Validity of the Generalized Brink-Axel Hypothesis in ^{238}Np

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We analyze primary γ-ray spectra of the odd-odd ^{238}Np nucleus extracted from $^{237}\mathrm{Np}(d, p\gamma)^{238}\mathrm{Np}$ coincidence data measured at the Oslo Cyclotron Laboratory. The primary γ spectra cover an excitation-energy region of $0 \leq E_i \leq 5.4$ MeV, and allow us to perform a detailed study of the γ-ray strength as a function of excitation energy. Hence, we can test the validity of the generalized Brink-Axel hypothesis, which, in its strictest form, claims no excitation-energy dependence on the γ strength. In this work, using the available high-quality ^{238}Np data, we show that the γ-ray strength function is to a very large extent independent of the initial and final states. Thus, for the first time, the generalized Brink-Axel hypothesis is experimentally verified for γ transitions between states in the quasicontinuum region, not only for specific collective resonances, but also for the full strength below the neutron separation energy. Based on our findings, the necessary criteria for the generalized Brink-Axel hypothesis to be fulfilled are outlined.

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Sixty years ago, Brink proposed in his Ph.D. thesis [1] that the photoabsorption cross section of the giant electric dipole resonance (GDR) is independent of the detailed structure of the initial state. In his thesis, he expressed his hypothesis as follows: "If it were possible to perform the photo effect on an excited state, the cross section for absorption of a photon of energy E would still have an energy dependence given by [Eq.] (15)," where Eq. (15) refers to a Lorentzian shape of the photoabsorption cross section. Brink's original idea, the Brink hypothesis, was first intended for the photoabsorption process on the GDR, but has been further generalized, applying the principle of detailed balance, to include absorption and emission of γ rays between resonant states [2,3]. In addition to assuming the independence of the excitation energy, there is no explicit dependence on initial and final spins except the obvious dipole selection rules, implying that all levels exhibit the same dipole strength regardless of their initial spin quantum number. We will refer to this as the generalized Brink-Axel (gBA) hypothesis. A review of the history of the hypothesis was given by Brink in Ref. [4].

The gBA hypothesis has implications for almost any situation where nuclei are brought to an excited state above $\approx 2\Delta$, where $\Delta \approx 1$ MeV is the pair-gap parameter. Here, the nucleus will typically deexcite via γ-ray emission and/or by emission of particles. In this context, it is usual to translate the γ-ray cross section $\sigma(E_\gamma)$ into the γ-ray strength function (γSF) by $f(E_\gamma) = (3\pi^2\hbar^2c^2)^{-1}\sigma(E_\gamma)/E_\gamma$.

To describe and model the electric dipole part of the γ-decay channel, the gBA hypothesis is frequently used, applying in particular the assumption of spin independence [5]. For example, a rather standard approach to calculating the $E1$ strength is to apply some variant of the quasiparticle random-phase approximation (QRPA) to obtain $B(E1)$ values as a function of excitation energy, assuming that this $E1$ distribution corresponds to the one in the quasicontinuum; see, e.g., Ref. [6] and references therein. Also for $M1$ transitions the gBA hypothesis has been utilized; see, e.g., Ref. [7]. Furthermore, the hypothesis is also often applied to β decay and electron capture for calculating Gamow-Teller and Fermi transition strengths; see, e.g., Ref. [8] and references therein. The main reason for its wide range of applications is the drastic simplification of the considered problem, and in some cases it is a key necessity to be able to perform the desired calculation [8]. Hence, the question of whether the hypothesis is valid or not, and under which circumstances, is of utmost importance for multiple reasons: its fundamental impact on nuclear structure and dynamics, and its pivotal role for the description of γ and β decay for applied nuclear physics, such as input for (n,γ) cross-section calculations relevant to the r-process nucleosynthesis in extreme astrophysical environments [9] and next-generation nuclear power plants [10].

However, it is not at all obvious either from experiment or theory that the gBA hypothesis is valid. From an experimental point of view, there are two main reasons for this: the hypothesis has primarily been tested at very high excitation energies or with only a few states included. In the first case, compilations show that the width of the GDR varies with temperature and spin, in contradiction to the gBA hypothesis [11]. However, the hypothesis was not originally considered for building the GDR on such highly excited states. Obviously, thermal fluctuations will affect the width of the GDR, but the GDR energy centroid stays rather fixed. Other test cases suffer from large Porter-Thomas (PT) fluctuations [12], since the γSF could not be averaged over a sufficient number of levels [13]. In particular, this is the case for lighter nuclei or if levels close to the ground state are considered.

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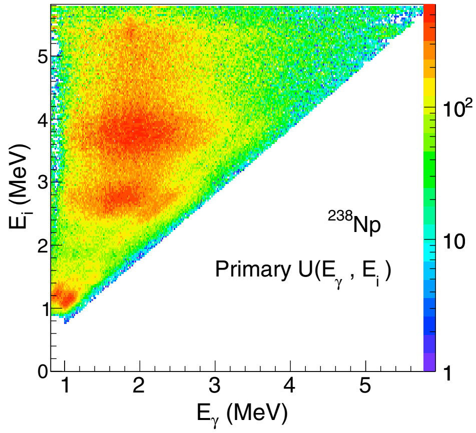

FIG. 1. Primary γ-ray matrix of ^{238}Np [27].

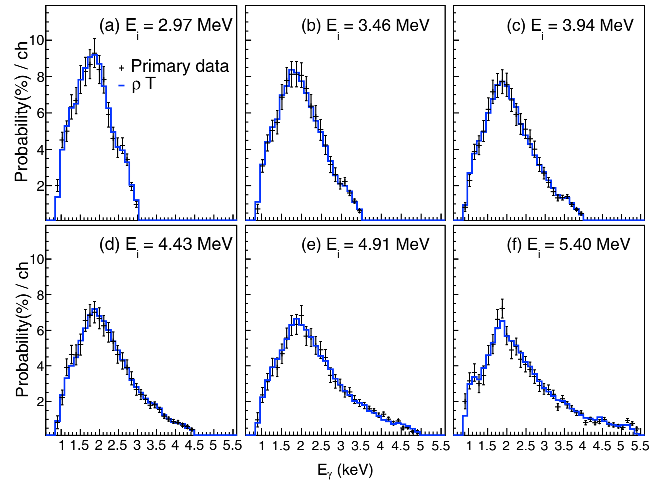

FIG. 2. Primary γ-ray spectra (data points) at various initial excitation energies compared to the product $\rho(E_i - E_\gamma)\mathcal{T}(E_\gamma)$ (blue curve).

In general, experimental data supporting the gBA hypothesis are rather scarce. For example, (n,γ) reactions give γSFs consistent with the gBA hypothesis, but in a limited γ-ray energy range [14–18]. Furthermore, data on the ^{89}Y$(p,\gamma)^{90}$Zr reaction point towards deviations from the gBA hypothesis [19]. There have also been various theoretical attempts to test the gBA hypothesis and modifications or even violations are found [20,21]. For some theoretical applications, the assumption of the gBA hypothesis is successfully applied [22–26]. We may learn from these experimental and theoretical attempts that the structure and dynamics of the system represent important constraints.

In this Letter, we address the gBA hypothesis from an experimental point of view, and we provide the needed criteria for the hypothesis to be valid for γ decay below the neutron threshold by a detailed analysis of the ^{238}Np γSF. The ^{238}Np nucleus is probably the ultimate case to test the gBA hypothesis, as it is an odd-odd system with an extremely high level density. Already a few hundred keV above the ground state, we find a level density of $\approx$200 MeV^{-1}, which increases to $\approx 43 \times 10^6$ MeV^{-1} at the neutron separation energy of $S_n = 5.488$ MeV. In a previous study [27], the level density and γSF were extracted from the distributions of primary γ rays measured in the ^{237}Np$(d, p\gamma)^{238}$Np reaction. This very rich data set represents ideal conditions for testing the gBA hypothesis where the PT fluctuations are negligible due to the high level density. In the following, we utilize the primary matrix of the initial excitation energy E_i versus the γ-ray energy [27].

Figure 1 shows the primary $U(E_\gamma, E_i)$ γ spectra (unfolded with the detector response functions) as a function of initial excitation energy E_i. We now normalize U to obtain the probability that the nucleus emits a γ ray with energy E_γ at the excitation energy E_i by $P(E_\gamma, E_i) = U(E_\gamma, E_i)/\sum_{E_\gamma} U(E_\gamma, E_i)$. The probability is assumed to be factorized into

$$P(E_\gamma, E_i) \propto \rho(E_i - E_\gamma)\mathcal{T}(E_\gamma). \qquad (1)$$

According to Fermi's golden rule [28,29], the decay probability P is proportional to the level density at the final energy $\rho(E_i - E_\gamma)$. The decay probability is also proportional to the squared transition matrix element $|\langle f|\hat{T}|i\rangle|^2$ between the initial $|i\rangle$ and final $\langle f|$ states, which is represented by the γ-ray transmission coefficient $\mathcal{T}$ when averaged over many transitions with the same transition energy E_γ. For now, let us assume that the transmission coefficient depends only on E_γ, in accordance with the gBA hypothesis.

The factorization given in Eq. (1) allows us to simultaneously extract the functions ρ and $\mathcal{T}$ from the two-dimensional probability landscape P. The technique used, the Oslo method [30], requires no models for these functions. In the present case [27], we have fitted each vector element of the two functions to the following region of the P matrix: $3.0 \leq E_i \leq 5.4$ MeV and $E_\gamma > 0.84$ MeV. The justification of this standard procedure has been experimentally tested for many nuclei by the Oslo group [31] and a survey of possible errors for the Oslo method was presented in Ref. [32].

The applicability of Eq. (1) and the quality of the least χ^2 fitting procedure are demonstrated in Fig. 2. The agreement is very satisfactory with $\chi^2_{\text{reduced}} = 0.81$, and indicates that the determination of the level density ρ and the transmission coefficient $\mathcal{T}$ works well. In the following we assume that ρ and $\mathcal{T}$ are normalized according to the procedure in Ref. [30]. Thus, we introduce a normalization factor N in Eq. (1), which only depends on the initial excitation energy, and rewrite

$$N(E_i)P(E_\gamma, E_i) = \rho(E_i - E_\gamma)\mathcal{T}(E_\gamma), \qquad (2)$$

which determines the normalization factor by

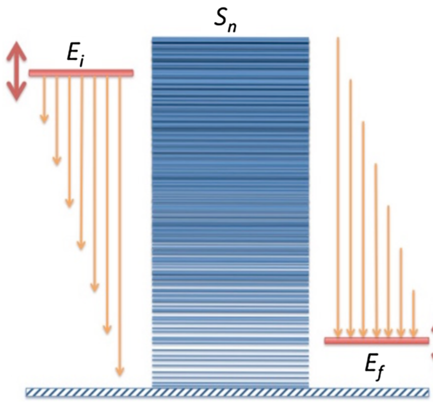

FIG. 3. The procedure to extract the γSFs as functions of the initial E_i (left) and final E_f (right) excitation energies. The blue-shaded region (middle) illustrates the exponentially increasing level density as a function of excitation energy. The two γSFs are limited to $E_\gamma < E_i$ and $E_\gamma < S_n - E_f$, respectively, where S_n is the neutron separation energy.

$$N(E_i) = \frac{\int_0^{E_i} \mathcal{T}(E_\gamma)\rho(E_i - E_\gamma)dE_\gamma}{\int_0^{E_i} P(E_\gamma, E_i)dE_\gamma}. \quad (3)$$

It is an open question if the transmission coefficient $\mathcal{T}$ actually changes with the excitation energy, as this procedure gives an average $\mathcal{T}$ for a rather wide range of initial excitation energies in the standard Oslo method. Hence, it is possible that variations of $\mathcal{T}$ as a function of the initial (or final) excitation energy might be masked. In the following, we will investigate whether $\mathcal{T}$ depends on the initial and final excitation energies in order to test the gBA hypothesis. For this we collect γ transitions from certain initial states or γ transitions into certain final states as illustrated in Fig. 3.

The idea is that, as the level density ρ is known, the γ transmission coefficient can be studied in detail per the excitation energy bin simply by NP/ρ from Eq. (2). More specifically, we get for the initial states

$$\mathcal{T}(E_\gamma, E_i) = \frac{N(E_i)P(E_\gamma, E_i)}{\rho(E_i - E_\gamma)}, \quad (4)$$

or alternatively for the final states

$$\mathcal{T}(E_\gamma, E_f) = \frac{N(E_f + E_\gamma)P(E_\gamma, E_f + E_\gamma)}{\rho(E_f)}, \quad (5)$$

where $E_f = E_i - E_\gamma$. One should note that the normalization factor N is calculated from the assumption that both $\mathcal{T}(E_\gamma, E_i)$ and $\mathcal{T}(E_\gamma, E_f)$ fluctuate on the average around the excitation-independent $\mathcal{T}(E_\gamma)$, see Eq. (3).

We now translate the γ transmission coefficient into the γSF by [18]

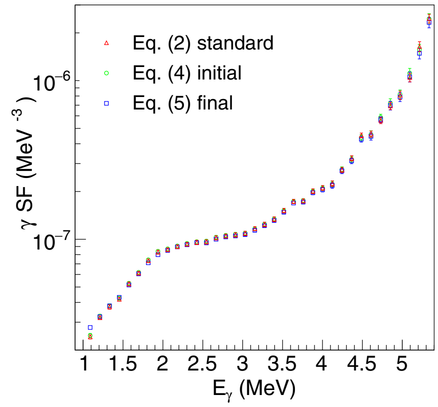

FIG. 4. Comparison between the three γSFs obtained by Eqs. (2), (4), and (5).

$$f(E_\gamma) = \frac{1}{2\pi}\frac{\mathcal{T}(E_\gamma)}{E_\gamma^{2L+1}}, \quad (6)$$

where we assume that dipole radiation ($L = 1$) dominates the γ decay in the quasicontinuum. This is motivated by known discrete γ-ray transitions from neutron capture [18] and angular distributions of primary γ-rays measured at high excitation energies [33].

To check that the normalization function $N(E_i)$ is reasonable, we compare the three γSFs obtained from Eqs. (2), (4), and (5), where $f(E_\gamma, E_i)$ and $f(E_\gamma, E_f)$ are averaged over the initial and final excitation energies by

$$f_i(E_\gamma) = \frac{1}{S_n - E_\gamma}\int_0^{S_n - E_\gamma} f(E_\gamma, E_i)dE_i, \quad (7)$$

$$f_f(E_\gamma) = \frac{1}{S_n - E_\gamma}\int_{E_\gamma}^{S_n} f(E_\gamma, E_f)dE_f, \quad (8)$$

respectively. Figure 4 demonstrates that the three extraction methods give $f(E_\gamma) = f_i(E_\gamma) = f_f(E_\gamma)$ within the experimental errors. This supports the normalization function N used in Eqs. (4) and (5).

We are now ready to compare our data with the gBA hypothesis. Figure 5 shows the initial excitation energy dependent $f(E_\gamma, E_i)$ compared with $f(E_\gamma)$ obtained with the standard Oslo method (blue curve). The excitation-energy bins are $\Delta E_i = 121$ keV wide, and only every fourth gate is shown. The overall agreement is excellent, and the same holds also for all the bins not shown. It is clear that each of these γSFs is built on a specific initial excitation-energy gate, but with no specific final state, as illustrated in Fig. 3. However, for a given E_γ and E_i, the final excitation energy is determined. Since all data points coincide with $f(E_\gamma)$, this also indicates the independence of the final state.

Any potential dependence of the final state is best analyzed by $f(E_\gamma, E_f)$ as given by Eq. (5) and shown in

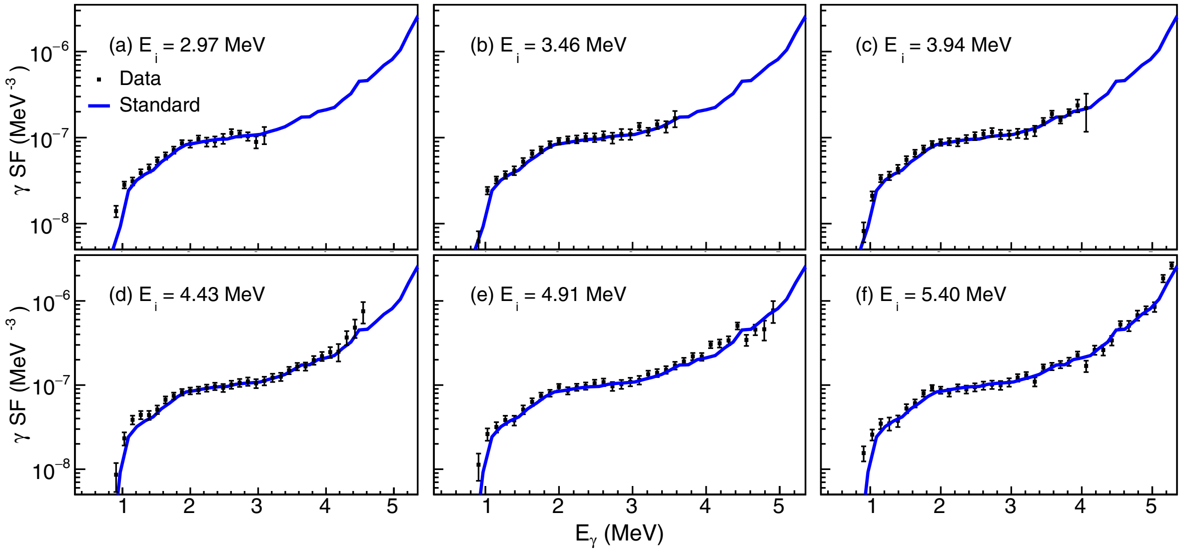

FIG. 5. The γSFs as functions of initial excitation energies (data points), see Eq. (4). The blue curve is obtained by the standard Oslo method, see Eq. (2). The excitation energy bins are 121 keV broad.

Fig. 6. Again, we find an excellent agreement between the various γSFs with γ transitions into specific final excitation-energy bins. However, there are discrepancies for $E_\gamma < 1$ MeV, which feed the final states below $\approx$1 MeV. At these energies, $f(E_\gamma, E_f)$ shows an increase compared to the average $f(E_\gamma)$. These γ transitions could possibly be due to vibrational modes built on the ground state and, if this is true, are not part of a general γSF extracted in the quasicontinuum with the standard Oslo method. Vibrational levels are strongly populated in inelastic scattering, such as the reactions ^{237,239}Np$(d,d')^{237,239}$Np performed by Thompson *et al.* [34]. In that work, levels built on vibration modes were seen for excitation energies in the $\approx$0.9- and $\approx$1.6-MeV regions. A similar population of vibrational states has been observed in the ^{238}U(^{16}O, ^{16}O$'$)^{238}U and ^{238}U$(\alpha,\alpha')^{238}$U reactions [35]. By means of $\alpha\gamma$ coincidences, a concentration of $E_\gamma \approx 1$ MeV transitions depopulating β, γ, and octupole vibrational bands has been seen. Thus, the enhanced γSF found in our data at low excitation energies with $E_\gamma \approx$ 1 MeV is likely due to the deexcitation of vibrational structures. Such structures might show up at higher excitations, but they are strongly fragmented and therefore difficult to observe.

The excellent agreement between the excitation energy dependent and independent γSFs indicates that PT fluctuations are small compared to the experimental errors for the system studied. For the χ^2_ν distribution, which governs the

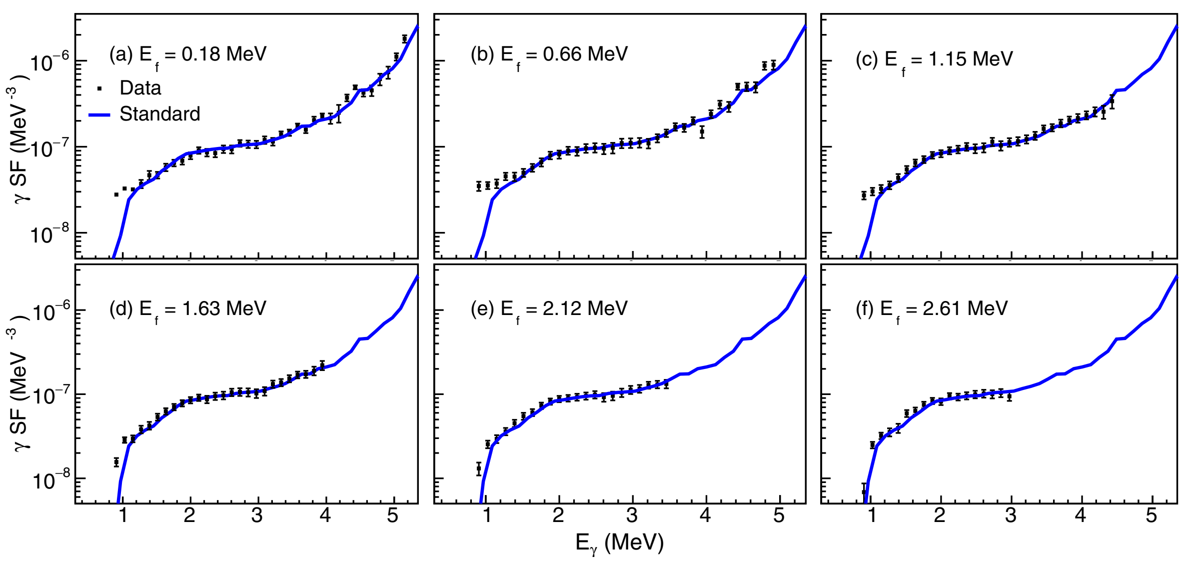

FIG. 6. The γSFs as functions of the final excitation energies (data points), see Eq. (5). See the text of Fig. 5.

PT fluctuations, the relative fluctuations are given by $r = \sqrt{(2/\nu)}$, where ν is the number of degrees of freedom [36]. Typically, we have at 2.0 and 4.0 MeV excitation energies, ≈1200 and ≈120 000 levels within the 121-keV excitation energy bins. Taking the number of levels as the number of degrees of freedom, we obtain $r = 4.1\%$ and 0.4%, respectively, which should be compared with the data error bars of Figs. 5 and 6 of typically 10%. Thus, in the ^{238}Np case, the PT fluctuations are smaller than the statistical errors and not significant. However, for systems with fewer than ≈200 levels per bin the PT fluctuations will exceed the experimental statistical error of 10%. This gives guidance for the necessary statistics and the required level density for the gBA hypothesis to be fulfilled.

In conclusion, we have studied the γ-ray strength function between well defined excitation energy bins in ^{238}Np. For the first time, the generalized Brink-Axel hypothesis has been verified in the nuclear quasicontinuum. The discrepancies seen in the low excitation energy region are probably caused by the decay of vibrational states built on the ground state. These excitation modes are not part of the γ-ray strength function of the quasicontinuum. The validity of the generalized Brink-Axel hypothesis requires that Porter-Thomas fluctuations be low by averaging over a sufficient number of levels compared to the experimental errors.

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