

## Top Quark Pair Production in Association with a Jet with Next-to-Leading-Order QCD Off-Shell Effects at the Large Hadron Collider

G. Bevilacqua,<sup>1</sup> H. B. Hartanto,<sup>2</sup> M. Kraus,<sup>2</sup> and M. Worek<sup>2</sup>

<sup>1</sup>*INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy*

<sup>2</sup>*Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*

(Received 2 October 2015; revised manuscript received 1 December 2015; published 5 February 2016)

We present a complete description of top quark pair production in association with a jet in the dilepton channel. Our calculation is accurate to next-to-leading order (NLO) in QCD and includes all nonresonant diagrams, interferences, and off-shell effects of the top quark. Moreover, nonresonant and off-shell effects due to the finite  $W$  gauge boson width are taken into account. This calculation constitutes the first fully realistic NLO computation for top quark pair production with a final state jet in hadronic collisions. Numerical results for differential distributions as well as total cross sections are presented for the Large Hadron Collider at 8 TeV. With our inclusive cuts, NLO predictions reduce the unphysical scale dependence by more than a factor of 3 and lower the total rate by about 13% compared to leading-order QCD predictions. In addition, the size of the top quark off-shell effects is estimated to be below 2%.

DOI: 10.1103/PhysRevLett.116.052003

*Introduction.*—Top quark studies are currently driven by the LHC experiments. An exploration of top quark production and decay dynamics is among the main physics goals of ATLAS and CMS. Besides the determination of the top quark mass, key measurements at the LHC include the total cross section, kinematic distributions, spin correlations, and top quark couplings to the  $W$  and  $Z$  bosons, a photon, and the standard model (SM) Higgs boson. Searches for rare top quark decays to probe physics beyond the SM also play a prominent role in research programs of both experimental collaborations. The top quark, however, is an extremely short-lived resonance and only its decay products can be detected experimentally. In general, for comparison with data, theoretical predictions must include top quark decays. In the SM, a top quark decays almost exclusively to a  $W$  boson and a  $b$  quark. The experimentally cleanest top quark decay channel comprises leptonic  $W$  gauge boson decays. The signature for this channel consists of two well isolated and oppositely charged leptons with high transverse momenta,  $p_T$ , large missing  $p_T$  from invisible neutrinos, and two jets, which originate from  $b$  quarks. Because of the large collision energy at the LHC,  $t\bar{t}$  pairs are abundantly produced with a large  $p_T$ ; hence, the probability of the top quark radiating gluons is enough to make the  $t\bar{t}j$  final state measurable with high statistics. In fact, for  $p_{Tj} > 40$  GeV, about half of the  $t\bar{t}$  events are expected to be accompanied by an additional hard jet. The correct description of  $t\bar{t}j$  production is, therefore, essential for studying the top quark pair production at the LHC. For example,  $t\bar{t}j$  can be employed in the measurement of the top quark mass by studying the normalized differential distribution cross section with respect to its invariant mass [1]. Moreover,  $t\bar{t}j$  constitutes an important background to processes with multijet final states, with the most prominent being the SM Higgs boson production in the

vector boson fusion with the following decay chain:  $H \rightarrow W^+W^- \rightarrow \ell^+\nu_\ell\ell^-\bar{\nu}_\ell$  [2,3]. The  $t\bar{t}j$  production plays a very important role in searches for physics beyond the SM. For example, it is one of the main backgrounds to processes such as supersymmetric particle production. Here, depending on the specific model, typical signals also include jets, charged leptons, and missing  $p_T$  due to the escaping lightest supersymmetric particle [4]. Anomalous production of additional jets accompanying a  $t\bar{t}$  pair could also be a sign of new physics beyond the SM [5].

The next-to-leading-order (NLO) corrections to  $t\bar{t}j$  with stable top quarks were first calculated in Refs. [6,7]. Afterwards, leading-order (LO) top quark decays in the narrow width approximation (NWA) have been included [8]. Subsequently, NLO top quark decays in the NWA, including  $t \rightarrow Wbj$ , have been added consistently [9]. A different path was taken in Refs. [10–12], where a matching to parton shower programs was worked out. In this case, however, only NLO corrections to the  $t\bar{t}j$  production with stable top quarks were taken into account, while top quark decays, if included, were modeled within parton shower frameworks.

In this Letter, we present a different approach. We drop altogether the approximation that top quarks are only produced on shell, and we concentrate on the fully realistic final state  $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$ . We consistently take into account resonant and nonresonant top quark contributions and all interference effects among them. In addition, nonresonant and off-shell effects due to the finite  $W$  gauge boson width are included. Because of its insignificance, we neglect flavor mixing as well as contributions from the suppressed initial bottom quark contributions. A few examples of Feynman diagrams contributing to the leading-order process at  $O(\alpha_s^3\alpha^4)$  are presented in Fig. 1. We stress here that contributions of the order  $O(\alpha_s\alpha^6)$  have not been included in our calculations. Full off-shell top quark effects

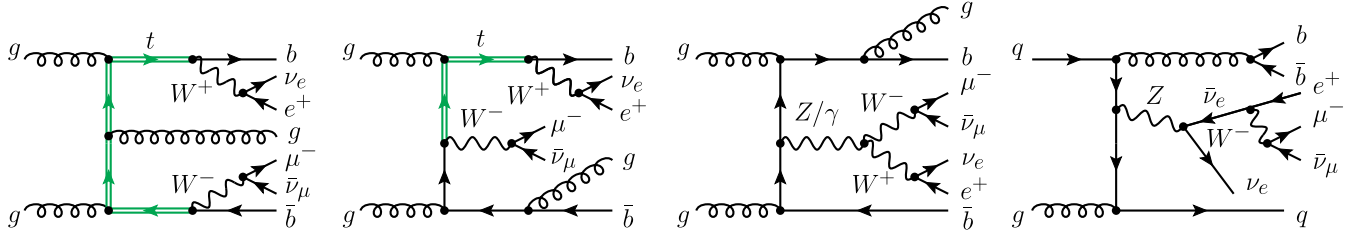


FIG. 1. Representative Feynman diagrams, involving two (first diagram), one (second diagram), and no top quark resonances (third diagram), contributing to the leading-order  $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j$  process at  $O(\alpha_s^3 \alpha^4)$ . The last diagram with a single  $W$  boson resonance contributes to the off-shell effects of the  $W$  gauge boson.

at NLO have already been considered in the literature for a simpler process, i.e., top quark pair production, first in Refs. [13,14], and subsequently in Refs. [15–18]. Quite recently, a first attempt to go beyond the NWA for  $2 \rightarrow 5$  processes was undertaken in Ref. [19], where NLO corrections to  $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} H$  were considered.

*Calculation.*—NLO QCD corrections to  $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j$  have been calculated with the `Helac-Nlo` Monte Carlo program [20]. This is the first such computation with five final states (the decay products of the  $W$ 's are not counted because they do not couple to color charged states) carried out within this framework. We compute the virtual corrections in the 't Hooft-Veltman version of the dimensional regularisation using `Helac-1Loop` [21] and `CutTools` [22], which are based on the Ossola-Papadopoulos-Pitau (OPP) reduction technique [23–25]. The most complicated one-loop diagrams in our calculations are heptagons. A number of optimizations have been devised in the algorithm of `Helac-1Loop` for the selection of loop topologies, which discard in advance all possibilities that are not compatible with the SM. This allowed us to substantially reduce the generation time. The process under consideration requires a special treatment of unstable top quarks, which is achieved within the complex mass scheme [26,27]. At the one-loop level, the appearance of a nonzero top quark width in the propagator requires the evaluation of scalar integrals with complex masses, which is supported by the `OneLOop` program used for the evaluation of the integrals [28]. For consistency, mass renormalization for the top quark is also done by applying the complex mass scheme in the well-known on-shell mass counter term. The preservation of gauge symmetries by this approach is explicitly checked by studying Ward identities up to the one-loop level. Reweighting techniques, helicity, and color sampling methods are additionally used in order to optimize the performance of our system. The singularities from soft or collinear parton emissions are isolated via subtraction methods for NLO QCD calculations. Specifically, two independent subtraction schemes are employed: the commonly used Catani-Seymour dipole subtraction [29–31] and a fairly new Nagy-Soper subtraction scheme [32], both implemented in the `Helac-Dipoles` software [31]. The implementation consists of a phase space integrator of subtracted real radiation and integrated subtraction terms for massless and massive cases. The phase space

integration is performed with the multichannel Monte Carlo generators `Phegas` [33] and `Kaleu` [34]. In the latter case, dedicated additional channels for each subtraction term have been added for both subtraction schemes to improve the convergence of the phase space integrals for the subtracted real contribution. Let us also note that we have implemented a new option in `Helac-Nlo` for automatically selecting the desired perturbative order in  $\alpha_s$  and  $\alpha$ , preserving at the same time the structure and the advantages of the Dyson-Schwinger recursive approach for the construction of the amplitude. This modification is particularly useful in the current project since we are interested in mixed contributions, i.e.,  $O(\alpha_s^3 \alpha^4)$  at LO and  $O(\alpha_s^4 \alpha^4)$  at NLO.

*Phenomenological application.*—In the following we present our numerical results for  $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X$  at the LHC at the center-of-mass energy of  $\sqrt{s} = 8$  TeV. Decays of the weak bosons to different lepton generations are considered to avoid virtual photon singularities arising from  $\gamma \rightarrow \ell^+ \ell^-$  decays. These effects are at the level of 0.5%, as checked by an explicit LO calculation. The SM parameters are set to

$$\begin{aligned} G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, & m_t &= 173.3 \text{ GeV}, \\ m_W &= 80.399 \text{ GeV}, & \Gamma_W &= 2.09974 \text{ GeV}, \\ m_Z &= 91.1876 \text{ GeV}, & \Gamma_Z &= 2.50966 \text{ GeV}, \\ \Gamma_t^{\text{LO}} &= 1.48132 \text{ GeV}, & \Gamma_t^{\text{NLO}} &= 1.3542 \text{ GeV}. \end{aligned}$$

The top quark width has been calculated according to Ref. [35]. We use the MSTW2008 set of parton distribution functions (PDFs) [36], i.e., MSTW2008lo68cl PDFs with a one-loop running  $\alpha_s$  at LO and MSTW2008nlo68cl with a two-loop running  $\alpha_s$  at NLO. All light quarks including  $b$  quarks, as well as leptons, are treated as massless. The suppressed contribution from  $b$  quarks in the initial state is neglected. When considering the total cross section at LO, this contribution amounts only to 0.8% of the total cross section. The renormalization and factorization scale is set to a common value  $\mu_R = \mu_F = \mu_0 = m_t$ . Let us notice that, while evaluating  $\Gamma_t^{\text{NLO}}$ , the value of  $\alpha_s$  at the scale  $m_t$  has been calculated from  $\alpha_s(m_Z) = 0.118$ . However, the  $\alpha_s(m_t)$  used within `Helac-Nlo` is obtained from the NLO MSTW2008 set that assumes  $\alpha_s(m_Z) = 0.12018$ . As a consequence, the corresponding  $\alpha_s(m_t)$  has a slightly different value. Should we use this value in the

calculation of the top quark width, we would instead get  $\Gamma_t^{\text{NLO}} = 1.35207$  GeV. The difference with respect to our value is at the level of one per mill only—therefore, completely negligible for our NLO QCD results. Jets are defined out of partons with pseudorapidity  $|\eta| < 5$  by the anti- $k_T$  jet algorithm [37] with the separation parameter  $R = 0.5$ . We require exactly two  $b$  jets, at least one light jet, two charged leptons, and missing  $p_T$ . Final states have to fulfill the following kinematical requirements:

$$\begin{aligned} p_{T\ell} &> 30 \text{ GeV}, & p_{Tj} &> 40 \text{ GeV}, \\ p_T^{\text{miss}} &> 40 \text{ GeV}, & \Delta R_{jj} &> 0.5, \\ \Delta R_{\ell\ell} &> 0.4, & \Delta R_{\ell j} &> 0.4, \\ |y_\ell| &< 2.5, & |y_j| &< 2.5, \end{aligned}$$

where  $\ell$  stands for  $\mu^-$ ,  $e^+$  and  $j$  corresponds to light and  $b$  jets. Results for the total cross sections are

$$\begin{aligned} \sigma_{\text{HELAC-NLO}}^{\text{LO}} &= 183.1^{+112.2(61\%)}_{-64.2(35\%)} \text{ fb}, \\ \sigma_{\text{HELAC-NLO}}^{\text{NLO}} &= 159.7^{+33.1(21\%)}_{-7.9(5\%)} \text{ fb}. \end{aligned} \quad (1)$$

At the central scale, the full  $pp$  cross section receives negative and moderate NLO corrections of 13%. Theoretical uncertainties, associated with neglected higher order terms in the perturbative expansion, have been estimated by varying the renormalization and factorization scales in  $\alpha_s$  and PDFs, up and down by a factor of 2 around the central scale of the process, i.e.,  $\mu_0$ . The scale uncertainties are evaluated to be 61% (48% after symmetrization) at LO and 21% (13% after symmetrization) at NLO. Thus, a reduction of the theoretical error by a factor of 3 is observed. The graphical presentation of the behavior of LO and NLO cross sections upon varying the scale by a factor of  $\xi$  with  $\xi \in \{0.125, \dots, 8\}$  is shown in Fig. 2.

In the next step, the size of the nonfactorizable corrections for our setup is assessed. To that end, the full result has been compared with the result in the NWA, which has been obtained by rescaling the  $t \rightarrow Wb$  coupling and  $\Gamma_t$  by a small factor to mimic the limit  $\Gamma_t \rightarrow 0$ . Finite top quark width effects change the cross section by less than 1% (2%) at LO (NLO), which is consistent with the expected uncertainty of the NWA, i.e., of the order of  $O(\Gamma_t/m_t)$ . We have also calculated the NLO cross section with a setup from Ref. [9], where NLO QCD corrections in the NWA have been evaluated for the  $pp \rightarrow e^+ \nu_e e^- \bar{\nu}_e b \bar{b} j$  final state. Instead of using the top quark width from Ref. [9], we have calculated it afresh to account for the off-shell effects of the  $W$  gauge boson and have obtained  $\Gamma_t = 1.31844$  GeV. In addition, we have included bottom quark contributions in the initial state and have required at least two  $b$  jets in the final state. Our finding is  $\sigma_{\text{HELAC-NLO}}^{\text{NLO}} = (275.5 \pm 0.6)$  fb. Comparing this to the result from Ref. [9], we observe a 4.5% difference, which is of the order of the NWA accuracy for the top quark and the  $W$  gauge boson. However, further investigation of the sources of the discrepancy would be desirable. We leave this for future study.

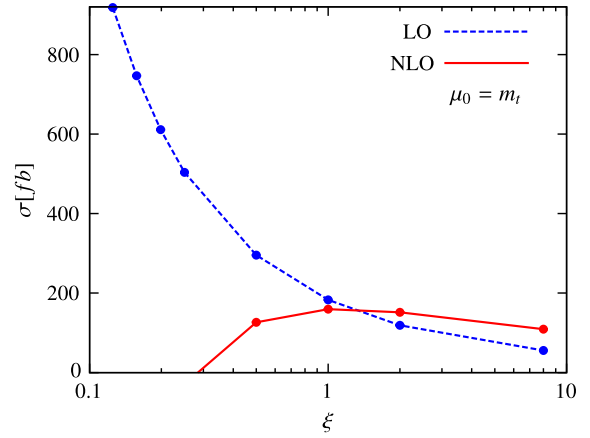


FIG. 2. Scale dependence of the LO and NLO cross sections for the  $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X$  process at the LHC for  $\sqrt{s} = 8$  TeV. The scale is set to a common value  $\mu_R = \mu_F = \xi \mu_0$ , where  $\mu_0 = m_t$ .

Representative differential distributions are presented in Fig. 3. We exhibit the transverse momentum of the hardest (in  $p_T$ ) light jet,  $p_{Tj_1}$ , the separation between charged leptons in the rapidity azimuthal angle plane,  $\Delta R_{e^+ \mu^-}$ , and the minimal invariant mass of the positron and the  $b$  jet,  $M_{be^+}$ . The dashed (blue) curve corresponds to the LO, the solid (red) one to the NLO result. The upper panels show the distributions themselves and the scale dependence bands that are constructed by calculating, bin by bin, a maximal and a minimal value out of the following set:  $\{m_t/2, m_t, 2m_t\}$ . The lower panels display the differential  $\mathcal{K}$  factor. Higher order corrections to  $p_{Tj_1}$  do not simply rescale the shape of the LO distribution, they instead induce distortions. With a fixed scale choice they reach  $-50\%$  within the plotted range. Thus, the  $p_{Tj_1}$  differential cross section can only be properly described when the higher order corrections are taken into account. Therefore, LO calculations together with a suitably chosen global  $\mathcal{K}$  factor would not approximate the full NLO QCD calculation well enough. However, a nearly constant  $\mathcal{K}$  factor can be achieved with a judicious choice of the dynamic scale. Negative NLO corrections in the high  $p_T$  tail means that the LO result is higher than the NLO one. The dynamic scale should depend on the  $p_T$  of the hardest jet, and its value should increase in the tail of the distribution. On the other hand, the asymptotic freedom guarantees that the value of  $\alpha_s$  becomes smaller there, resulting in lower NLO and LO cross sections. Because of the different dependence on the scale (see Fig. 2), the LO cross section, which in general is much more sensitive to the variation of the scale, will change more rapidly than the NLO curve, driving a positive NLO-LO ratio in this region. We leave the search for such a scale for the future. On the contrary, for the  $\Delta R_{e^+ \mu^-}$  distribution, negative, moderate, and quite stable corrections are visible. This can be explained by the dimensionless nature of the observable. Certainly,  $d\sigma/d\Delta R_{e^+ \mu^-}$  receives contributions from all scales, most notably from those that are sensitive to the threshold for the  $t\bar{t}j$  production. Indeed, for our scale

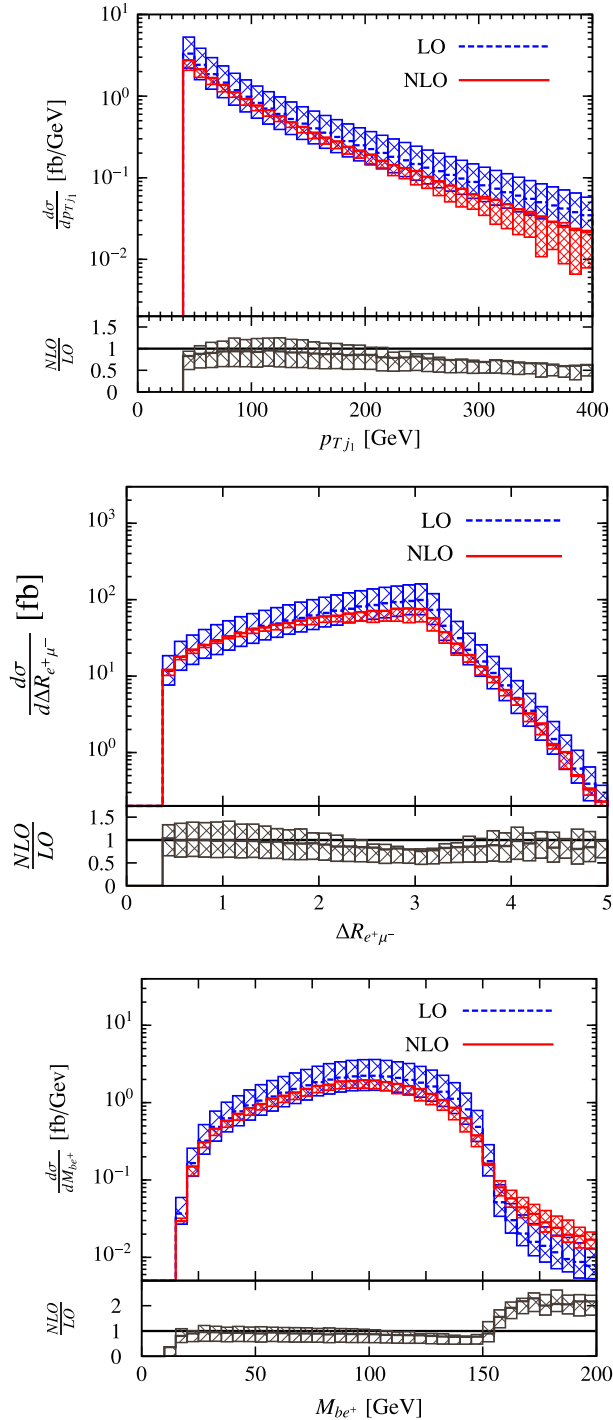


FIG. 3. Transverse momentum of the hardest light jet,  $\Delta R_{e^+\mu^-}$ , and the minimal  $M_{be^+}$  for  $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$  at the LHC, with  $\sqrt{s} = 8$  TeV. The uncertainty bands depict the scale variation. The lower panel displays the differential  $\mathcal{K}$  factor and its uncertainty band.

choice, effects of the phase space regions close to this threshold dominate and a dynamic scale will not alter this behavior. Finally, the invariant mass distribution of the positron and the  $b$  jet is shown. In general, one cannot determine which  $b$  jet should be paired with the positron. To increase the probability that both final states come from the

decay cascade initiated by the same top quark, we select the  $be^+$  pair that returns the smallest invariant mass [38]. In the case of  $t\bar{t}$  production, this observable has proved to be particularly important for extracting  $m_t$  with a very high precision [39,40]. The top quark mass can be determined either from the shape of the distribution away from the kinematical end point, defined as  $M_{be^+} = \sqrt{m_t^2 - m_W^2} \approx 153.5$  GeV, or from the behavior of the observable in the vicinity of that point. In the former case, off-shell effects are negligible, in the latter they might even reach 50% [41]. When the top quark and the  $W$  gauge boson decay on shell, the end point is represented by a sharp cut. However, additional radiation and off-shell effects introduce a smearing to the region, which is highly sensitive to the details of the description of the process. Thus, off-shell effects might prove to be very important for  $t\bar{t}j$  as well, should top quark mass measurements be carried out using  $M_{be^+}$ .

*Summary and outlook.*—In this Letter, NLO QCD corrections to  $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$  with complete off-shell and interference effects have been calculated for the first time. We have shown that NLO QCD corrections to the total cross section are moderate (13%). Nevertheless, their impact on some differential distributions is much larger. We have presented two cases,  $p_{Tj_1}$  and  $M_{be^+}$ , where higher order corrections are indispensable for correctly describing the whole range of the observable. We have also estimated the size of the top quark off-shell effects at NLO for the total cross section and have confirmed that they are of the order of  $O(\Gamma_t/m_t)$ . On the other hand, their influence on differential distributions might be much stronger, as has already been suggested by studies for the  $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$  production process [41]. We leave further comparisons for the future.

We would like to thank A. van Hameren for providing us with a new version of the KALEU software and C. G. Papadopoulos for the useful discussions. The work of H. B. Hartanto was supported by the German Research Foundation (DFG) under Grant “*Subtraction schemes at next-to-next-to-leading order with applications to top-quark and jet physics*”. M. W. and M. K. acknowledge support from the DFG under Grant No. WO 1900/1-1, “*Signals and Backgrounds Beyond Leading Order. Phenomenological studies for the LHC.*”

- [1] S. Alioli, P. Fernandez, J. Fuster, A. Irlles, S. O. Moch, P. Uwer, and M. Vos, A new observable to measure the top-quark mass at hadron colliders, *Eur. Phys. J. C* **73**, 2438 (2013).
- [2] D. L. Rainwater and D. Zeppenfeld, Observing  $H \rightarrow W^*W^* \rightarrow e^+\mu^\mp p_T$  in weak boson fusion with dual forward jet tagging at the CERN LHC, *Phys. Rev. D* **60**, 113004 (1999); **61** 099901(E) (2000).
- [3] N. Kauer, T. Plehn, D. L. Rainwater, and D. Zeppenfeld,  $H \rightarrow W^+W^-$  as the discovery mode for a light Higgs boson, *Phys. Lett. B* **503**, 113 (2001).

- [4] M. L. Mangano, Standard Model backgrounds to supersymmetry searches, *Eur. Phys. J. C* **59**, 373 (2009).
- [5] M. I. Gresham, I. W. Kim, and K. M. Zurek, Searching for top flavor violating resonances, *Phys. Rev. D* **84**, 034025 (2011).
- [6] S. Dittmaier, P. Uwer, and S. Weinzierl, NLO QCD Corrections to  $t\bar{t}$  + jet Production at Hadron Colliders, *Phys. Rev. Lett.* **98**, 262002 (2007).
- [7] S. Dittmaier, P. Uwer, and S. Weinzierl, Hadronic top-quark pair production in association with a hard jet at next-to-leading order QCD: Phenomenological studies for the Tevatron and the LHC, *Eur. Phys. J. C* **59**, 625 (2009).
- [8] K. Melnikov and M. Schulze, NLO QCD corrections to top quark pair production in association with one hard jet at hadron colliders, *Nucl. Phys.* **B840**, 129 (2010).
- [9] K. Melnikov, A. Scharf, and M. Schulze, Top quark pair production in association with a jet: QCD corrections and jet radiation in top quark decays, *Phys. Rev. D* **85**, 054002 (2012).
- [10] A. Kardos, C. Papadopoulos, and Z. Trocsanyi, Top quark pair production in association with a jet with NLO parton showering, *Phys. Lett. B* **705**, 76 (2011).
- [11] S. Alioli, S. O. Moch, and P. Uwer, Hadronic top-quark pair-production with one jet and parton showering, *J. High Energy Phys.* **01** (2012) 137.
- [12] M. Czakon, H. B. Hartanto, M. Kraus, and M. Worek, Matching the Nagy-Soper parton shower at next-to-leading order, *J. High Energy Phys.* **06** (2015) 033.
- [13] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, Next-to-Leading-Order QCD Corrections to  $W^+W^-b\bar{b}$  Production at Hadron Colliders, *Phys. Rev. Lett.* **106**, 052001 (2011).
- [14] G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos, and M. Worek, Complete off-shell effects in top quark pair hadroproduction with leptonic decay at next-to-leading order, *J. High Energy Phys.* **02** (2011) 083.
- [15] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, NLO QCD corrections to off-shell top-antitop production with leptonic decays at hadron colliders, *J. High Energy Phys.* **10** (2012) 110.
- [16] R. Frederix, Top Quark Induced Backgrounds to Higgs Production in the  $WW^{(*)} \rightarrow ll\nu\nu$  Decay Channel at Next-to-Leading-Order in QCD, *Phys. Rev. Lett.* **112**, 082002 (2014).
- [17] F. Cascioli, S. Kallweit, P. Maierhöfer, and S. Pozzorini, A unified NLO description of top-pair and associated  $Wt$  production, *Eur. Phys. J. C* **74**, 2783 (2014).
- [18] G. Heinrich, A. Maier, R. Nisius, J. Schlenk, and J. Winter, NLO QCD corrections to  $W^+W^-b\bar{b}$  production with leptonic decays in the light of top quark mass and asymmetry measurements, *J. High Energy Phys.* **06** (2014) 158.
- [19] A. Denner and R. Feger, NLO QCD corrections to off-shell top-antitop production with leptonic decays in association with a Higgs boson at the LHC, *J. High Energy Phys.* **11** (2015) 209.
- [20] G. Bevilacqua, M. Czakon, M. V. Garzelli, A. van Hameren, A. Kardos, C. G. Papadopoulos, R. Pittau, and M. Worek, HELAC-NLO, *Comput. Phys. Commun.* **184**, 986 (2013).
- [21] A. van Hameren, C. G. Papadopoulos, and R. Pittau, Automated one-loop calculations: A proof of concept, *J. High Energy Phys.* **09** (2009) 106.
- [22] G. Ossola, C. G. Papadopoulos, and R. Pittau, CUTTOOLS: A program implementing the OPP reduction method to compute one-loop amplitudes, *J. High Energy Phys.* **03** (2008) 042.
- [23] G. Ossola, C. G. Papadopoulos, and R. Pittau, Reducing full one-loop amplitudes to scalar integrals at the integrand level, *Nucl. Phys.* **B763**, 147 (2007).
- [24] G. Ossola, C. G. Papadopoulos, and R. Pittau, On the rational terms of the one-loop amplitudes, *J. High Energy Phys.* **05** (2008) 004.
- [25] P. Draggiotis, M. V. Garzelli, C. G. Papadopoulos, and R. Pittau, Feynman rules for the rational part of the QCD 1-loop amplitudes, *J. High Energy Phys.* **04** (2009) 072.
- [26] A. Denner, S. Dittmaier, M. Roth, and D. Wackerth, Predictions for all processes  $e^+e^- \rightarrow 4$  fermions  $+\gamma$ , *Nucl. Phys.* **B560**, 33 (1999).
- [27] A. Denner, S. Dittmaier, M. Roth, and L. H. Wieders, Electroweak corrections to charged-current  $e^+e^- \rightarrow 4$  fermion processes: Technical details and further results, *Nucl. Phys.* **B724**, 247 (2005); **B854**, 504(E) (2012).
- [28] A. van Hameren, ONELOOP: For the evaluation of one-loop scalar functions, *Comput. Phys. Commun.* **182**, 2427 (2011).
- [29] S. Catani and M. H. Seymour, A general algorithm for calculating jet cross-sections in NLO QCD, *Nucl. Phys.* **B485**, 291 (1997); **B510**, 503(E) (1998).
- [30] S. Catani, S. Dittmaier, M. H. Seymour, and Z. Trocsanyi, The dipole formalism for next-to-leading order QCD calculations with massive partons, *Nucl. Phys.* **B627**, 189 (2002).
- [31] M. Czakon, C. G. Papadopoulos, and M. Worek, Polarizing the dipoles, *J. High Energy Phys.* **08** (2009) 085.
- [32] G. Bevilacqua, M. Czakon, M. Kubocz, and M. Worek, Complete Nagy-Soper subtraction for next-to-leading order calculations in QCD, *J. High Energy Phys.* **10** (2013) 204.
- [33] C. G. Papadopoulos, PHEGAS: A phase space generator for automatic cross-section computation, *Comput. Phys. Commun.* **137**, 247 (2001).
- [34] A. van Hameren, KALEU: A general-purpose parton-level phase space generator, [arXiv:1003.4953](https://arxiv.org/abs/1003.4953).
- [35] M. Jezabek and J. H. Kuhn, QCD corrections to semi-leptonic decays of heavy quarks, *Nucl. Phys.* **B314**, 1 (1989).
- [36] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Parton distributions for the LHC, *Eur. Phys. J. C* **63**, 189 (2009).
- [37] M. Cacciari, G. P. Salam, and G. Soyez, The anti- $k_t$  jet clustering algorithm, *J. High Energy Phys.* **04** (2008) 063.
- [38] M. Beneke *et al.*, Top quark physics, standard model physics (and more) at the LHC, [arXiv:hep-ph/0003033](https://arxiv.org/abs/hep-ph/0003033).
- [39] S. Chatrchyan *et al.* (CMS Collaboration), Measurement of masses in the  $t\bar{t}$  system by kinematic endpoints in  $pp$  collisions at  $\sqrt{s} = 7$  TeV, *Eur. Phys. J. C* **73**, 2494 (2013).
- [40] G. Aad *et al.* (ATLAS Collaboration), Measurement of the top quark mass in the  $t\bar{t} \rightarrow$  lepton + jets and  $t\bar{t} \rightarrow$  dilepton channels using  $\sqrt{s} = 7$  TeV ATLAS data, *Eur. Phys. J. C* **75**, 330 (2015).
- [41] J. Alcaraz Maestre *et al.*, The SM and NLO Multileg and SM MC working groups: Summary report, [arXiv:1203.6803](https://arxiv.org/abs/1203.6803).