Quantum Optics Theory of Electronic Noise in Coherent Conductors

Arne L. Grimsmo,[1] Farzad Qassemi,[1] Bertrand Reulet,[1] and Alexandre Blais[1,2]
[1]*Départment de Physique, Université de Sherbrooke, 2500 boulevard de l'Université, Sherbrooke, Québec J1K 2R1, Canada*
[2]*Canadian Institute for Advanced Research, Toronto, Canada*
(Received 15 July 2015; published 27 January 2016)

We consider the electromagnetic field generated by a coherent conductor in which electron transport is described quantum mechanically. We obtain an input-output relation linking the quantum current in the conductor to the measured electromagnetic field. This allows us to compute the outcome of measurements on the field in terms of the statistical properties of the current. We moreover show how under ac bias the conductor acts as a tunable medium for the field, allowing for the generation of single- and two-mode squeezing through fermionic reservoir engineering. These results explain the recently observed squeezing using normal tunnel junctions [G. Gasse *et al.*, Phys. Rev. Lett. 111, 136601 (2013); J.-C. Forgues *et al.*, Phys. Rev. Lett. 114, 130403 (2015)].

DOI: 10.1103/PhysRevLett.116.043602

More than sixty years ago, Glauber showed that the electromagnetic radiation produced by a classical electrical current is itself classical [1,2]. The situation can however be different in mesoscopic conductors at low temperature. Indeed, in such conductors electron transport should no longer be considered classical and current is represented by an operator. Because this operator does not commute with itself when evaluated at different times or frequencies, Glauber's results no longer apply. One may then wonder if a "quantum current" may generate a nonclassical electromagnetic field. This is the central question addressed in this Letter: how does the quantum properties of current in a coherent conductor imprint on the properties of the electromagnetic field it radiates?

This question was partly addressed in Refs. [3–5] where it was shown, for example, that the statistics of photons emitted by a quantum conductor can deviate from the Poissonian statistics of a coherent state. While the photon statistics are most naturally revealed by power detection, measurements on quantum conductors are more typically realized with linear (i.e., voltage) detectors revealing quadratures of the electromagnetic field radiated by the sample. As a result, Refs. [3–5] only partly answer the question.

More recently it was predicted that, under ac bias, the electromagnetic field radiated by a coherent conductor can be squeezed [6]. The field is then characterized by fluctuations along one of two quadratures being smaller than the vacuum level. These quantum states of the electromagnetic field are typically associated with the presence of a nonlinear element, such as a Kerr medium in the optical frequency range [7] or a Josephson junction at microwave frequencies [8]. In contrast, squeezing was experimentally observed using a tunnel junction with linear current-voltage characteristics [9,10]. Here squeezing results, not from a coherent nonlinear interaction, but rather from the incoherent quantum shot noise of the junction under ac driving. The predictions of Ref. [6], however, only consider correlation functions of the current inside the conductor, not the properties of the emitted field that is squeezed and ultimately measured.

In this Letter, instead of focusing on the current in the coherent conductor, we determine the properties of the field that it radiates. Given that currents and voltages in electrical circuits are nothing more than another representation of electromagnetic fields, the theoretical methods of quantum optics are particularly well suited. By deriving an input-output relation [11–13] directly connecting the radiated electromagnetic field to the current, we can compute measurable properties of the radiated field. We then go a step further and consider the fermionic degrees of freedom of the conductor as an environment for the electromagnetic field of a microwave resonator. Tracing out the conductor's degrees of freedom leads to a Lindblad master equation for the electromagnetic field in a squeezed bath, thus giving to access to its squeezing and multimode entanglement properties. Depending on the choice of ac bias for the junction, one can induce both single- and two-mode squeezing of the resonator modes. This shows how the electrons in the coherent conductor act as an effective, tunable medium for the field, and provides clear insight into the mechanism responsible for squeezing of the field. In contrast to conventional schemes based on coherent interactions with nonlinearities, the fermionic degrees of freedom here provide a broadband squeezed bath for the resonator, where dissipation can be used to cool modes of the resonator into a squeezed state [14].

Our first step is to model the electromagnetic environment of the sample as a semi-infinite transmission line of characteristic impedance $Z_0 = \sqrt{L_0/C_0}$, with L_0 and C_0 the inductance and capacitance per unit length, respectively. The position-dependent flux $\hat{\phi}_{\mathrm{tl}}(x,t)$ along the transmission line is [11,12,15]

$$\hat{\phi}_{\rm tl}(x,t) = \alpha \int_0^\infty \frac{d\omega}{\sqrt{\omega}} (\hat{a}_{\rm in}[\omega] e^{-i\omega(t+x/v)} + \hat{a}_{\rm out}[\omega] e^{-i\omega(t-x/v)} + {\rm H.c.}), \quad (1)$$

where $v = 1/\sqrt{L_0 C_0}$ is the speed of light in the transmission line and $\alpha = \sqrt{\hbar Z_0/4\pi}$. The subscripts "in" and "out" denote components moving towards and away from the sample, respectively. The corresponding annihilation operators satisfy $[\hat{a}_{\rm in}[\omega], \hat{a}^\dagger_{\rm in}[\omega']] = \delta(\omega - \omega')$ and similarly for $\hat{a}_{\rm out}$. Finally, current at position x in the transmission line is given in terms of the flux by $\hat{I}_{\rm tl}(x,t) = L_0^{-1}\partial_x \hat{\phi}_{\rm tl}(x,t)$, while voltage is $\hat{V}_{\rm tl}(x,t) = \partial_t \hat{\phi}_{\rm tl}(x,t)$.

With the sample located at $x = 0$, current conservation imposes that

$$\hat{I}_{\rm s}(t) = \hat{I}_{\rm tl}(x = 0, t), \quad (2)$$

where $\hat{I}_s$ is the sample's electron current operator in the presence of the transmission line and of classical voltage bias. This equality links the bosonic operators of the line to the fermionic degrees of freedom of the sample. In the frequency domain, this takes the form [16]

$$\hat{a}_{\rm out}[\omega] = \hat{a}_{\rm in}[\omega] - i\sqrt{\frac{Z_0}{\pi\hbar\omega}} \hat{I}_s[\omega] \quad (3)$$

which relates the field travelling away from the conductor $\hat{a}_{\rm out}$ to the incoming field $\hat{a}_{\rm in}$ and the condutor's current operator $\hat{I}_s$ [17]. This is akin to an input-output boundary condition in quantum optics [11,12,18,19]. An expression similar to Eq. (3) can be found in Ref. [3] for the case of a quantum conductor coupled to the electromagnetic field freely propagating in three dimensions.

Since $\hat{I}_s[\omega]$ depends on the current evaluated at all times, it does not commute with $\hat{a}_{\rm in}[\omega]$. Care must therefore be taken when evaluating moments of $\hat{a}_{\rm out}[\omega]$. In Ref. [3], this problem was avoided by neglecting the influence of the field's vacuum fluctuations on the current $\hat{I}_s$. This is justified for a sample of impedance much larger than Z_0, thus very poorly matched to the transmission line, and does not correspond to usual experimental conditions where impedance matching is preferable. Here, we address the problem of noncommutativity by writing Eq. (3) in terms of the quantum conductor's bare current operator $\hat{I}$ in the absence of the electromagnetic environment, rather than the full current $\hat{I}_s$ containing the influence of the field. This is done by going to the Heisenberg picture and solving for the current operator perturbatively in the light-matter coupling α. This linear response treatment is justified for typical low impedance electromagnetic environments such that $Z_0 \ll R_K$ with $R_K = h/e^2 \sim 26$ kΩ the quantum of resistance. For the common experimental value $Z_0 = 50\ \Omega$, one indeed has $e\alpha/\hbar = \sqrt{Z_0/2R_K} \sim 0.03 \ll 1$. For the low impedance transmission line and when the sample can be treated in the lumped-element limit, we take the interaction between the line's and samples's degrees of freedom to be of the form $H_{\rm I}(t) = \hat{I}(t)\hat{\phi}_{\rm tl}(x = 0, t)$ [16]. To first order in the interaction we then find [16,20]

$$\hat{I}_s[\omega] = \hat{I}[\omega] + \hat{V}_{\rm tl}[\omega]/Z[\omega], \quad (4)$$

with $Z[\omega]$ the impedance of the sample. In this expression, $\hat{I}[\omega]$ is the Fourier transform of $\hat{I}(t)$, the electronic current operator evolving according to the bare quantum conductor Hamiltonian [16]. In principle, this free Hamiltonian can contain disorder, interactions, etc., as well as the effect of the classical dc and ac bias voltage, $V_{\rm dc} + V_{\rm ac}\cos\omega_{\rm ac}t$, applied to the conductor.

Combining Eqs. (3) and (4) directly leads to

$$\hat{a}_{\rm out}[\omega] = r[\omega]\hat{a}_{\rm in}[\omega] - it[\omega]\frac{\hat{I}[\omega]}{\sqrt{4\pi\hbar\omega Z^{-1}[\omega]}}, \quad (5)$$

with $r = (Z - Z_0/Z + Z_0)$ the reflection coefficient and $t = (2\sqrt{ZZ_0}/Z + Z_0)$ the transmission coefficient with $|r|^2 + |t|^2 = 1$. In contrast to Eq. (3), the bare current operator $\hat{I}$ entering Eq. (5) commutes at all times with the incoming field $\hat{a}_{\rm in}$ that has not yet interacted with the conductor. Arbitrary correlation functions of the outgoing field can thus easily be evaluated with this input-output boundary condition.

As examples, we now discuss the results for different types of common measurements. For simplicity we restrict the discussion to the practically important case of an ideally matched sample, $Z[\omega] = R = Z_0$. Then $r = 0$ and the outgoing field takes the simple form $\hat{a}_{\rm out}[\omega] = -i\hat{I}[\omega]/\sqrt{4\pi S_{\rm vac}(\omega)}$ where $S_{\rm vac}(\omega) = \hbar\omega/R$ is the current noise spectral density of vacuum noise. Measurable properties of the output field are then fully determined by the current. In particular, second order moments of the output field are given in terms of current-current correlation functions which under ac excitation obey [16,21,22]

$$\langle I[\omega']I[\omega]\rangle = 2\pi[\tilde{S}(\omega') + S_{\rm vac}(\omega')]\delta(\omega' + \omega) + 2\pi\sum_{p\neq 0} X(\omega')\delta(\omega' + \omega - p\omega_{\rm ac}). \quad (6)$$

In this expression,

$$\tilde{S}(\omega) = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_{\rm ac}}{\hbar\omega_{\rm ac}}\right) S\left(V_{\rm dc} + \frac{n\hbar\omega_{\rm ac}}{e}, \omega\right), \quad (7)$$

is the photoassisted noise, $S(V,\omega) = F[S_0(V + \hbar\omega/e) + S_0(V - \hbar\omega/e)]/2 + (1 - F)S_0(\hbar\omega/e)$ the noise spectral density of current fluctuations in the conductor, $S_0(V) = R^{-1}eV\coth(eV/2k_BT)$, and F the Fano factor [23]. Moreover,

$$X(\omega) = \frac{F}{2}\sum_n J_n J_{n+p}\left[S_0\left(V_{\rm dc} + \frac{\hbar}{e}(\omega + n\omega_{\rm ac})\right) + (-1)^p S_0\left(V_{\rm dc} - \frac{\hbar}{e}(\omega + n\omega_{\rm ac})\right)\right], \quad (8)$$

characterizes the noise dynamics [24]. For brevity we have here omitted the argument of the Bessel functions J_n that is the same as in Eq. (7).

We first consider photodetection of the output field in the experimentally relevant situation where the signal is band-pass filtered before detection. This can be taken into account by defining a filtered output field

$$\hat{b}_{\rm out}(t) = \frac{1}{\sqrt{2\pi B}} \int_B d\omega e^{-i(\omega-\omega_0)t} \hat{a}_{\rm out}[\omega], \tag{9}$$

where B refers to a measurement bandwidth centered at the observation frequency $\omega_0 \gg 2\pi B$. With this definition, the filtered photocurrent is [16]

$$\langle \hat{b}^\dagger_{\rm out}(t)\hat{b}_{\rm out}(t)\rangle = \frac{\tilde{S}(\omega_0) - S_{\rm vac}(\omega_0)}{2S_{\rm vac}(\omega_0)}, \tag{10}$$

where we have assumed a small filter bandwidth and dropped fast rotating terms [16]. As expected, a photodetector is sensitive to the spectral density of the current noise *emitted* by the conductor [25–27]. In practice, this can be measured by separating the emission and absorption noise [28,29]. A more common detection scheme is to measure the time-averaged power of the emitted electromagnetic field. Again assuming a small measurement bandwidth, we find from Eq. (1) that $\overline{\langle \hat{V}_{\rm tl,f}(t)^2\rangle}/R = B\tilde{S}(\omega_0)R$, with $\hat{V}_{\rm tl,f}$ the filtered transmission-line voltage and where we have omitted a contribution from the vacuum noise of the in-field [16]. In contrast to photodetection, measurement of the power of the electromagnetic field is related to the *symmetric* current-current correlator containing both emission and absorption [27]. This is not in contradiction with the fact that a passive detector cannot detect vacuum fluctuations [26]. Power measurements are indeed performed using active devices like amplifiers and mixers.

Following the experiments of Refs. [9,10], we now consider measurement of field quadratures as obtained by homodyne detection [7]. Defining quadratures of the output field in the frequency domain as $\hat{X}_{\rm out}[\omega] = \hat{a}^\dagger_{\rm out}[\omega] + \hat{a}_{\rm out}[\omega]$ and $\hat{Y}_{\rm out}[\omega] = i(\hat{a}^\dagger_{\rm out}[\omega] - \hat{a}_{\rm out}[\omega])$, and using Eq. (5), we immediately find for the variance of these quantities [16]

$$\Delta\hat{X}^2_{\rm out}[\omega] = \frac{\langle\{\hat{I}[-\omega],\hat{I}[\omega]\}\rangle - 2\langle\hat{I}[\omega]^2\rangle}{4\pi S_{\rm vac}(\omega)},$$
$$\Delta\hat{Y}^2_{\rm out}[\omega] = \frac{\langle\{\hat{I}[-\omega],\hat{I}[\omega]\}\rangle + 2\langle\hat{I}[\omega]^2\rangle}{4\pi S_{\rm vac}(\omega)}. \tag{11}$$

In practice, $\langle I[\omega]^2\rangle$ is only nonzero in the presence of ac bias on the sample. Indeed, as expressed by Eq. (6), modulation of the bias voltage at frequency $\omega_{\rm ac}$ induces correlations between Fourier components of the current separated by $p\omega_{\rm ac}$, with p an integer [21,22]. For $p\omega_{\rm ac} = 2\omega_0$, and defining filtered output quadratures, $\hat{X}_{\rm out,f}(t) = \hat{b}^\dagger_{\rm out}(t) + \hat{b}_{\rm out}(t)$ and $\hat{Y}_{\rm out,f}(t) = i[\hat{b}^\dagger_{\rm out}(t) - \hat{b}_{\rm out}(t)]$, we find [16]

$$\Delta\hat{X}^2_{\rm out,f}(t) = 2\left(N(\omega_0) + \frac{1}{2} - M(\omega_0)\right),$$
$$\Delta\hat{Y}^2_{\rm out,f}(t) = 2\left(N(\omega_0) + \frac{1}{2} + M(\omega_0)\right), \tag{12}$$

where

$$N(\omega) = \frac{\tilde{S}(\omega) - S_{\rm vac}(\omega)}{2S_{\rm vac}(\omega)}, \qquad M(\omega) = \frac{X(\omega)}{2S_{\rm vac}(\omega)}. \tag{13}$$

Clearly, the X quadrature of the output field is squeezed when $M(\omega_0) > N(\omega_0)$, equivalently $X(\omega_0) > \tilde{S}(\omega_0) - S_{\rm vac}(\omega_0)$, with $M(\omega)$ and $N(\omega)$ bounded from the Heisenberg inequality by $N(\omega)[N(\omega)+1] \geq M(\omega)^2$ [7]. The same condition for squeezing was found in Refs. [6,9] by directly postulating the link between the field quadratures and the current operator for a normal tunnel junction. The squeezing generated by such a junction is, however, moderate; Gasse *et al.*'s experiment reporting a value of 1.3 dB is in excellent agreement with our theoretical predictions, see Fig. 2 of Ref. [9]. The degree of squeezing can be tuned through $V_{\rm ac}$ and $V_{\rm dc}$. As shown in Ref. [16], we find the squeezing to be maximal for $eV_{\rm dc} = \hbar\omega_0$, $\omega_{\rm ac} = 2\omega_0$, with a maximum squeezing of ~2 dB at zero temperature.

To better understand the mechanism responsible for squeezing by the quantum conductor we now derive an equation of motion for the state $\rho(t)$ of the field. This is done by following the standard quantum optics approach: the bath is integrated out invoking the Born-Markov approximation to obtain a Lindblad master equation describing the dynamics of the field only [7]. The crucial difference from the usual treatment is that here the fermionic degrees of freedom of the sample play the role of bath for the bosonic modes of the field. Moreover, it is possible to engineer the system-bath interaction with the ac modulation frequency, leading to different field steady states.

To simplify the discussion and because it is experimentally relevant [30–32], we consider the setup illustrated in Fig. 1 where a normal tunnel junction is fabricated at the end of a $\lambda/4$ transmission line resonator of characteristic impedance Z_0. The case of a Josephson junction has been considered in Refs. [15,33–35]. The effect of the junction on the resonator is easily found by decomposing the resonator flux in terms of normal modes $\phi(x,t) = \sum_m \phi_m(t)u_m(x)$, with $u_m(x)$ the mode envelope [16,36]. The full line in Fig. 1 illustrates $|u_1(x)|$ for a junction impedance $R < Z_0$, while the dashed line corresponds to $\mathrm{Re}[u_1(x)]$ for $R > Z_0$. As expected, in the former case the junction acts as a short to ground and the mode envelope

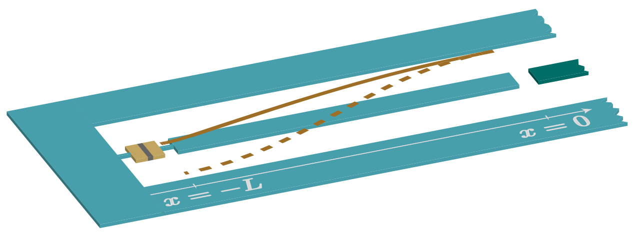

FIG. 1. Transmission line resonator (blue) terminated by a normal tunnel junction. The first resonator mode envelopes are illustrated for $R/Z_0 = 0.1$ (full brown line) and $R/Z_0 = 2000$ (dashed brown line). The output field can be measured via the capacitive coupling to the output port (dark blue).

approaches that of a $\lambda/4$ resonator, except for a small gap $|u_1(-L)| \sim R/Z_0$ at the location of the junction. On the other hand, for large tunnel resistance the junction acts as an open and the resonator's bias on the junction is larger with $|u_1(-L)| \sim 1 + (Z_0/R)^2$.

Having characterized the resonator mode in the presence of the junction, we now obtain the master equation assuming $Z_0 \ll R_K$. In this limit, the interaction Hamiltonian reads $\hat{H}_I = \sum_m \alpha_m \hat{I}(\hat{a}_m^\dagger + \hat{a}_m)$, where $\alpha_m = \sqrt{\hbar Z_m} u_m(-L)$ with Z_m the effective impedance of the resonator's mth mode and $\hat{a}_m^{(\dagger)}$ the annihilation (creation) operator for the same mode [16]. As above, the current operator $\hat{I}$ takes into account the presence of classical dc and ac bias on the junction.

We first focus on the situation where the ac frequency is, as above, $p\omega_{\rm ac} = 2\omega_m$ where p is an integer and ω_m now the frequency of the mth resonator mode. In the rotating-wave approximation we find [16]

$$\dot{\rho}(t) = \kappa_m(N_m + 1)\mathcal{D}[\hat{a}_m]\rho + \kappa_m N_m \mathcal{D}[\hat{a}_m^\dagger]\rho + \kappa_m M_m \mathcal{S}[\hat{a}_m]\rho + \kappa_m M_m \mathcal{S}[\hat{a}_m^\dagger]\rho, \tag{14}$$

with $\mathcal{D}[\hat{a}]\rho = \hat{a}\rho\hat{a}^\dagger - \{\hat{a}^\dagger\hat{a}, \rho\}/2$ and $\mathcal{S}[X]\rho = \hat{a}\rho\hat{a} - \{\hat{a}^2, \rho\}/2$. Equation (14) is the standard master equation of a bosonic mode in a squeezed bath [7] where $\kappa_m = u_m(-L)^2 \omega_m Z_m/R$ is the cavity damping rate caused by the tunnel junction resistance [37]. The thermal photon number $N_m = N(\omega_m)$ and the quantity $M_m = M(\omega_m)$ responsible for squeezing are the same as in Eq. (13). Evolution under Eq. (14) leads to steady-state variances of the intracavity quadratures $\hat{X}_m = \hat{a}_m^\dagger + \hat{a}_m$ and $\hat{Y}_m = i(\hat{a}_m^\dagger - \hat{a}_m)$ taking the form $\Delta X_m^2 = 2N_m + 1 - 2M_m$ and $\Delta Y_m^2 = 2N_m + 1 + 2M_m$. In other words, intracavity squeezing is identical to what was found in Eq. (12) in the absence of the resonator.

The form of the above master equation clearly illustrates the dissipative nature of squeezing by a tunnel junction. This type of squeezing by dissipation has been explored theoretically in various systems and, in particular, in Ref. [14] where it was shown that modulating the quality factor of a linear cavity could lead to ideal and unbounded squeezing. A similar mechanism is in action here with the periodic modulation of the Fermi level of the tunnel junction by the ac bias. The achievable squeezing is, however, neither pure nor unbounded, with the purity $p = S_{\rm vac}/\sqrt{\tilde{S}^2 - X^2} < 1$. At zero temperature, the highest expected purity is $p \sim 0.91$ corresponding to the 2 dB of squeezing mentioned above. This conclusion also applies to the cavity output field. Indeed, taking into account an output port (illustrated by the capacitor on the right-hand side of Fig. 1) reduces intracavity squeezing by adding vacuum noise. This additional contribution is, however, absent from the cavity output field if the decay rates at the two ends of the resonator are matched [13], leaving the degree of squeezing unchanged from the above input-output theory without cavity [16].

Taking advantage of the multimode structure of the resonator, other choices of ac drive can lead to entangled steady states. In particular, taking $p\omega_{\rm ac} = \omega_m + \omega_n$ results in [16]

$$\begin{aligned}\dot{\rho}(t) = \sum_{l=n,m} \{\kappa_l(N_l + 1)\mathcal{D}[\hat{a}_l]\rho + \kappa_l N_l \mathcal{D}[\hat{a}_l^\dagger]\rho\} \\ + \sqrt{\kappa_n\kappa_m} M_{nm}(\hat{a}_n\rho\hat{a}_m + \hat{a}_m\rho\hat{a}_n - \{\hat{a}_n\hat{a}_m, \rho\}) \\ + \sqrt{\kappa_n\kappa_m} M_{nm}(\hat{a}_n^\dagger\rho\hat{a}_m^\dagger + \hat{a}_m^\dagger\rho\hat{a}_n^\dagger - \{\hat{a}_n^\dagger\hat{a}_m^\dagger, \rho\}),\end{aligned} \tag{15}$$

where κ_l and N_l are the same as above and $M_{nm} = X(\omega_n)/2\sqrt{S_{\rm vac}(\omega_n)S_{\rm vac}(\omega_m)}$. This master equation leads to two-mode squeezing. Indeed, in steady state the variance of the joint quadratures $\hat{X}_\pm = \hat{X}_n \pm \hat{X}_m$ and $\hat{Y}_\pm = \hat{Y}_n \pm \hat{Y}_m$ are

$$\begin{aligned}\Delta X_+^2 = \Delta Y_-^2 = 2(N_n + N_m + 1 - 2M_{nm}), \\ \Delta X_-^2 = \Delta Y_+^2 = 2(N_n + N_m + 1 + 2M_{nm}),\end{aligned} \tag{16}$$

where we have assumed $\kappa_n = \kappa_m$ for simplicity. Similarly to the above single-mode case, the pairs of commuting quadratures ΔX_+^2 and ΔY_-^2 are squeezed for $2M_{nm} > N_n + N_m$.

It is interesting to point out that these quadratures are entangled when $\Delta X_+^2 + \Delta Y_-^2 < 4$ [39]. This type of two-mode squeezing generated by a normal tunnel junction under the above ac modulation frequency was already experimentally reported in Ref. [10]. Equation (15) shows how this entanglement generation can be understood as resulting from correlated absorbtion (emission) of entangled excitations from (into) the bath, without any direct coupling between the modes.

Finally, it is interesting to also consider the situation where the ac modulation is such that $p\omega_{\rm ac} = |\omega_n - \omega_m|$. Rather than two-mode squeezing, this leads to correlated decay where emission by mode n stimulates emission from mode m, and vice versa. Details are given in the Supplemental Material [16]. For this choice of ac bias, the variance of the above joint quadratures keeps the same form, except for ΔX_+^2 and ΔX_-^2 whose roles are exchanged. Since $[\hat{X}_-, \hat{Y}_-] = 4i$, the variance of these two quadratures must respect $\Delta X_- \Delta Y_- \geq 2$ implying that $N_n + N_m \geq 2M_{nm}$. In other words, these quadratures cannot be squeezed below the vacuum level when the ac-bias frequency is matched to the frequency difference of the two modes, also implying that the two modes are not entangled.

In summary, we have shown how the electromagnetic field radiated by a quantum conductor is related to its electronic properties. This establishes a common language between quantum optics and mesoscopic physics. In particular, recent experimental observations of squeezing and entanglement produced by a tunnel junction are well understood within our framework. These results have been obtained assuming a simple quantum conductor in a low impedance electromagnetic environment. A next step is to go beyond the assumptions made here by considering conductors with internal dynamics, such as quantum dots, or conductors placed in a high impedance environment. By engineering such environments, for which the dynamical Coulomb blockade becomes prominent, one can expect different mechanisms producing squeezing, entanglement, and even the generation of non-Gaussian electromagnetic fields.

We thank Julien Gabelli and Karl Thibault for useful discussions. This work was supported by the Canada Excellence Research Chairs program, NSERC, FRQNT via INTRIQ and the Université de Sherbrooke via EPIQ.

Note added.—Recently, we became aware of an alternate description of squeezing by tunnel junction in a resonator [40].

[1] R. J. Glauber, Phys. Rev. **84**, 395 (1951).
[2] R. J. Glauber, Phys. Rev. **131**, 2766 (1963).
[3] C. W. J. Beenakker and H. Schomerus, Phys. Rev. Lett. **86**, 700 (2001).
[4] C. W. J. Beenakker and H. Schomerus, Phys. Rev. Lett. **93**, 096801 (2004).
[5] A. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. B **81**, 155421 (2010).
[6] A. Bednorz, C. Bruder, B. Reulet, and W. Belzig, Phys. Rev. Lett. **110**, 250404 (2013).
[7] D. Walls and G. J. Milburn, *Quantum Optics* 2nd ed (Springer, Berlin, 2008).
[8] B. Yurke, P. G. Kaminsky, R. E. Miller, E. A. Whittaker, A. D. Smith, A. H. Silver, and R. W. Simon, Phys. Rev. Lett. **60**, 764 (1988).
[9] G. Gasse, C. Lupien, and B. Reulet, Phys. Rev. Lett. **111**, 136601 (2013).
[10] J.-C. Forgues, C. Lupien, and B. Reulet, Phys. Rev. Lett. **114**, 130403 (2015).
[11] B. Yurke and J. S. Denker, Phys. Rev. A **29**, 1419 (1984).
[12] B. Yurke, *Quantum Squeezing* (Springer, New York, 2004), Chap. 3.
[13] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[14] N. Didier, F. Qassemi, and A. Blais, Phys. Rev. A **89**, 013820 (2014).
[15] J. Leppäkangas, G. Johansson, M. Marthaler, and M. Fogelström, Phys. Rev. Lett. **110**, 267004 (2013).
[16] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.116.043602 for more information.
[17] We use the Fourier transform convention $f(t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} f[\omega]/2\pi$
[18] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[19] C. Gardiner and P. Zoller, *Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics* (Springer, New York, 2004).
[20] R. Graham, Z. Phys. B **76**, 265 (1989).
[21] J. Gabelli and B. Reulet, Proc. SPIE Int. Soc. Opt. Eng. **6600**, 66000T (2007).
[22] J. Gabelli and B. Reulet, arXiv:0801.1432.
[23] Y. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[24] J. Gabelli and B. Reulet, Phys. Rev. Lett. **100**, 026601 (2008).
[25] G. Lesovik and R. Loosen, J. Exp. Theor. Phys. Lett. **65**, 295 (1997).
[26] U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. B **62**, R10637 (2000).
[27] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).
[28] R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).
[29] R. Deblock, E. Onac, L. Gurevich, and L. P. Kouwenhoven, Science **301**, 203 (2003).
[30] J. R. Souquet, M. J. Woolley, J. Gabelli, P. Simon, and A. A. Clerk, Nat. Commun. **5**, 5562 (2014).
[31] C. Altimiras, O. Parlavecchio, P. Joyez, D. Vion, P. Roche, D. Esteve, and F. Portier, Phys. Rev. Lett. **112**, 236803 (2014).
[32] O. Parlavecchio, C. Altimiras, J.-R. Souquet, P. Simon, I. Safi, P. Joyez, D. Vion, P. Roche, D. Esteve, and F. Portier, Phys. Rev. Lett. **114**, 126801 (2015).
[33] M. Hofheinz, F. Portier, Q. Baudouin, P. Joyez, D. Vion, P. Bertet, P. Roche, and D. Esteve, Phys. Rev. Lett. **106**, 217005 (2011).
[34] V. Gramich, B. Kubala, S. Rohrer, and J. Ankerhold, Phys. Rev. Lett. **111**, 247002 (2013).
[35] A. D. Armour, M. P. Blencowe, E. Brahimi, and A. J. Rimberg, Phys. Rev. Lett. **111**, 247001 (2013).
[36] J. Bourassa, F. Beaudoin, J. M. Gambetta, and A. Blais, Phys. Rev. A **86**, 013814 (2012).
[37] The Markov approximation is valid when the environment's time scale is short with respect to κ^{-1} [38] or in other words for $\kappa^{-1} \gg \mathrm{Min}[\hbar/k_B T, \hbar/eV_{\mathrm{dc}}]$. As expected, this implies that the sample resistance should not be matched to Z_0.
[38] H. J. Carmichael, *Statistical Methods in Quantum Optics 1* (Springer, New York, 1999).
[39] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).
[40] U. C. Mendes and C. Mora, New J. Physs. **17**, 113014 (2015).