Evading the Vainshtein Mechanism with Anomalous Gravitational Wave Speed: Constraints on Modified Gravity from Binary Pulsars

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By using observations of the Hulse-Taylor pulsar, we constrain the gravitational wave (GW) speed to the level of 10^{-2}. We apply this result to scalar-tensor theories that generalize Galileon 4 and 5 models, which display anomalous propagation speed and coupling to matter for GWs. We argue that this effect survives conventional screening due to the persistence of a scalar field gradient inside virialized overdensities, which effectively "pierces" the Vainshtein screening. In specific branches of solutions, our result allows us to directly constrain the cosmological couplings in the effective field theory of dark energy formalism.

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Modifications of general relativity (GR) that explain the acceleration of the Universe can display a gravitational wave (GW) speed $c_T \neq 1$ (we use units $\hbar = c = 1$). What are the observational constraints on this parameter? In some given model, c_T can be expressed as a specific function of the (post-Newtonian) parameters of the theory and, thus, constrained indirectly with Solar System tests (see, e.g., [1]). On the other hand, cosmological observations limit c_T to the 10% level (e.g., [2]). In Ref. [3], Moore and Nelson observe that subluminal GWs would be Cherenkov-radiated by particles traveling faster than c_T. By looking at high-energy cosmic ray data, the authors manage to constrain this effect to the impressive level of 10^{-15}. We notice, however, that the typical energy of the corresponding radiated gravitons, $\sim 10^{10}$ GeV, is well above any reasonable cutoff of the modified gravity theories for cosmic acceleration. It is not difficult to envision, e.g., Goldstone modes in spontaneous Lorentz-breaking situations that are subluminal at low frequencies and recover relativistic propagation above the symmetry-breaking scale [4]. With binary pulsar timing data, in this Letter, we obtain for c_T looser limits ($\sim 10^{-2}$), which however apply to frequencies that are relevant for an effective theory of dark energy.

One obvious objection is that scalar-tensor theories generally come equipped with screening mechanisms, allowing us to recover the stringent tests of gravity in the Galaxy and in the Solar System. Among these, the *Vainshtein screening* [5] is particularly efficient and relevant for those scalar-tensor theories that display anomalous GWs speed. What screening guarantees, however, is the suppression of the contribution of the scalar field ϕ to the total gravitational attraction between bodies in the Newtonian approximation. In a screened situation, the fluctuations of the metric field—gravitons—are left as the only mediators of long-range interactions. But not necessarily do they behave as in GR. The point is that the background value of the scalar ϕ_0, although not directly participating in gravitational interactions, generally maintains a nonvanishing gradient that spontaneously breaks Lorentz symmetry. In such a situation, the effective gravitational Lagrangian need not be that of GR, even if it involves only massless gravitons. The Vainshtein screen is *pierced*.

The same mechanism is responsible for other violations of the screening considered in the literature. In simple cases, deviations from GR boil down to a spacetime variation of the Newton constant G_N. References [6,7] use lunar-laser ranging to constrain this effect, obtaining limits on modified gravity models that are comparable in size to those obtained here. Preferred-frame effects [8] and possibly anomalous values of the gravitational slip parameter $\gamma_{\rm PPN}$ [9,10] (see also the following on this) have also been discussed in the literature.

Quadrupole formula, revisited.—For the sake of generality, we will consider a twofold modification of GR encoded in the following Lagrangian for the GW sector:

$$\mathcal{L} = \frac{1}{64\pi G_{\rm gw}} \sum_{\alpha=+,\times} \left[\frac{1}{c_T^2} \dot{\gamma}_\alpha^2 - |\vec{\nabla}\gamma_\alpha|^2 \right], \qquad (1)$$

where $+$ and $\times$ represent the two polarizations of the GWs.

First, we allow for a coupling of GWs to matter, $G_{\rm gw}$, possibly different than Newton's constant G_N inferred in the Newtonian limit via the Poisson equation. Indeed, in addition to the radiating gravitons described by the above Lagrangian, we have the potential gravitons [11], responsible for the bound of the binary system. In modified gravity theories with an additional scalar degree of freedom, the scalar sector also becomes radiative. We will rely on Vainshtein screening while assuming that the contribution of the radiated scalar to the variation of the binary system period is negligible, as was shown to be the case in specific models [12].

The second modification that we consider is that GWs can propagate at a speed c_T different from the speed of light. We assume here that such a speed is constant and direction- and polarization-independent. This statement is exact in the limit of a constant gradient for the background scalar field ϕ_0 and in the reference frame where such a gradient is along the time direction. In the following, we will quantify the corrections due to the presence of a spatial component of the gradient and argue that in realistic situations such a component is negligible.

It is interesting to revisit, step by step, the standard derivation of the quadrupole formula (e.g., [13]) in light of these modifications. First, we want to estimate the energy flux of a GW across a spherical surface at large distance r from the source. The standard expression can be modified, essentially, by dimensional analysis (e.g., by rescaling the time as $\partial_t = c_T \partial_{t'}$ so that the GR formulas can be applied straightforwardly). We find

$$\frac{dE}{dt} = \frac{r^2}{32\pi c_T G_{\rm gw}} \int d\Omega \langle \partial_t \gamma_{ij} \partial_t \gamma_{ij} \rangle, \quad (2)$$

where $\langle \ldots \rangle$ means an average over a region of spacetime much larger than the GW wavelength. On the other hand, at the lowest (quadrupole) order in the velocity expansion, the radiated amplitude of GWs from a given source is obtained with the usual formulas, barring the replacement $G_N \to G_{\rm gw}$ and the different retarded time at which the source is evaluated,

$$[\gamma_{ij}]_{\rm quad} = \frac{2G_{\rm gw}}{r} \ddot{Q}_{ij}^{\rm TT}\left(t - \frac{r}{c_T}\right), \quad (3)$$

where $Q_{ij}^{\rm TT}$ is the transverse-traceless projection of the quadrupole moment Q_{ij} of the source. Note that $\ddot{Q}_{ij}^{\rm TT}$ appears in (3) after using the energy-momentum conservation of matter. Since the matter sector has the usual Lorentz symmetry, the time derivatives acting on $Q_{ij}^{\rm TT}$ do not introduce any additional factors of c_T. As in the standard calculation, the way such a projection is made depends on the direction of the GW, and this should be taken into account when calculating the surface integral (2). This results in the following total power emitted:

$$P_{\rm quad} = \frac{G_{\rm gw}}{5c_T} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle. \quad (4)$$

This expression coincides with the formula obtained in [14] for Horava gravity.

Binary pulsar constraints.—By the above modified quadrupole formula, binary pulsars observations will allow us to constrain the combination $c_T G_{\rm gw}$, modulus some assumptions on the expressions of the Keplerian parameters of the bound system that we detail in the following. The emission of GWs results in a decrease of the orbital period P_b [13]. *Mutatis mutandis*, we get

$$\dot{P}_b = -\left(\frac{G_{\rm gw}}{G_N}\frac{c}{c_T}\right)\frac{192\pi G_N^{5/3}}{5c^5}\left(\frac{P_b}{2\pi}\right)^{-5/3}(1-e^2)^{-7/2} \times \left(1 + \frac{73e^2}{24} + \frac{37e^4}{96}\right) m_p m_c (m_p + m_c)^{-1/3}, \quad (5)$$

TABLE I. Orbital parameters for PSR B1913 + 16 from Ref. [16].

Parameter	Description	Value
e	Eccentricity	0.617 133 4(5)
P_b (days)	Period	0.322 997 448 911(4)
$\dot{w}$ (deg/yr)	Periastron advance	4.226 598(5)
γ (ms)	Einstein delay	4.2992(8)
$\dot{P}_b$	Period decay	$-2.423(1) \times 10^{-12}$

where e is the eccentricity of the Keplerian orbit, m_p and m_c are the masses of the pulsar and its companion, respectively, and we have temporarily reintroduced [just here and in (6)] the dimensional speed of light c. As explained below Eq. (1), we assume potential gravitons and radiative gravitons to couple to matter with different strengths. Note the different roles in the derivation played by G_N, coming from the formula of the orbits, and $G_{\rm gw}$, coming from the actual emission of gravitational waves.

We use the most accurate available data on $\dot{P}_b$, those of the Hulse-Taylor pulsar (PSR B1913 + 16) [15], with the orbital parameters shown in Table I. [Equation (5) is calculated in the orbiting system reference frame which is accelerated with respect to the Solar System barycenter frame [17]. This effect, known as the Shklovskii effect, gives an extra $\Delta \dot{P}_{b,{\rm Gal}} = -0.027 \pm 0.005 \times 10^{-12}$ which should be subtracted.] Before using this information, we need the standard expressions for the advance of the periastron $\dot{\omega}$ and the amplitude of the Einstein delay γ [18], which also depend on the Keplerian parameters e and P_b and on the masses m_p and m_c. We can thus use the binary pulsar data to constrain the combination $c_T G_{\rm gw}$, in addition to the two masses.

While the expression of γ is derived, essentially, in the Newtonian approximation, a comment regarding the parameter $\dot{\omega}$ is in order here. In a modified gravity setup, such a quantity depends on both post-Newtonian parameters $\gamma_{\rm PPN}$ and $\beta_{\rm PPN}$ [18,19]. Since we are aiming (see below) to a precision of 10^{-2}, we rely on Solar System tests, which constrain $\gamma_{\rm PPN}$ and $\beta_{\rm PPN}$ at the levels of 10^{-5} and 10^{-3}, respectively, and $\dot{\omega}$ directly and independently, for Mercury, at the level of 10^{-3} [18,20,21].

We can now proceed to construct the mass-mass diagram and the corresponding constraints on $c_T G_{\rm gw}$ as shown in Fig. 1. We see that the binary pulsar data meet in a small region of the (m_p, m_c) plane. GR predictions fall in the intersection of $\dot{\omega}$ and γ within about 1σ confidence. By imposing compatibility of the three constraints at the 1σ level, we obtain the following bound:

$$0.995 \lesssim \frac{G_{\rm gw}}{G_N}\frac{c}{c_T} \lesssim 1.00. \quad (6)$$

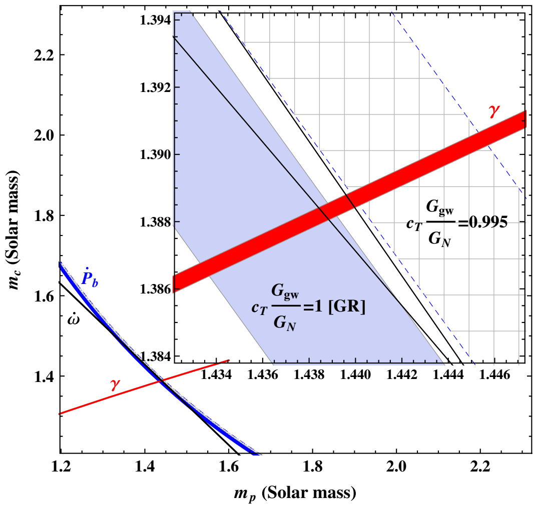

FIG. 1. Mass-mass diagram for PSR B1913 + 16 (the Hulse-Taylor pulsar) based on the post-Keplerian parameters $\dot{w}$ (black), γ (red), and $\dot{P}_b$ (blue). Varying the combination $c_T G_{\rm gw}/G_N$ amounts to shifting the 1σ stripe of $\dot{P}_b$.

Symmetries and scalar field gradients.—We now discuss the implications of the above bound on concrete scalar-tensor models for dark energy. First, it is helpful to consider the basic structure of the simplest scenario that displays an anomalous GW speed, the *quartic* Galileon model, with Lagrangian

$$\mathcal{L}_4^{\rm Gal} = -\frac{X}{\Lambda^6}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]. \tag{7}$$

In the above, $X \equiv \partial_\mu\phi\partial^\mu\phi$ and Λ is some energy scale of the order of $\Lambda \simeq (M_p H_0^2)^{1/3}$ with H_0 the Hubble parameter today. By inspection of the second term inside the square brackets, we see that the covariant derivatives generate a term quadratic in the Christoffel symbols. In the presence of a background field ϕ_0 with a nonvanishing timelike gradient, such a term contributes to the quadratic Lagrangian for the gravitons h_{ij} as $\sim\dot{h}_{ij}^2$, thus modifying the propagation speed of the GW. It is immediate to see that c_T is dependent only on the gradient of ϕ_0 in this case. Only when $\nabla_\mu\phi_0$ vanishes does c_T go to one.

But for theories enjoying *shift symmetry* $\phi \to \phi + {\rm const}$, the actual value of the scalar is irrelevant, and there is no evident mechanism for it to detach from cosmic evolution and become constant inside a virialized object. This means that we do expect, in general, a nonvanishing scalar gradient inside screened environments—a "local remnant" of the expansion of the Universe.

Indeed, the profile of a cosmologically evolving scalar field in the presence of a matter source is easily estimated for those theories that enjoy a further, Galileon, symmetry [22], which makes a constant gradient of ϕ, and not only its actual value, irrelevant. In Minkowski space, this is defined as the invariance under $\nabla_\mu\phi \to \nabla_\mu\phi + b_\mu$ with b_μ a constant vector. Let $\phi_0^{\rm cosm}(t)$ be the cosmological solution obtained under the assumption of homogeneity and isotropy. Well inside the Hubble radius, where the metric is similar to Minkowski, this is effectively a field configuration of constant gradient. Once we find a suitable radial solution $\phi_0^{\rm astro}(r)$ vanishing at infinity around some localized matter source, in virtue of Galileon symmetry, we can simply add the two solutions:

$$\phi_0(r,t) \simeq \phi_0^{\rm cosm}(t) + \phi_0^{\rm astro}(r). \tag{8}$$

Galileon theories are a combination of five Lagrangian terms with an increasing number of fields ϕ.

As a case study, let us consider the quartic Galileon (7). The cosmological gradient for this theory is given by $\dot{\phi}_0^{\rm cosm} \sim H_0 M_p \sim \Lambda^3 H_0^{-1}$. On the other hand, the analysis of the Vainshtein mechanism near a spherically symmetric object shows that the radial gradient of the scalar inside the screened region for a quartic Galileon is constant, $(\phi_0^{\rm astro})' \sim (M/M_p)^{1/3}\Lambda^2 \sim r_V\Lambda^3$, where we have introduced the Vainshtein radius $r_V \simeq (M/M_p)^{1/3}\Lambda^{-1}$ and M is the mass of the matter source. In summary,

$$\frac{\phi_0'}{\dot{\phi}_0} \sim \frac{r_V}{H_0^{-1}}, \tag{9}$$

which shows that a localized source contributes a very mild radial component to the total gradient of the field. For example, the Sun has $r_V \sim 1$ kpc, so this ratio is of the order of $\sim 10^{-6}$. In comparison, our peculiar velocity with respect to the cosmic microwave background (CMB) gives a much larger effect ($\sim 10^{-3}$). Our estimates are in agreement with the explicit numerical calculations of Ref. [23].

Gravity inevitably breaks the symmetry $\nabla_\mu\phi \to \nabla_\mu\phi + b_\mu$, if anything, because there is no such thing as a constant vector b_μ in a general spacetime. However, we can apply the above estimates to all scalar-tensor theories that reduce to Galileon in the decoupling limit, formally defined as $M_P \to \infty$ while keeping Λ constant. Among these, theories with weakly broken Galileon symmetry [24] have their Lagrangians protected against quantum corrections.

Cosmological EFT operators.—We have just shown that $\phi_0' \ll \dot{\phi}_0$ (even) inside the Vainshtein radius, where the nonlinearities in the scalar can become important but the metric is very close to Minkowski. The most general quadratic Lagrangian for the metric fluctuations in the presence of a background scalar field of constant timelike gradient is conveniently studied within the effective field theory (EFT) formalism for cosmological perturbations [25–27]. By choosing the time coordinate to be proportional to the scalar field (unitary gauge), all degrees of freedom are transferred to the metric, chosen to be the one minimally coupled to matter (*Jordan frame*). A limited number of operators capture the linear dynamics of the most general scalar-tensor theory with an equation of motion of at most second order for the propagating scalar fluctuation [26]. Among such operators, only three affect the pure graviton sector:

$$\mathcal{L} \supset \frac{M^2}{2}[R + \epsilon_4(\delta K^{ij}\delta K_{ij} - \delta K^2) - \tilde{\epsilon}_4{}^{(3)}R\delta N], \quad (10)$$

where R is the Ricci scalar, δK^{ij} the perturbation of the extrinsic curvature K^{ij} of the $t = \text{const}$ hypersurfaces, ${}^{(3)}R$ their Ricci scalar, and δN the perturbation of the lapse function. M, ϵ_4, and $\tilde{\epsilon}_4$ are time-dependent coefficients. In GR, $M = \text{const}$, $\epsilon_4 = \tilde{\epsilon}_4 = 0$. The above operators arise, e.g., in the class of models introduced in Ref. [28] as a generalization of the *Horndeski theory*, which is the most general scalar-tensor theory with equations of motion of at most second order [29,30]. We refer the reader to Ref. [31] for the expressions of ϵ_4 and $\tilde{\epsilon}_4$ as functions of the full beyond-Horndeski Lagrangians.

To study the effects of the terms (10), it is convenient to switch to the Newtonian gauge on a Minkowski background. By forcing a time-diffeomorphism $t \to t + \pi$, the fluctuations of the scalar field π reappear in the action, after which we can fix the metric to have the form

$$ds^2 = -(1 + 2\Phi)dt^2 + [(1 - 2\Psi)\delta_{ij} + \gamma_{ij}]dx^i dx^j, \quad (11)$$

where Φ and Ψ are the two Newtonian potentials and γ_{ij} represents the transverse-traceless graviton. At highest order in derivatives, the quadratic Lagrangian reads [31]

$$\begin{aligned}\mathcal{L} = \frac{1}{2}g^{\mu\nu}T_{\mu\nu} + M^2\bigg\{&\frac{1}{4c_T^2}[\dot{\gamma}_{ij}^2 - c_T^2(\vec{\nabla}\gamma_{ij})^2] \\ &- 3c_T^{-2}\dot{\Psi}^2 + (\vec{\nabla}\Psi)^2 - 2c_T^{-2}(1 + \alpha_H)\vec{\nabla}\Phi\vec{\nabla}\Psi \\ &+ c_1\dot{\pi}^2 - c_2(\vec{\nabla}\pi)^2 + \text{mixing terms}\bigg\}, \end{aligned} \quad (12)$$

where we have defined the GW speed $c_T^2 = (1 + \epsilon_4)^{-1}$ and the beyond-Horndeski parameter $\alpha_H = \tilde{\epsilon}_4 - \epsilon_4$. In the Jordan frame there is no direct coupling of π to the matter fields, but the scalar-metric mixing terms schematically indicated in (12), of the type $\vec{\nabla}\Phi\vec{\nabla}\pi$, $\vec{\nabla}\Psi\vec{\nabla}\pi$, and $\dot{\Psi}\dot{\pi}$. When $\alpha_H \neq 0$, the higher derivative term $\vec{\nabla}\Psi\vec{\nabla}\dot{\pi}$ also appears [26]. The explicit form of the last line of (12) depend on all the operators of the EFT—i.e., also on those omitted in (10)—as well as on the time derivatives of ϵ_4 and $\tilde{\epsilon}_4$ and is responsible for the rich linear phenomenology of dark energy, in which the π fluctuations play a dominant role [32–34].

However, in the vicinity of a localized matter source, the π fluctuations become irrelevant because of the screening, and we can thus forget about the third line of (12). As long as the on-shell gravitons γ_{ij} and the Newtonian potentials Φ and Ψ can be considered as short-wavelength fluctuations on top of a constant background scalar field gradient, the first two lines of (12) can be borrowed from cosmology and applied to general setups. This is the case for GWs of wavelengths much shorter than the distance from the source. From the first line of (12) we can read off $G_{\text{gw}} = c_T^2/(8\pi M^2)$. For a given (shift-symmetric) theory, the cosmological value of c_T (equivalently, of the EFT parameter ϵ_4) can be calculated as a function of $X = -\dot{\phi}_0^2$ [26,31]. If such a gradient acquires a spatial component ϕ'—either along the radius from a matter source or in the direction of our motion with respect to the CMB frame—c_T simply transforms as a velocity under a boost of speed $v = \phi'/\dot{\phi}$ and becomes direction dependent. Along the two principal directions, the boosted velocity reads

$$c_T^{\text{astro}} = \frac{c_T(X) \pm v}{1 \pm c_T(X)v}. \quad (13)$$

We are left with the second line of (12), which can be used to describe the dynamics of the scalar potential gravitons in the Newtonian approximation. However, its applicability to general screened situations is more subtle. Since the Newtonian potentials and the background field ϕ_0 are generated by the same source, they are of the same typical wavelengths, and the constant gradient approximation for ϕ_0 is not guaranteed to work. By substituting $g^{\mu\nu}T_{\mu\nu} \simeq -2\Phi\rho_m$, one would obtain the relation between the two Newtonian potentials $\gamma_{\text{PPN}} \equiv \Psi/\Phi$ and the Newton constant by the Poisson equation:

$$\gamma_{\text{PPN}} = \frac{1 + \alpha_H}{c_T^2}, \qquad G_N = \frac{c_T^4}{8\pi M^2(1 + \alpha_H)^2}. \quad (14)$$

The study of spherically symmetric configurations in the full beyond-Horndeski models confirms that the above always correspond to one available branch of solutions [9,10]. Theories with terms up to $(\nabla^2\phi)^2$ (type 4) show a total of three branches, in two of which the GR result $\gamma_{\text{PPN}} \simeq 1$ is recovered inside the Vainshtein radius. However, for beyond Horndeski of type 5 [terms up to $(\nabla^2\phi)^3$], there appears to be now a way to recover the GR value in any of available branches. We would like to emphasize that the branch corresponding to (14), always present, also develops nonlinearities inside the Vainshtein radius. A closer inspection of the solutions in Ref. [9] shows, however, that for this branch the relevant nonlinearities are in the mixed $\pi - \Phi$ and $\pi - \Psi$ sectors, and not in the self-interactions of the scalar, as it is usually assumed.

The branch recovering $\gamma_{\text{PPN}} \simeq 1$, when available, is often taken as the appropriate solution inside virialized objects [35,36], also because it matches the asymptotically flat solutions in some specific cases [37]. However, which branch applies to realistic scenarios is ultimately selected by the time evolution. The point is to understand, case by case, which solution continuously evolves from the (unique) linear configuration describing a tiny perturbation in the early Universe, and this will depend, in general, on the details of the theory.

Observational constraints and discussion.—Within the "linear branch" of solutions (14), the cosmological EFT

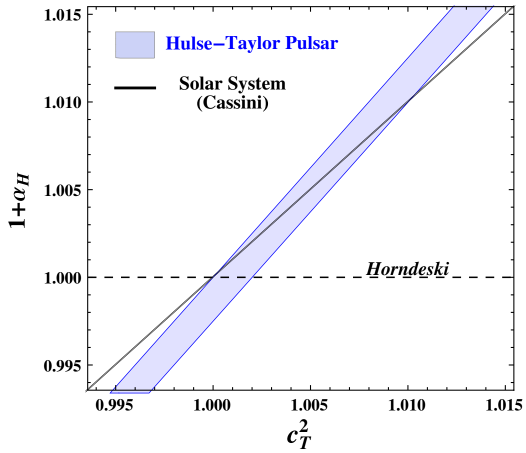

FIG. 2. Combined constraints in the (c_T^2, α_H) parameter space for the linear branch of solutions (14). The (tight) bound is the Cassini measurement [20] (black curve) and the light-blue stripe corresponding to the Hulse-Taylor pulsar bound obtained from the top panel. Within Horndeski theories, because $G_N = c_T^2 G_{\rm gw}$, the bound (6) turns into a slight preference for superluminal propagation.

parameters ϵ_4 and α_H are tightly constrained. First, the bound (6) turns into a constraint for the combination of parameters $(1+\alpha_H)^2/c_T$. At the same time, as already noted, e.g., in Ref. [9], the value of the slip parameter in the linear branch (14) is powerfully constrained by the Cassini spacecraft experiment [20]: $\gamma_{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$. This combines with our binary pulsar result as in Fig. 2.

We would like to stress, however, that beyond the details related to the specific branch of solutions, the bound on the GW speed is very general: Hulse-Taylor pulsar observations constrains c_T at the level of 10^{-2}, barring remarkable and unlikely cancellations with the (linear and nonlinear) physics that determines the orbits of the bound system. Our result applies to all dark energy models in which gravity is modified enough to display a different speed for GWs. Within scalar-tensor theories, in particular, we have considered Galileon 4 and 5 type models and its generalizations and argued that the effect is not screened in general, because it is related to the persistence of the (cosmological) scalar field gradient even inside conventionally Vainshtein-screened regions.

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