Tunable Polarons of Slow-Light Polaritons in a Two-Dimensional Bose-Einstein Condensate

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When an impurity interacts with a bath of phonons it forms a polaron. For increasing interaction strengths the mass of the polaron increases and it can become self-trapped. For impurity atoms inside an atomic Bose-Einstein condensate (BEC) the nature of this transition is not understood. While Feynman's variational approach to the Fröhlich model predicts a sharp transition for light impurities, renormalization group studies always predict an extended intermediate-coupling region characterized by large phonon correlations. To investigate this intricate regime and to test polaron physics beyond the validity of the Fröhlich model we suggest a versatile experimental setup that allows us to tune both the mass of the impurity and its interactions with the BEC. The impurity is realized as a dark-state polariton (DSP) inside a quasi-two-dimensional BEC. We show that its interactions with the Bogoliubov phonons lead to photonic polarons, described by the Bogoliubov-Fröhlich Hamiltonian, and make theoretical predictions using an extension of a recently introduced renormalization group approach to Fröhlich polarons.

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When a mobile impurity interacts with an atomic Bose-Einstein condensate (BEC) it forms a polaron [1–3]. These quasiparticles were first introduced by Landau and Pekar [4,5] when they studied the electron-phonon interaction in polarizable crystals on the basis of the Fröhlich Hamiltonian. One of the key predictions was the possibility of self-trapping of the impurity in its surrounding phonon cloud. For self-trapped impurities, the fluctuations $\langle \boldsymbol{r}^2 \rangle$ of the impurity position are strongly suppressed. In addition, the polaron mass—a quantity of central interest in this paper—becomes large. The Fröhlich Hamiltonian also provides a good description of an impurity interacting with a condensate, when phonon-phonon scattering is negligible. Using Feynman's variational approach to the Fröhlich Hamiltonian [6], it was predicted more recently that self-trapping can also take place for impurities in a BEC [3]. However, the nature of the self-trapping in this system is the subject of an ongoing debate [7].

For sufficiently light impurities, Feynman's variational approach to the Fröhlich Hamiltonian predicts a sharp self-trapping transition in three dimensions [3,8], indicated by a nonanalyticity of the polaron mass. Using more sophisticated theoretical methods it has recently been claimed that, rather than undergoing a sharp transition, the polaron mass depends analytically on the coupling strength and there exists an extended regime of intermediate couplings before the impurity becomes self-trapped [9–11]. In this peculiar regime, phonons become correlated due to phonon-phonon interactions mediated by the impurity. Their strength is determined not only by the impurity-phonon coupling constant α but also by the inverse impurity mass M^{-1}. We show in Fig. 1 that the same is true for a quasi-two-dimensional BEC, where Feynman's approach predicts a sharp transition for ratios of impurity to host-atom mass M/m less than 0.01. Renormalization group (RG) calculations [9,11,12] in contrast always predict a smooth crossover.

At present, only little is known about the polaron at intermediate couplings, also because it has not yet been realized experimentally. Understanding this regime, dominated by quantum fluctuations, is of fundamental interest and may lead to applications in material science. For example, polaronic effects may be important in the

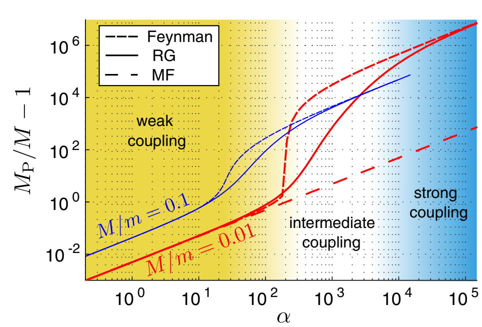

FIG. 1. Ratio of the polaron mass M_p to the bare impurity mass M as a function of the dimensionless coupling constant α in a quasi-two-dimensional BEC for different ratios of impurity to host-atom mass M/m. Feynman's approach predicts a sharp transition for $M/m \lesssim 0.01$, in contrast to predictions from mean-field (MF) theory and an extended renormalization group (RG) approach introduced in Ref. [12].

high-T_c cuprate superconductors [13], and intermediate coupling physics may play a role here.

Furthermore, when phonon-phonon interactions become relevant, the Fröhlich model is no longer sufficient to describe the physics of a mobile impurity in a BEC. In this regime the condensate wave function is locally deformed and a bubble polaron can form, where the impurity is self-trapped in a comoving mean-field potential. Although there has been some theoretical work based on mean-field approximations [14], Monte Carlo studies [15], self-consistent T-matrix calculations [16], variational studies [17,18], and perturbative analysis [19], a complete understanding of this regime is lacking.

Here, we propose a versatile experimental setup for studying polarons in a BEC at intermediate couplings for small impurity masses. The impurity is realized by coupling the condensate to a quantized mode of the electromagnetic field in a slow-light (or electromagnetically induced transparency, EIT [20]) configuration, see Fig. 2(a). Here, the impurity is a dark-state polariton (DSP) [21,22] with an effective mass M that can be varied by the control laser. We show that this tuning knob can be used to study the transition all the way from weak, through intermediate, to strong couplings. Absorption spectroscopy allows us to directly measure the full spectral function $I(\omega)$ of the polaron, from which most of its characteristics can be obtained [16,23,24]. Although we concentrate here on polarons described within the Fröhlich model, the proposed experimental setup is also well suited to study the impurity-BEC interaction when there is a sizable condensate depletion. As argued in [14], self-trapping of the impurity in the effective mean-field potential of the deformed condensate is particularly pronounced if the mass ratio of impurity to condensate atoms becomes small, a regime easily accessible with our scheme. Likewise, it was shown in [19] that the corrections to polaron energy, quasiparticle weight, and polaron mass becomes larger for smaller mass ratios. Furthermore, three-body Efimov physics not accounted for in the Fröhlich model may become relevant here [18].

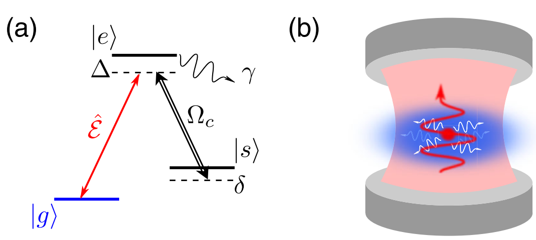

FIG. 2. Setup for realizing tunable Fröhlich polarons of photons in a BEC: A quasi-two-dimensional BEC of ground state atoms (b) coupled to lasers in a Λ scheme (a). By exciting the driven atoms using a probe field $\hat{\mathcal{E}}$, a mobile impurity (a long-lived DSP) can be created. Its interactions with the Bogoliubov phonons lead to polaron formation. The mass of the impurity, as well as the polaronic coupling constant, can be tuned by changing the Rabi frequency Ω_c of the control laser.

We study a system where cavity photons are coupled to a BEC, and the quantum nature of the light field becomes important. More generally, systems where quantum properties both of the light and the atoms becomes important have been studied before [25,26].

System.—We consider ultracold atoms with two internal metastable states $|g\rangle$ and $|s\rangle$. They are coupled by a two-photon optical transition through a short-lived excited state $|e\rangle$ (decay rate γ), see Fig. 2(a). When the two-photon detuning δ is within the EIT linewidth, the nondecaying eigenmodes of this system are DSPs [21,22], propagating with a group velocity v_g much smaller than the vacuum speed of light c_0 [21,27].

We assume that the atoms form a BEC in the internal ground state. Although $v_g \ll c_0$ can become as small as a few meters per second, it is much larger than the speed of sound c of Bogoliubov excitations in the BEC (c is of the order of a few mm/s). To avoid the emission of Cherenkov radiation, we thus confine the longitudinal motion of DSPs to a single longitudinal cavity mode with wave number k_0, see Fig. 2(b). To minimize interaction-induced losses caused by scattering into excited motional states of atoms, we furthermore introduce a strong longitudinal confinement for the atoms, leading to a quasi-two-dimensional (2D) BEC [28].

Now we describe how the DSPs interact with Bogoliubov phonons. Details are presented in the Supplemental Material [29]. The microscopic Hamiltonian $\hat{\mathcal{H}}$ contains the matter fields $\hat{\psi}_\mu(\boldsymbol{r})$, where $\mu = g, s, e$ denotes the internal states and $\boldsymbol{r}$ is the transverse coordinate. The internal states $|g\rangle$ and $|e\rangle$ are coupled by a quantized cavity field $\hat{\mathcal{E}}(\boldsymbol{r})$, normalized such that $\hat{\mathcal{E}}^\dagger\hat{\mathcal{E}}$ is a 2D number density. g_{2D} denotes the vacuum Rabi frequency on the $|g\rangle - |e\rangle$ transition, which is reduced by a Franck-Condon overlap due to the 2D confinement of the atoms (see Supplemental Material [29] for details). The transition between $|e\rangle$ and $|s\rangle$ is driven by a control field of Rabi-frequency Ω_c.

For two-photon resonance, the DSP is given by

$$\hat{\Psi}(\boldsymbol{r}) = \sin\theta\hat{\psi}_s(\boldsymbol{r}) - \cos\theta\hat{\mathcal{E}}(\boldsymbol{r}). \quad (1)$$

Up to nonadiabatic corrections, the DSP is decoupled from the bright-state polariton $\hat{\Phi}(\boldsymbol{r}) = \cos\theta\hat{\psi}_s(\boldsymbol{r}) + \sin\theta\hat{\mathcal{E}}(\boldsymbol{r})$ which is subject to losses. Here, $\tan\theta = g_{2D}\sqrt{n_0}/|\Omega_c|$, with $n_0 = N_0/L^2$ denoting the 2D BEC density, L being the linear system size, and N_0 the number of atoms in the condensate.

We assume that atoms in internal states μ and ν interact via contact interactions with strengths $g_{\mu\nu}^{2D}$, tunable by Feshbach resonances [34]. For our purposes it is sufficient to use the relations to the scattering lengths $a_{\mu\nu}$ (in three dimensions) valid in the infrared limit, $g_{\mu\nu}^{2D} = \sqrt{8\pi}a_{\mu\nu}/\ell_z$, see Refs. [9,28]. Here, $\ell_z \gg a_{gg}$ is the extent of the quasi-two-dimensional BEC in the strongly confined region.

Using Bogoliubov theory the elementary excitations are modeled by phonons $\hat{a}_{\boldsymbol{k}}$. The atomic scattering as well as the atom-light interactions give rise to couplings of the DSP to Bogoliubov phonons. We find the corresponding Hamiltonian to be of the Fröhlich [35] type (see the Supplemental Material [29]), which forms the basis of all of the following theoretical investigations ($\hbar = 1$):

$$\hat{\mathcal{H}}_F = \int d^2\boldsymbol{k}\left\{\omega_k \hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}} + \hat{\Psi}^\dagger_{\boldsymbol{k}}\left[\frac{\boldsymbol{k}^2}{2M} + \mu - i\kappa\cos^2\theta\right]\hat{\Psi}_{\boldsymbol{k}}\right\} + \int d^2\boldsymbol{r}\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\int d^2\boldsymbol{k} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} V_k(\hat{a}_{\boldsymbol{k}} + \hat{a}^\dagger_{-\boldsymbol{k}}). \quad (2)$$

Here, nonadiabatic couplings to the bright-state polariton $\hat{\Phi}$ and the excited state $\hat{\psi}_e$ were neglected, but they are derived in the Supplemental Material [29]. The first term in Eq. (2) describes free phonons, where the Bogoliubov dispersion is given by $\omega_k = ck\sqrt{1 + k^2\xi^2/2}$. $\xi = (2mg^{2D}_{gg}n_0)^{-1/2}$ is the healing length. The speed of sound reads $c = (g^{2D}_{gg}n_0/m)^{1/2}$. The second term in Eq. (2) corresponds to the dispersion relation of a free DSP. κ is the cavity linewidth and the transverse mass M of the DSP is determined by

$$M^{-1} = \cos^2\theta M^{-1}_{\mathrm{ph}} + \sin^2\theta m^{-1}. \quad (3)$$

Here $M_{\mathrm{ph}} = k_0/c_0$ is the transverse mass of cavity photons. The chemical potential μ is derived in the Supplemental Material [29]. The last term in Eq. (2) describes the impurity-phonon interaction

$$V_k = g_{\mathrm{eff}}\frac{\sqrt{n_0}}{2\pi}\left(\frac{k^2\xi^2}{2 + k^2\xi^2}\right)^{1/4}, \qquad g_{\mathrm{eff}} = \sin^2\theta g^{2D}_{gs}. \quad (4)$$

The Bogoliubov-Fröhlich Hamiltonian (2) is characterized by two dimensionless numbers [11]

$$\alpha = \frac{g^2_{\mathrm{eff}}n_0}{\pi c^2} \quad \text{and} \quad \frac{m}{M}, \quad (5)$$

quantifying the impurity-phonon interaction and the mass ratio of the bosons in the condensate and the impurity, respectively. For realistic parameters [27,36] we estimate $m/M_{\mathrm{ph}} \approx 10^{11}$. By changing θ, the mass ratio $m/M \approx \cos^2\theta m/M_{\mathrm{ph}}$ can be tuned over a wide range. Typically, the impurity is much lighter than the underlying bosons, due to its photonic component, but in the ultra-slow-light regime mass ratios on the order of unity should be accessible.

Phase diagram.—The following discussion of the phase diagram is based on an extension of the renormalization group (RG) approach to Fröhlich polarons introduced in Refs. [9,11]. The key idea behind the earlier RG scheme is to decouple fast and slow phonon degrees of freedom perturbatively in every momentum shell. In Ref. [12] we extended this approach by performing a global mean-field (MF) shift after every RG step, corresponding to an inclusion of infinitely many diagrams. The extended method is not only more accurate for strong couplings, but it is also necessary to calculate the effective polaron mass in a regime where the impurity is light [12].

In Fig. 3 we present the full phase diagram of the 2D Bogoliubov-Fröhlich polaron (details being discussed below). We distinguish three different regimes of weak coupling (where Lee-Low-Pines MF theory [37] applies), strong coupling (where Landau and Pekar's strong coupling approximation applies [4,5]), and intermediate coupling (where neither of the two approaches is accurate). They are connected by smooth crossovers. It can be shown analytically (see Ref. [12]) that MF theory is not only asymptotically exact in the commonly discussed limits $\alpha \to 0$ and $M \to \infty$, but also in the limit where $M \to 0$.

In the case of BEC polarons realized with bare atoms, different polaron regimes can be accessed only by tuning α, while the mass ratio is fixed around a value between ~0.1 to ~10. For the photonic setup, on the other hand, the impurity mass M can be used as a tuning parameter. In particular, extremely light impurities can easily be created and regimes of the phase diagram inaccessible to bare atoms can be addressed. This versatility makes the photonic setup superior to purely atomic systems, for the investigation of the transition from weak through intermediate couplings.

Now we turn to a more detailed discussion of how the phase diagram in Fig. 3 was obtained from the extended RG approach [12]. We expect that in the suggested experimental setup our calculations can be put to a test. In Fig. 1 we show an example of how the effective polaron

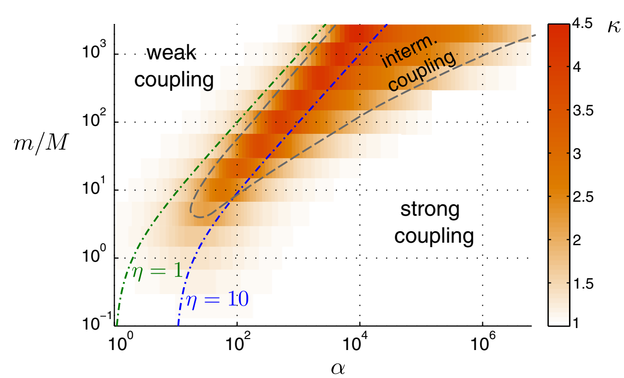

FIG. 3. Full phase diagram of the two-dimension Bogoliubov-Fröhlich model: For sufficiently light impurities and large enough α, an extended regime of intermediate couplings is found (as a guide to the eye, the dashed line indicates where the crossover takes place). The color plot shows the exponent κ of the power law $M_p/M - 1 \sim \alpha^\kappa$, determined from the slope of curves as in Fig. 1. Parameters used in the RG simulations were $\Lambda_0 = 2000/\xi$ and $P = 0.01Mc$ (for their definition, see Ref. [12]). We also plotted the maximum values α_{max} below which the Fröhlich model is valid, for $\eta = 1$, 10 as defined in the text (dash-dotted lines).

mass depends on α, for a light impurity ($M/m = 0.01$). As found previously using the perturbative RG [9,10], a smooth crossover takes place from a quasifree polaron to a self-trapped polaron. For small couplings, M_p increases linearly with α according to the MF prediction and crosses over into the intermediate coupling regime with a nonlinear growth, $M_p(\alpha)/M - 1 \sim \alpha^\kappa$. Eventually the strong-coupling regime is entered where M_p increases linearly with α again, but with a different slope. We determined the exponent $\kappa(\alpha)$ (defined as the slope of the double-logarithmic curves in Fig. 1) to distinguish the different regimes in the phase diagram in Fig. 3.

Experimental considerations.—DSPs in ultracold BECs have been observed experimentally in the slow-light limit [27,38]. By performing similar experiments in quasi-two-dimensional BECs [28] with light confined to a cavity, see Fig. 2, photonic Fröhlich polarons can be realized. By varying the intensity of the control laser Ω_c, the effective mass of the DSP can be tuned, and using Feshbach resonances [34] the coupling strength α can be varied. This should allow us to explore the phase diagram shown in Fig. 3. Realistic experimental parameters are provided in the Supplemental Material [29].

Next, we discuss conditions when the Fröhlich Hamiltonian (2) is valid. In the derivation of the model [9,11] we neglected phonon-phonon scattering induced by the impurity, which is justified when $\epsilon := \sqrt{n_{\rm ph}/n_0} \ll 1$. Here, $n_{\rm ph}$ denotes the (real-space) phonon density [2,9] which we estimate by $n_{\rm ph} \approx N_{\rm ph}/\xi^2$. The phonon number (at zero total momentum $P = 0$) can be calculated from MF theory and the condition $\epsilon \ll 1$ becomes $\epsilon = |g_{\rm eff}|(m/\sqrt{\pi})\sqrt{(m/M + m)} \ll 1$. Demanding an upper bound $\epsilon < \epsilon_{\rm max}$ thus constrains $|g_{\rm eff}|$, which yields an upper bound for the coupling strength,

$$\alpha_{\rm max} = \frac{\epsilon_{\rm max}^2}{g_{\rm gg}^{2D}}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{\epsilon_{\rm max}^2}{\sqrt{8\pi}}\frac{\ell_z}{a_{gg}}\left(1 + \frac{m}{M}\right). \tag{6}$$

To estimate which range of parameters in the Fröhlich polaron phase diagram can be accessed in an experiment, we plotted $\alpha_{\rm max}$ for $\eta = (\epsilon_{\rm max}^2/\sqrt{8\pi})(\ell_z/a_{gg}) = 1$ and 10 in Fig. 3. One recognizes that the validity of the Fröhlich model for BECs extends into the intermediate coupling regime, while going to strong coupling may require us to go beyond the Fröhlich model. Ultimately, experiments need to clarify how the system behaves at intermediate couplings, and we believe that the proposed setup is well suited to explore this.

Experimental signatures.—We proceed by discussing possible signatures of polaron formation. In order to create a DSP polaron, one can envision first storing a weak probe pulse, ideally containing a single or a few photons, in the BEC using the storage protocol of [39,40] and subsequently restoring an intracavity DSP with a small photonic component, i.e., $0 < \cos^2\theta \ll 1$. Most strikingly, the effective mass of the polaron M_p significantly increases as compared to the bare mass M, see Fig. 1. One way to measure this effect is to observe dipole oscillations [41] of a polaron wave packet inside a harmonic potential $M\omega_0^2\boldsymbol{r}^2/2$ seen by the DSP. The weak harmonic confinement with an oscillator length $\ell = (M_p\omega_0)^{-1/2} \gg \xi$ can easily be implemented using spherical cavity mirrors.

A more powerful method for analyzing photonic polarons is absorption spectroscopy upon driving the cavity by an external laser at a frequency ω and with the momentum $\boldsymbol{P}$, i.e., with an amplitude $\mathcal{E} \sim \mathcal{E}_0 e^{i\boldsymbol{P}\cdot\boldsymbol{r} - i\omega t}$. The absorption rate Γ of photons from the laser is given by the spectral function $I(\omega, \boldsymbol{P})$, $\Gamma(\omega, \boldsymbol{P}) \sim I(\omega, \boldsymbol{P})$, in complete analogy to the radio-frequency spectroscopy discussed, e.g., in Ref. [24].

The momentum-resolved spectral function of the photonic polaron has a characteristic delta-function peak $I_{\rm coh}(\omega, \boldsymbol{P}) = Z\delta(\omega - E_0(\boldsymbol{P}))$, which is located at the polaron energy $\omega = E_0(\boldsymbol{P})$. By measuring the momentum dependence of the polaron energy (around $P = 0$) the polaron mass can be obtained. Using the sum rule $\int d\omega I(\omega, \boldsymbol{P}) = 1$, also the quasiparticle residue Z can be obtained from the spectral function.

Another option is to investigate the Bose system directly and observe the polaron's phonon cloud. This can be done, for example, by measuring correlations in time-of-flight experiments [10]. Using Bragg spectroscopy, the spatial structures of the polaron cloud could be studied.

Summary.—In this article we suggested a realistic experimental setup for exploring the polaron formation of mobile impurities inside a BEC. By coupling the atoms to lasers in a slow-light setting, we showed that DSP impurities with a tunable mass can be realized. Their interaction with the Bogoliubov phonons of a BEC can be modeled by a Fröhlich Hamiltonian. One of the main motivations to study this system is to explore the self-trapping transition experimentally, with the impurity mass serving as a flexible tuning parameter. The physics of this transition, dominated by phonon correlations, is poorly understood. The theoretical analysis presented suggests a smooth crossover rather than a sharp phase transition as may be expected from Feynman's variational approach [3]. Ultimately, experiments are needed to clarify how the polaron becomes self-trapped.

The suggested setup furthermore raises new questions, including how the polaron properties change in a regime where the Fröhlich Hamiltonian is no longer valid [15,16,19]. There, the formation of bubble polarons [14] as well as interesting few-body physics [17,18] may be expected. Although the validity of the theoretical analysis presented here is questionable in this parameter range, the ability to tune both the coupling strength α and the impurity mass M at will makes our system appealing to the search for new physical effects.

Finally, in solid-state systems polarons have almost exclusively been studied under equilibrium conditions.

Ultracold quantum gases provide long coherence times and allow us to study dynamical effects. This includes the possibility to measure the full spectral function [23,24], which is possible in our system using absorption spectroscopy. Also, the dynamics of polaron formation can be studied in real time in the suggested experiments. The use of photons coupled to short-lived atomic states, moreover, opens the possibility of studying polarons in driven-dissipative systems far from equilibrium.

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