

Nonlocal Measurements via Quantum Erasure

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Nonlocal observables play an important role in quantum theory, from Bell inequalities and various postselection paradoxes to quantum error correction codes. Instantaneous measurement of these observables is known to be a difficult problem, especially when the measurements are projective. The standard von Neumann Hamiltonian used to model projective measurements cannot be implemented directly in a nonlocal scenario and can, in some cases, violate causality. We present a scheme for effectively generating the von Neumann Hamiltonian for nonlocal observables without the need to communicate and adapt. The protocol can be used to perform weak and strong (projective) measurements, as well as measurements at any intermediate strength. It can also be used in practical situations beyond nonlocal measurements. We show how the protocol can be used to probe a version of Hardy's paradox with both weak and strong measurements. The outcomes of these measurements provide a nonintuitive picture of the pre- and postselected system. Our results shed new light on the interplay between quantum measurements, uncertainty, nonlocality, causality, and determinism.

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Many fundamental questions in quantum mechanics concern measurements and their effects. Much progress has been made regarding the measurability of various, formally defined, “observables” under realistic constraints, with a special emphasis on relativistic and temporal constraints [1–5], but many questions remain open. In light of relativistic constraints, it is known that measurements cannot violate causality; this limits the types of instantaneous projective measurements that can be made on spacelike separated systems [6,7]. Such instantaneous measurements are of interest for a number of reasons. From a fundamental perspective, we are often interested in spacelike separated subsystems, such as EPR (Einstein-Podolsky-Rosen) pairs, where communication would rule out the nonlocal aspect of an argument. From a practical perspective, we want to avoid adaptive schemes, even at the cost of nondeterministic protocols, e.g., linear optics schemes with postselection [8].

While only a few nonlocal observables can be measured instantaneously with a projective measurement [2], many others can be measured in a destructive way [9,10]. The latter schemes produce the desired probabilities for the outcomes of the measurement. However, they give an unfavorable information gain–disturbance trade-off and usually have a random state at the output, independent of the input state and measurement result. In this Letter we present the erasure scheme for effectively creating the von Neumann measurement Hamiltonian for a large class of nonlocal and other nonstandard observables. It can be used for making strong projective (Lüders [11]) measurements,

weak measurements, and measurements at any intermediate strength. Although it can be used for measuring a wide verity of observables, we focus on nonlocal product observables due to their significance.

We call an operator Ω on a bipartite system (or Hilbert space) $\mathcal{H}_S = \mathcal{H}_A \otimes \mathcal{H}_B$ a nonlocal product observable when $\Omega = A \otimes B$ and A, B are Hermitian operators on \mathcal{H}_A and \mathcal{H}_B , respectively. We usually consider two observers Alice (\mathcal{A}) and Bob (\mathcal{B}) with access to \mathcal{H}_A and \mathcal{H}_B , respectively, such that in the relevant time interval \mathcal{A} and \mathcal{B} are spacelike separated. As a consequence the subsystems cannot interact and Ω cannot be measured directly. We call the instantaneous measurement of an observable on a spacelike separated system a nonlocal measurement.

Nonlocal product observables play a significant role in quantum theory, for example, CHSH (Clauser-Horne-Shimony-Holt) observables [12], semicausal measurements [1], nonlocality without entanglement [13], and stabilizer codes [14]. In some cases, such as the CHSH experiment, it is sufficient to extract the result by making local measurements of A and B . Such local measurements disturb the system more than the ideal nonlocal measurement (see Ref. [15] for examples) and cannot be used in other cases such as quantum error correction and state discrimination, where the outgoing state is as important as the result. In the case of weak measurements, the correlations between local measurements are of second order and a local method does not give the desired result [25,26]. Weak measurements of nonlocal product observables also play an important role in our understanding of quantum mechanics. Examples

include Bell tests [27–29], nonlocality via postselection [30], and the quantum pigeonhole principle [30,31]. They also play a role in other scenarios such as quantum computing [32]. Here, we demonstrate their significance with a variant of Hardy’s paradox [33]. Despite various attempts to find a scheme for nonlocal measurements with a weak limit [25,34,35] the erasure scheme below is the first scheme that has both a weak and a strong limit for a wide variety of nonlocal and other general observables.

The von Neumann scheme.—The standard quantum mechanical model for a measurement was introduced by von Neumann and later improved by Lüders [11] for degenerate observables. To measure an observable Ω^S on a system \mathcal{S} we need to couple it to a second quantum system, the meter \mathcal{M} , that will register the result of the measurement by the shift of a pointer variable Q^M [36]. The coupling Hamiltonian is

$$H_I = f(t)\Omega^S P^M, \quad (1)$$

where P^M is the conjugate momentum to Q^M and $f(t)$ is usually an impulse function, which is nonvanishing only around the time of the measurement. The interaction strength is $g = \int_0^\tau f(t)dt$. While formally one can write this Hamiltonian for any Hermitian operator Ω^S on \mathcal{S} , it may be impossible to implement it physically, e.g., when Ω^S is a nonlocal product observable. It is, however, possible to replace the unitary evolution $U = e^{ig\Omega^S P^M}$ with an isometry V such that, for a fixed initial meter state $|0\rangle^M$ we get $V|\psi\rangle^S|0\rangle^M = U|\psi\rangle^S|0\rangle^M = \sum_k a_k |k\rangle^S |\lambda_k\rangle^M$, where $|\psi\rangle^S = \sum_k a_k |k\rangle^S$ is an arbitrary system state and the $|k\rangle^S$ are eigenstates of Ω^S , $\Omega^S|k\rangle^S = \lambda_k|k\rangle^S$. While the implementation of V induces the desired dynamics, it may have two drawbacks: first, it may depend on the initial state of the meter; second, it might not have a free parameter corresponding to the measurement strength g . Both appear in standard nonlocal measurement schemes such as modular measurements [2].

After the measurement, the system state is dephased in the eigenbasis of Ω^S ; however, if Ω^S is degenerate, each degenerate subspace remains coherent. The measurement is usually followed by reading out the state of the pointer Q^M . When the shift in Q^M is large compared to the uncertainty Δ_Q , i.e., $\langle \lambda_k | \lambda_l \rangle \approx \delta(\lambda_k - \lambda_l)$, the measurement is strong and the result of a single measurement is unambiguous; thus, dephasing is complete. When the possible shift in Q is much smaller than Δ_Q , we have a weak measurement. Within the von Neumann model this can be achieved by choosing the coupling strength g to be small enough, or by increasing Δ_Q . As a result of the weak measurement, the system is only slightly dephased.

Weak measurements allow us to ask questions about a system at an intermediate time between an initial preparation of the state $|\psi\rangle$ (preselection) and a final projective

measurement leading to $|\phi\rangle$ (postselection), without making counterfactual statements. The result is a complex number called the weak value

$$\{\Omega\}_w = \frac{\langle \phi | \Omega | \psi \rangle}{\langle \phi | \psi \rangle}. \quad (2)$$

Although the read-out requires many identical experiments, in each experiment the result is encoded in a quantum meter whose dynamical evolution is dictated by a weak potential term in the Hamiltonian $H_w = \{\Omega\}_w P^M$ [37].

Quantum erasure.—A description of a quantum eraser [38–41] is simple when the meter has a discrete Hilbert space and $\{|\lambda_k\rangle^N\}$ is an orthonormal basis. Before the read-out stage it is possible to undo or erase the measurement locally in \mathcal{N} by measuring in the conjugate basis to $\{|\lambda_k\rangle^N\}$ and postselecting the result corresponding to the POVM element $\Pi_{+M}^N = | + M \rangle \langle + M |^N$ with $| + M \rangle^N = \sum_k |\lambda_k\rangle^N$

$$\Pi_{+M}^N [U|\psi\rangle^S|0\rangle^N] \propto |\psi\rangle^S | + M \rangle^N. \quad (3)$$

The erasure procedure is probabilistic, but we can make it deterministic by considering all POVM elements and adding a unitary operation at the end.

Note: A scheme for erasing weak measurements [42] and a relation between weak measurement and erasure [43] were recently proposed. In contrast, our method below utilizes the quantum erasure of a strong measurements as a tool for performing general measurements.

Main result.—The erasure scheme below involves two meters: \mathcal{N} and \mathcal{M} . The pointer for \mathcal{N} is Q^N so $Q^N|\lambda_k\rangle^N = \lambda_k|\lambda_k\rangle^N$; likewise, Q^M is the pointer for \mathcal{M} .

Proposition 1 It is possible to induce the von Neumann coupling between \mathcal{S} and \mathcal{M} by making a strong measurement of \mathcal{S} with \mathcal{N} and erasing the result.

Proof.—Let $\sum_k a_k |k\rangle^S |\lambda_k\rangle^N$ be the system meter state after a strong measurement. The second meter \mathcal{M} is in the arbitrary initial state $|0\rangle^M$. We now let \mathcal{M} interact with \mathcal{N} using the unitary $e^{igQ^N P^M}$

$$e^{igQ^N P^M} \sum_k a_k |k\rangle |\lambda_k\rangle |0\rangle = \sum_k a_k |k\rangle |\lambda_k\rangle e^{ig\lambda_k P^M} |0\rangle$$

and then erase using Π_{+M}

$$\Pi_{+M} \sum_k a_k |k\rangle |\lambda_k\rangle e^{ig\lambda_k P^M} |0\rangle \propto \sum_k a_k |k\rangle | + M \rangle e^{ig\lambda_k P^M} |0\rangle.$$

This is the dynamics induced by the Hamiltonian (1). \square

In the following we show how this method can be used for measurements of nonlocal product observables.

Measurement of product observables.—The challenge with measuring a nonlocal observable is to couple to the degenerate subspaces according to the Lüders rule. Given a bipartite system and two local Hermitian operators X on \mathcal{H}_A and Y on \mathcal{H}_B , the degenerate subspaces of XY are generally different from those of the local observables X and Y .

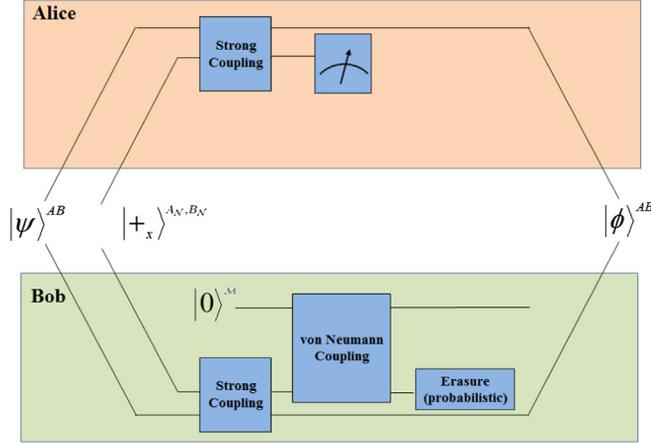


FIG. 1. Nonlocal measurement based on quantum erasure. The strong measurement requires Alice and Bob to locally couple their system to an entangled meter (the ancilla \mathcal{N}). Alice then measures her part \mathcal{N}_A , effectively “pushing” the result of the strong measurement to Bob’s \mathcal{N}_B . Bob performs the weak measurement of \mathcal{N}_B using \mathcal{M} and then erases the information encoded on \mathcal{N} (undoing the initial coupling). If successful they induce the von Neumann Hamiltonian (1)

The erasure procedure above can be used to remove the redundant information encoded locally. Below we combine this method with a remote measurement to produce the Hamiltonian (1) with $\Omega^S \rightarrow XY$ (see Fig. 1).

\mathcal{N} is prepared in an entangled state on the Hilbert space $\mathcal{H}_{\mathcal{N}} = \mathcal{H}_{A_{\mathcal{N}}} \otimes \mathcal{H}_{B_{\mathcal{N}}}$, which depends on the properties of X and Y . \mathcal{M} is local at Bob’s side and has an initial state $|q=0\rangle$. Alice locally couples the entangled \mathcal{N} to her subsystem, performing a strong measurement with the result encoded nonlocally (i.e., Alice cannot access the result alone). Alice then reads out the state of her strong meter. This teleports the result to Bob (possibly with a known offset). Next, Bob performs the procedure outlined in the proof of Proposition 1. The resulting dynamics is $U = e^{igXYP}$.

Details: We define the sets of orthogonal projectors $\{\hat{X}_k\}$, $\{\hat{Y}_l\}$ such that $X = \sum_{k=1}^{|x|} x_k \hat{X}_k$ and $Y = \sum_{l=1}^{|y|} y_l \hat{Y}_l$, where $\{x_k\}$ and $\{y_l\}$ are the sets of distinct eigenvalues of X and Y with cardinality $|x|$ and $|y|$, respectively. Assume (without loss of generality) that $|x| \leq |y|$. \mathcal{N} will have dimension $|x| \times |x|$ and will be prepared in the initial entangled state $|+_x\rangle^{\mathcal{N}} = (1/\sqrt{|x|}) \sum_{m=0}^{|x|-1} |m\rangle^{A_{\mathcal{N}}} |m\rangle^{B_{\mathcal{N}}}$. The system is initially in the unknown state $|\psi\rangle = \sum_{i,k} \alpha_{i,k} |i, k\rangle^{A,B}$. We denote the global $(A, B, \mathcal{N}, \mathcal{M})$ initial state by $|\Psi_0\rangle$. We also define $K_{\mu} = \sum_k (k - \mu) X_k$. The scheme is as follows.

1. Alice couples K_0 and \mathcal{N} using $U_s |i\rangle |m\rangle = |i\rangle |m+i\rangle$, producing

$$|\Psi_1\rangle = \sum_m \sum_{i,k} \alpha_{i,k} |i, k\rangle^{A,B} |m+i, m\rangle^{A_{\mathcal{N}}, B_{\mathcal{N}}} |q=0\rangle.$$

2. Alice reads out \mathcal{N}^A and gets a result corresponding to $|\mu\rangle$. Thus,

$$|\Psi_2\rangle = \sum_{i,k} \alpha_{i,k} |i, k\rangle^{A,B} |\mu\rangle^{A_{\mathcal{N}}} |\mu-i\rangle^{B_{\mathcal{N}}} |q=0\rangle. \quad (4)$$

Note that the label $\mu-i$ is modular, i.e., $|\mu-i\rangle^{B_{\mathcal{N}}} = |\mu-i \pm |x|\rangle^{B_{\mathcal{N}}}$.

3. Bob now has access to the operator K_{μ} , so he can couple to $\Omega_{\mu}^{\mathcal{N}} = [\sum_k x_{k-\mu} X_k] \otimes Y$ using the local interaction Hamiltonian $Y^B Q^{B_{\mathcal{N}}} P^{\mathcal{M}}$, where $Q^{B_{\mathcal{N}}} |\mu-i\rangle^{B_{\mathcal{N}}} = x_{i-\mu} |\mu-i\rangle^{B_{\mathcal{N}}}$.

4. Bob erases Alice’s measurement with probability $1/|x|$. If he succeeds, the effective $\mathcal{S} - \mathcal{M}$ dynamics is $U = e^{ig\Omega_{\mu}^{\mathcal{S}} P^{\mathcal{M}}}$.

For $\mu=0$ this is the desired observable, and in some special cases it is a simple rescaling for all μ [15]. The worst case measurement will succeed with probability $1/|x|^2$ (both erasure and $\mu=0$ are required) while the best case will succeed with probability $1/|x|$ (only erasure is required). In either case failure would correspond to a nontrivial (but known) unitary evolution during the interval between pre- and postselection. For a more detailed description see Ref. [15].

Determinism and nonlocality.—The protocol is probabilistic; however, it can be turned into a deterministic protocol if Alice and Bob are allowed to communicate. This is to be expected since the von Neumann Hamiltonian of a product operator measurement (or even the less general isometry V) can be used for signaling between Alice and Bob [6,7]. The entanglement and communication resources for our scheme are at most equivalent to a single round of teleportation. This can be compared to the naive strategy of teleporting, measuring, and teleporting back. In the example below, and the one in Ref. [15], we show that the communication cost of our scheme saturates the lower bound imposed by causality.

However, the motivation for the protocol is the fact that it can be implemented without communication or adaptive components. The nonlocal paradox below is a good example of a situation where communication is not allowed by assumption, as is the case with the Bell inequality. From a practical perspective we can easily imagine other situations such as linear optics, where the resources necessary for an adaptive scheme that requires communication outweigh the advantage of a deterministic protocol [8].

In a postselected scenario with a future boundary condition $|\phi\rangle^{A,B}$, it is possible to include the postselection requirement for the measurement in the future boundary conditions. The preselected system would then be $|\psi\rangle^{A,B} |+_x\rangle^{A_{\mathcal{N}}, B_{\mathcal{N}}}$; the postselection would be $|\phi\rangle^{A,B} |0\rangle^{A_{\mathcal{N}}} |M\rangle^{B_{\mathcal{N}}}$. Taking U_s into account gives

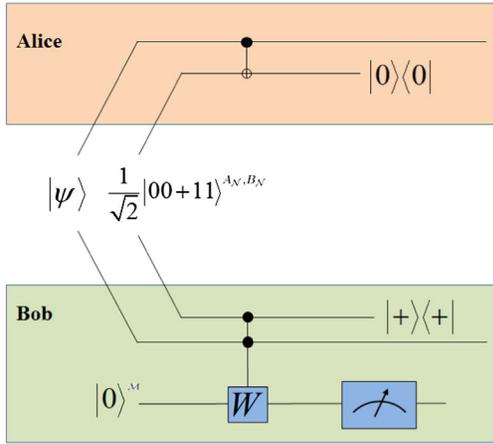


FIG. 2. Instantaneous measurement of $\Pi_{1,1}$. To measure the nonlocal observable $\Pi_{1,1} = |11\rangle\langle 11|$ Alice and Bob need an entangled ancilla \mathcal{N} and a local meter \mathcal{M} (on Bob's side). They locally couple using a CNOT on Alice's side and a controlled-controlled W on Bob's side with $W = e^{igP^{\mathcal{M}}}$. The value of g determines the measurement strength. After a successful (local) postselection of the state $|0+\rangle$ for the ancilla the effective system-meter coupling will be $e^{ig\Pi_{1,1}P^{\mathcal{M}}}$.

$$\{YQ^{B_{\mathcal{N}}}\}_w = \frac{\langle +M | \langle 0 | \langle \phi | U_s^\dagger Y Q^{B_{\mathcal{N}}} | \psi \rangle | +_x \rangle}{\langle +M | \langle 0 | \langle \phi | U_s^\dagger | \psi \rangle | +_x \rangle} = \{XY\}_w.$$

Example.—Consider a two qubit system, where Alice and Bob each have local access to a single qubit. The observables of interest are the local projectors $\Pi_{m,\cdot} = |m\rangle\langle m| \otimes \mathbb{1}$, $\Pi_{\cdot,m} = \mathbb{1} \otimes |m\rangle\langle m|$ and the nonlocal projector $\Pi_{m,n} = \Pi_{\cdot,m}\Pi_{n,\cdot} = |m\rangle\langle m| \otimes |n\rangle\langle n|$ with $m, n \in \{0, 1\}$. Now let \mathcal{M} be a meter with conjugate momentum $P^{\mathcal{M}}$ located on Bob's side. Our scheme allows us to create the effective interaction Hamiltonian $H = \Pi_{m,n}P^{\mathcal{M}}$ with probability $\frac{1}{4}$ the maximal probability allowed by causality constraints [15].

The explicit scheme is as follows (see Fig. 2): The ancilla \mathcal{N} is prepared in the entangled state $|+_x\rangle = \frac{1}{\sqrt{2}}|00 + 11\rangle^{A_{\mathcal{N}},B_{\mathcal{N}}}$; U_s is a CNOT between $A_{\mathcal{N}}$ and Alice's subsystem. The interaction with \mathcal{M} is a controlled-controlled W between $B, B_{\mathcal{N}}$ and \mathcal{M} with $W = e^{igP^{\mathcal{M}}}$. Following the interactions, Alice and Bob postselect the state $|0\rangle^{A_{\mathcal{N}}}|+\rangle^{B_{\mathcal{N}}}$ on the ancilla (with probability $\frac{1}{4}$). The induced transformation is $U = e^{ig\Pi_{1,1}P^{\mathcal{M}}}$. Bob can choose g to make the measurement weak or strong.

In principle, Alice and Bob do not need to coordinate their actions. They can each freely choose which operator to couple without notifying the other. Moreover, Bob can choose $W = e^{igP^{\mathcal{M}}}$ without notifying Alice.

A nonlocal paradox.—In the EPR scenario, a bipartite system has a definite state with respect to a nonlocal observable but has random marginals [44]. In a postselected regime it is possible to observe the opposite behavior, i.e., a system with definite local properties but uncertain nonlocal ones. Let $|\psi_H\rangle = \frac{1}{\sqrt{2}}[|0\rangle|-\rangle - |1\rangle|+\rangle]$ be a

preselected state and $|\phi_H\rangle = |+\rangle|+\rangle$ be the postselection. If either Alice or Bob make a local measurement of $\Pi_{1,\cdot}$ or $\Pi_{\cdot,1}$, respectively, they will expect the result 1 with certainty. This follows from the Aharonov-Bergman-Lebowitz formula for calculating probabilities on pre- and postselected systems [45]. If this were a classical scenario, it would have implied that a measurement of $\Pi_{1,1}$ should also produce the outcome 1 deterministically. However, the probability of obtaining the outcome 1 for a measurement of $\Pi_{1,1}$ is $\frac{1}{2}$.

The scheme presented in the previous section allows us to directly measure $\Pi_{1,1}$. If instead we measure $\Pi_{1,1}$ indirectly via $\Pi_{1,\cdot}$ and $\Pi_{\cdot,1}$, we will get the results 1,1 with probability $\frac{1}{4}$ (see Ref. [15] for details).

One may see the paradox as a result of measurement disturbance. Weak measurements let us avoid this issue. The local weak values are $\{\Pi_{1,\cdot}\}_w = \{\Pi_{\cdot,1}\}_w = 1$, while the nonlocal one is $\{\Pi_{1,1}\}_w = \frac{1}{2}$. Here, we see the full power of our scheme. It allows the first direct measurement of these weak values.

Nonlocal weak measurements were previously used to provide an elegant solution to Hardy's paradox [33,46]. The same logic applies in the example of above. The nonlocal weak values are $\{\Pi_{1,1}\}_w = \{\Pi_{0,1}\}_w = \{\Pi_{1,0}\}_w = -\{\Pi_{0,0}\}_w = \frac{1}{2}$. The last weak value is negative and ensures the weak values add up to 1. In Hardy's experiment it can be associated with negative occupation numbers in an interferometer. Using Pusey's construction [47] it is possible to show that the negative weak value is a result of contextuality. In this case the context is the information of the measurement regarding local observables.

Generalizations.—It is possible to generalize the erasure scheme to other types of operators Ω . If the measured operator is separable, like the Bell operator, it is possible to measure each product operator and add the results on a single meter [25]. It is also possible to perform more general measurements of a degenerate observable by decomposing the measurement into extremal POVMs [48] and using the erasure technique to coarse grain the outcome. Another class of measurable operators is non-Hermitian operators resulting from sequential measurements. These measurements are natural in various settings such as tests of contextuality and Leggett-Garg inequalities and measurements of quantum trajectories [28,49–51]. The specifics of an erasure based sequential measurement scheme are given elsewhere [52].

Finally, measurements are only one possible application of the Hamiltonian (1). The erasure scheme can be modified to generate this Hamiltonian under a wide set of constraints that prevent the direct coupling of a system \mathcal{M} to a degenerate operator Ω^S . In the Supplemental Material [15], we show how to use the erasure technique to construct a generic controlled-controlled-unitary gate in

cases where the relevant qubits cannot interact, a common restriction for photonic qubits.

Conclusions.—We presented the erasure scheme for effectively creating a von Neumann measurement Hamiltonian for a nonlocal observable. It is based on the fact that it is possible to perform a von Neumann measurement by making a strong measurement and erasing the result (Proposition 1).

The scheme has a number of advantages over known schemes for instantaneous nonlocal measurements. It can be used in the strong, intermediate, and weak regimes; so far this range was only possible for the special case of sum observables [25]. It is also versatile in terms of the types of observables that can be measured; other schemes such as the modular measurement [2] or the Kedem-Vaidman [35] and Resch-Steinberg (RS) [34] schemes can only be used for specific subsets of nonlocal observables and cannot be further generalized [15]. Another advantage is that it can be used for a much wider class of observables than those presented here, for example, multipartite observables and non-Hermitian operators [52].

Causality constraints imply a probabilistic scheme. However, with clever postprocessing and correction techniques it can be used to get a result on every possible run. In some cases, causality constraints rule out the possibility that the outcome would always correspond to the desired observable. “Failing” postselection would either give a result for a different operator and/or act like a known unitary in the intermediate time between the pre- and postselection. It is possible to avoid these constraints in postselected scenarios by including the probabilistic element in the postselection. The limitations of the scheme, and the possible ways to overcome them, demonstrate the subtle interplay between causality, determinism, and quantum measurement.

The scheme has many potential uses. Here, we highlighted its role in tests of quantum foundations in nonlocal scenarios. In a future publication [52] we will show that the scheme can be used for sequential experiments such as those used in tests of contextuality and Leggett-Garg inequalities. In these sequential scenarios the measurement is not instantaneous but the causality constraints are stricter since they explicitly involve communication backwards in time.

Regarding experimental realizations, the scheme is feasible in optics and other platforms such as NMR [53] and atomic spontaneous emission [54]. For a weak measurement it has a significant advantage over the RS scheme [34,55] since the resources required are linear in g as opposed to the RS scheme that scales quadratically [15]. It would be interesting to see if current methods can be used to perform the full version of these techniques including the correction and postprocessing steps. It would also be interesting to find further applications for the erasure method such as improved experimental accuracies [56]

or protective tomography [57] at the weak limit or error correction at the strong limit [58], or for generating many-body interactions under realistic constraints [15].

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