum criticality [17,18], while the quantum nature of the novel critical regime sharply sets apart our scenario from other NEQ fixed points, as occurring in surface growth [26], directed percolation [27], or turbulence [28–31].

The results presented here are obtained within a functional renormalization group approach [32–34] based on the Keldysh path integral associated to the Lindblad quantum master equation. The nature of the new FP precludes the use of more conventional *classical* dynamical field theories pioneered by Hohenberg and Halperin [35], and fully necessitates our *quantum* dynamical field theory approach.

A platform for nonequilibrium quantum criticality.—The starting point for our RG program is the quantum master equation governing the evolution of the density operator $\hat{\rho}$ of a one-dimensional ($d = 1$) bosonic field $\hat{\phi}(x)$;

$$\partial_t \hat{\rho} = -i[H, \hat{\rho}] + \mathcal{L}[\hat{\rho}]. \tag{1}$$

Two-body collisions of strength λ, among bosons (of mass m), are encoded in the Hamiltonian H, while the Liouvillian can be decomposed into the sum of four dissipative channels $\mathcal{L} = \sum_a \mathcal{L}_a$ $(a = p, l, t, d)$, with $\mathcal{L}_a[\rho] = \gamma_a \int_x (\hat{L}_a(x)\hat{\rho}\hat{L}_a^\dagger(x) - \frac{1}{2}\{\hat{L}_a^\dagger(x)\hat{L}_a(x), \hat{\rho}\})$, where local Lindblad operators incoherently create (destroy) single particles $\hat{L}_p(x) = \hat{\phi}^\dagger(x)(\hat{L}_l(x) = \hat{\phi}(x))$, respectively, with rates γ_p (γ_l), or destroy two particles, $\hat{L}_t(x) = \hat{\phi}(x)^2$, with rate γ_t. The key element of our analysis is the Lindblad operator $\hat{L}_d(x) = \partial_x \hat{\phi}(x)$, which is responsible for single particle diffusion with rate γ_d, and which can be realized through microcavity arrays [9,10] as portrayed in Fig. 1 (see also Ref. [36]).

In a simple mean field description, when the gain of single particles is balanced by two-body losses, a condensate $\phi_0 = \langle \hat{\phi}(x) \rangle$ with spontaneously chosen global phase can emerge [38–40]. A crucial ingredient is now the absence of particle number conservation due to pump and loss processes. For this reason, there is no sound mode with dispersion $\omega \sim |q|$ $(z = 1)$, and, instead, the canonical dynamic exponent is $z = 2$. The effective phase space dimension is then $D = d + z = 3$, allowing for a condensation transition in one dimension even when fluctuations beyond mean field are taken into account.

A parameter regime of strong quantum diffusion can, indeed, disclose a quantum critical behavior analogous to

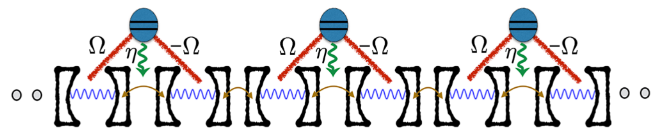

FIG. 1. A one-dimensional array of microwave resonators, coupled to an array of superconducting qubits (blue dots), which can decay with rate η (cavity bosons can tunnel among neighbouring sites—yellow arrows). Each pair of adjacent photonic modes interact with a single qubit via the dipole term, $\mathcal{H} \sim \Omega\sigma_j^+(b_i - b_{i+1}) + \mathrm{H.c.}$; b_i are the bosonic annihilation operators for the cavity modes and the local qubit Hamiltonians are proportional to the σ_j^z Pauli matrix. For an energy scale separation $\eta \gg \Omega$, the qubit dynamics can be adiabatically eliminated [37]. This gives rise to Lindblad operators proportional to $\sim b_i - b_{i+1}$, which in the continuum limit yields $L_d(x)$. It imprints an additional diffusion on the propagation of bosons and, crucially for this work, gives rise to a scaling of noise level $\sim q^2$, as discussed after Eq. (2).

zero temperature quantum criticality, as we are going to glean in the following, recasting the nonunitary quantum evolution encoded in Eq. (1) into an equivalent Keldysh functional integral formulation of dynamics [41,42]. The quadratic part of the action, occurring in the Keldysh partition function, reads

$$S_{\mathrm{kin}} = \int_{t,x} (\bar{\phi}_c^*, \bar{\phi}_q^*) \begin{pmatrix} 0 & \bar{P}^A \\ \bar{P}^R & \bar{P}^K \end{pmatrix} \begin{pmatrix} \bar{\phi}_c \\ \bar{\phi}_q \end{pmatrix}, \tag{2}$$

where $\bar{\phi}_c$ and $\bar{\phi}_q$ are the so-called classical and quantum fields, defined by the symmetric and antisymmetric combinations of the fields on the forward and backward parts of the Keldysh contour [41]. In Eq. (2), $\bar{P}^R = (\bar{P}^A)^\dagger = i\partial_t + (\bar{K}_R - i\bar{K}_I)\partial_x^2 + i\bar{\chi}$ is the retarded (advanced) inverse Green's function, while $\bar{P}^K = i(\bar{\gamma} - 2\bar{\gamma}_d\partial_x^2)$ is the Keldysh inverse Green's function. In Eq. (2) we relabeled the parameters in view of RG applications: at the microscopic scale, k_{UV} (the ultraviolet scale where our RG starts), they are expressed in terms of the couplings entering Eq. (1), $\bar{K}_R|_{k_{\mathrm{UV}}} \equiv 1/2m$, $\bar{K}_I|_{k_{\mathrm{UV}}} = \bar{\gamma}_d|_{k_{\mathrm{UV}}} \equiv \gamma_d$, $\bar{\chi}|_{k_{\mathrm{UV}}} = (\gamma_l - \gamma_p)/2$ and $\bar{\gamma}|_{k_{\mathrm{UV}}} = \gamma_p + \gamma_l$. The existence of two independent Green's functions, $\bar{G}^{R/A}$ and $\bar{G}^K$—an exclusive aspect of NEQ dynamics [41,43]—allows for a distinction between a "retarded mass," $\bar{\chi}$, which controls the distance from the condensation transition, and a "Keldysh mass," $\bar{\gamma}$, which will play in the following the role of a temperature and which microscopically corresponds to a constant Markovian noise level induced by the environment.

The role of canonical scaling of a non-Markovian quantum noise, $\bar{P}^K_{\mathrm{eq}}(\omega) \sim |\omega|$ (responsible for zero-temperature bosonic quantum phase transitions [41]), can be taken by the Markovian diffusive driving, $\bar{P}^K_{\mathrm{neq}}(q) \sim 2\bar{\gamma}_d q^2$, in a model with dynamical critical exponent $z = 2$ $(\omega \sim q^z)$. In our system, such a quantum scaling regime and its associated NEQ fixed point, appear then in the simultaneous limit $\gamma_l \to \gamma_p$, and $\gamma_l, \gamma_p \to 0$, where diffusion becomes dominant over $\bar{\gamma}$ in P^K, and the mass gap closes ($\bar{\chi} \to 0$). This double fine-tuning is analogous to the simultaneous tuning of the spectral gap and temperature to zero at equilibrium quantum critical points [25], and it opens the door to the realization of a driven analogue of quantum criticality in our system. In passing, we mention that the gapless nature of the NEQ drive $\bar{P}^K_{\rm neq}(q \to 0) \to 0$, is due to the existence of a many-body dark state [11,12]—a mode decoupled from noise, located at $q = 0$ in our case.

In a strongly diffusive, near critical regime, spectral and Keldysh components of the Gaussian action, Eq. (2), scale then alike, $\bar{P}^{R/A/K}(q) \sim q^2$ (in compact notation $[\bar{P}^{R/A/K}] = 2$). This fixes the canonical classical and quantum field dimensions to $[\bar{\phi}_c] = [\bar{\phi}_q] = d/2$ and sets the canonical scaling dimension of quartic couplings to $2 - d$ (upper critical dimension, $d_c = 2$).

We now point out the two key scales which delimit the scaling regime introduced here and the novel NEQ quantum critical point discussed later. The first can be gleaned from an analogy with equilibrium: In the quantum-classical crossover at finite temperature, the quantum scaling associated to equilibrium quantum phase transitions persists at scales smaller than the de Broglie length, $L_{\rm dB} \sim (1/T^{1/z})$ [25], where temperature T cuts off coherent quantum fluctuations. Analogously, the novel NEQ quantum critical regime is delimited at low momenta by the Markov momentum scale Λ_M—the threshold where constant Markovian noise prevails over single particle diffusion, spoiling the diffusive scaling of the noise component of the quadratic action ($\bar{P}^K$). For Λ_M we find the upper bound $\Lambda_M \lesssim 0.2\Lambda_G$ (see Supplemental Material [44]) and at distances larger than Λ_M^{-1}, critical properties are governed by a FP in the Kardar-Parisi-Zhang universality class [47]. The second key scale is the Ginzburg momentum scale, $\Lambda_G \simeq \gamma_t/\gamma_d$: at momenta lower than this scale corrections to canonical scaling become effective, indicating the breakdown of a mean-field description [48]. According to this analysis, the novel critical behavior manifests then in the momenta window $\Lambda_M \lesssim q \lesssim \Lambda_G$.

Nonequilibrium functional renormalization.—We now aim at determining the universality class, i.e., the full set of critical exponents associated to the quantum NEQ critical regime, which is technically characterized by the so-called Wilson-Fisher FP of the RG equations [48]. To this end, we dress the microscopic coefficients of Eq. (1) with RG corrections, employing a functional RG (FRG) suited for open NEQ quantum many body systems [21] (and previously developed for NEQ closed settings [49–51]). FRG allows us to interpolate from the microscopic dissipative action to the infrared effective action, introducing an infrared regulator $\bar{R}_k$, which suppresses stepwise fluctuations with momenta less than an infrared cutoff scale k. In this way we can approach smoothly the critical point where infrared divergences govern the physics. The FRG flow is based on a functional differential equation [32] for the effective action Γ_k, $\partial_k \Gamma_k = (i/2){\rm Tr}[(\Gamma_k^{(2)} + \bar{R}_k)^{-1} \partial_k \bar{R}_k]$, where the trace operation, Tr, denotes summation over internal degrees of freedom as well as summation over frequencies and momenta, and $\Gamma_k^{(2)}$ the second functional derivative of the effective action with respect to the fields. In order to convert the functional differential equation for Γ_k into a closed set of nonlinear differential equations for the RG running of the couplings (the beta functions [32,48]), we provide a functional ansatz for $\Gamma_k \equiv S_{Q,k} = S_{\rm kin} + S_{\rm int}$, where we systematically take into account in $S_{\rm int} \equiv S_h + S_a$ all operators which are classified relevant according to the quantum power counting discussed above:

$$S_h = -\int_{x,t} \frac{1}{2}\left[\frac{\partial \bar{\mathcal{U}}_c}{\partial \bar{\phi}_c}\bar{\phi}_q + \frac{\partial \bar{\mathcal{U}}_c^*}{\partial \bar{\phi}_c^*}\bar{\phi}_q^* + \frac{\partial \bar{\mathcal{U}}_q}{\partial \bar{\phi}_q}\bar{\phi}_c + \frac{\partial \bar{\mathcal{U}}_q^*}{\partial \bar{\phi}_q^*}\bar{\phi}_c^*\right],$$

$$S_a = \int_{x,t} i\bar{g}_1\left(\bar{\phi}_c^*\bar{\phi}_c - \frac{\bar{\rho}_0}{2}\right)\bar{\phi}_q^*\bar{\phi}_q + i\bar{g}_2(\bar{\phi}_q^*\bar{\phi}_q)^2 + -\frac{1}{4}[\bar{g}_3(\bar{\phi}_c^*\bar{\phi}_q)^2 - \bar{g}_3^*(\bar{\phi}_c\bar{\phi}_q^*)^2]. \qquad (3)$$

S_h and S_a are, respectively, the Hermitian and anti-Hermitian parts of the interaction action. The potentials $\bar{\mathcal{U}}_c = \frac{1}{2}\bar{u}_c(\bar{\phi}_c^*\bar{\phi}_c - \bar{\rho}_0)^2$ and $\bar{\mathcal{U}}_q = \frac{1}{2}\bar{u}_q(\bar{\phi}_q^*\bar{\phi}_q)^2$ have associated complex couplings $\bar{u}_{c,q} \equiv \bar{\lambda}_{c,q} + i\bar{\kappa}_{c,q}$, which microscopically coincide with the parameters entering the master equation ($\bar{\lambda}_c|_{k_{\rm UV}} = \bar{\lambda}_q|_{k_{\rm UV}} = \lambda$, $\bar{\kappa}_c|_{k_{\rm UV}} = \bar{\kappa}_q|_{k_{\rm UV}} = \gamma_t$, $\bar{g}_1|_{k_{\rm UV}} = 2\gamma_t$). The couplings $\bar{g}_2$ and $\bar{g}_3 \equiv \bar{\lambda}_3 + i\bar{\kappa}_3$ are, instead, only generated in the course of renormalization. In Eq. (3) we introduced the condensate density (resulting from balance of particles gain and losses) $\bar{\rho}_0$, since in our practical calculations we approach the transition from the ordered phase, taking the limit of the stationary state condensate $\bar{\rho}_0 = \bar{\phi}_c^*\bar{\phi}_c|_{ss} = \bar{\phi}_0^*\bar{\phi}_0 \to 0$. In this way, we capture two-loop effects [32] necessary to compute the full set of critical exponents and thus determine the universality class. We also rewrite the inverse R/A propagators allowing for a complex wave-function renormalization coefficient Z, $\bar{P}^R = iZ^*\partial_t + \bar{K}^*\partial_x^2$, with $\bar{K} \equiv \bar{K}_R + i\bar{K}_I$ and $Z \equiv Z_R + iZ_I$, [52], whose anomalous dimension can acquire a real and imaginary part, $\eta_Z \equiv \eta_{ZR} + i\eta_{ZI} \equiv -\partial_t Z/Z$. We mark that the quantum dynamical field theory $S_{Q,k}$ has a richer RG operator content than a conventional equilibrium Martin-Siggia-Rose action [41,43] or the semiclassical model in three dimensions for driven-dissipative condensation [21] (where $\bar{\gamma}_d = 0$, and $\bar{u}_q = \bar{g}_1 = \bar{g}_2 = \bar{g}_3 = 0$ from the outset—on the basis of their RG irrelevance).

Rescaling all the couplings $\{g\}$ of $S_{Q,k}$ by the quantum canonical power counting, we find a FP solution $\{\tilde{g}^*\}$ of the FRG beta functions in terms of the rescaled variables $\{\tilde{g}\}$ (see Supplemental Material Ref. [44]). The analysis in the

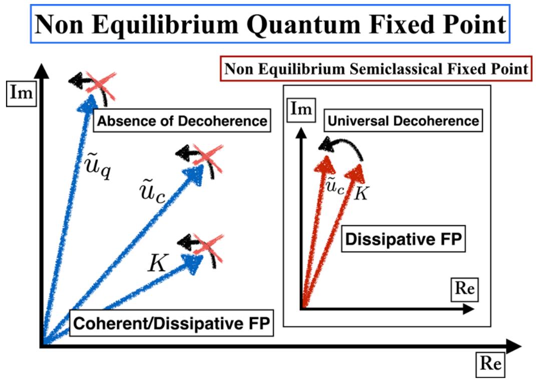

FIG. 2. In the quantum problem, all the rescaled couplings (we portayed some of the $\{\tilde{g}\}$) keep a nonvanishing real part at the FP and their RG flow freezes in the complex plane (as indicated by the red cross on the curved arrows). In the semiclassical problem, instead, decoherence forces asymptotically all the couplings to flow onto the imaginary axis, and dynamics at infrared scales becomes purely dissipative.

vicinity of the FP gives access to the full set of critical exponents. We will use as a benchmark for the salient physical features of the quantum FP in $d = 1$ dimensions ($D = 3$), its semiclassical driven Markovian counterpart in $d = 3$ dimensions [21].

A nonequilibrium quantum universality class.—(i) *Nonequilibrium fixed point.*—The key new property of the quantum FP is its mixed nature with coexistent coherent and dissipative processes, as shown in Fig. 2. This new FP is less stable than the finite temperature FP, or the semiclassical one, since the additional fine tuning of the Markovian noise level is necessary to reach the quantum FP—as discussed above.

In the domain of equilibrium phase transitions, the universality classes of d-dimensional critical quantum systems and of their classical $d + z$ dimensional counterparts [25,53,54] coincide. Table I compares the full set of critical exponents of the quantum transition and of its semiclassical driven-dissipative counterpart [21], elucidating that the analogy does not hold in the case of our NEQ setting. In the vicinity of the transition, the exponent (ν) controlling the divergence of the correlation length of the Bose field, exhibits, for instance, the mismatch among the two critical characters.

(ii)*Absence of asymptotic decoherence.*—Persistence of quantum mechanical facets at criticality (for length scales shorter than Λ_M^{-1}), is a common feature between our FP and equilibrium quantum critical points [25,53,54]. The low energy anomalous dispersion relation of critical modes, $\omega_k \sim k^{2-\eta_{K_I}}(c_1 - ic_2)$, encodes coherent effects ($c_{1,2}$ are two positive constants depending on the quantum FP), in contrast to the purely diffusive leading behavior of ω_k in the vicinity of the dissipative FP of the semiclassical model, $\omega_k \sim -ik^{2-\eta_{K_I}}$ [21]. From an RG point of view, the exponent degeneracy $\eta_{K_R} = \eta_{K_I} = -0.025$ allows for a finite ratio of coherent propagation (K_R) versus diffusion (K_I) $r = (K_R/K_I) \sim k^{-\eta_{K_R}+\eta_{K_I}}$, which is thus fully consistent with the results of Fig. 2, and, in particular, indicates the absence of decoherence at long distances.

(iii)*Absence of asymptotic thermalization.*—The persistence of NEQ character at macroscales and the associated nonthermal character of the distribution function, constitute the strongest evidence that the quantum universality class found in this Letter cannot be related to its semiclassical driven Markovian counterpart in $d + z$ dimensions, or to an equilibrium FP.

To see this point, we note that the fluctuation-dissipation relation demands, in the 3D driven-dissipative model, that the effective temperature $T_C = |Z|\gamma$—extracted from the infrared bosonic distribution function $F_C(\omega, k) \sim (T_C/\omega)$, is scale invariant. This expresses the principle of detailed balance of thermal equilibrium states (invariance of temperature under the system partition) in a RG language [21,55]. Such circumstance occurs at the semiclassical FP via the emergent exponent degeneracy $\eta_\gamma = -\eta_{ZR}$ (cf. Table I)—the system thermalizes asymptotically.

In the same spirit, if thermalization were to ensue close to the quantum FP, scale invariance of the low-frequency distribution function $F_Q(\omega, k) \sim [T_Q(k)/\omega](1 + \tilde{\gamma}^*/2)$ ($T_Q(k) \equiv |Z|\gamma_d k^2$) must be expected as a necessary condition. Specifically, replacing the bare scaling of the frequency $\omega \sim k^z$ in $F_Q(\omega, k)$, insensitivity to system's partition would manifest in the *exact* scaling relation $F_Q \sim k^0$. The absence of exponent degeneracy, $\eta_{\gamma_d} \neq -\eta_{ZR}$ (cf. Table I), signals scaling violation in the infrared behavior of $F_Q \sim k^{-(\eta_{\gamma_d}+\eta_{ZR})}$, and, accordingly, the absence of infrared thermalization at the quantum FP.

(iv)*RG limit cycle of Z.*—Finally, we notice that the peak of the spectral density—the imaginary part of the retarded single particle dynamical response $A(\omega = \mathrm{Re}\omega_k) = [\mathrm{Re}(Z)/|Z|^2](1/\mathrm{Im}\omega_k)$ is sensitive to oscillations present in $Z \sim k^{-\eta_{ZR}} e^{-i\eta_{ZI} t}$, induced by a nonvanishing η_{ZI}

TABLE I. Comparison between the critical exponents of the quantum and semiclassical driven dissipative models (taken from Ref. [21]). In the semiclassical scaling $\gamma \sim k^0$ and the Markovian noise can acquire an anomalous dimension, η_γ.

Crit. Exps.	ν	η_{K_R}	η_{K_I}	η_{ZR}	η_{ZI}	η_{γ_d}	η_γ
Quantum	0.405	−0.025	−0.025	0.08	0.04	−0.26	×
Semi-classical	0.72	−0.22	−0.12	0.16	0	×	−0.16

[$t = \log(k/\Lambda_G)$ is the RG flow parameter]. Even if these RG limit-cycle oscillations occur with a huge period, $2\pi/\eta_{ZI}$, they are a remarkable signature of the novel critical behavior, since they prevent the possibility to have a real wave-function renormalization Z, contrary to what happens for purely dissipative relaxational models [35] or for the semiclassical FP, where, instead, $\eta_{ZI} = 0$ (cf. Table I) [21].

Conclusions.—We have shown that both quantum mechanical coherence and the microscopic driven nature of open quantum systems can persist close to a critical point, in striking contrast with classical equilibrium and semiclassical NEQ fixed points. We have discussed the impact of this novel critical behavior on the correlation length of the order parameter on the distribution function and on the spectral density. An important perspective direction is to study the effect of additional symmetries and conservation laws on the dynamical fine structure of this novel universal behavior.

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