High Precision Determination of the β Decay Q_{EC} Value of ^{11}C and Implications on the Tests of the Standard Model

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We report the determination of the Q_{EC} value of the mirror transition of ^{11}C by measuring the atomic masses of ^{11}C and ^{11}B using Penning trap mass spectrometry. More than an order of magnitude improvement in precision is achieved as compared to the 2012 Atomic Mass Evaluation (AME2012) [Chin. Phys. C **36**, 1603 (2012)]. This leads to a factor of 3 improvement in the calculated $\mathcal{F}t$ value. Using the new value, $Q_{EC} = 1981.690(61)$ keV, the uncertainty on $\mathcal{F}t$ is no longer dominated by the uncertainty on the Q_{EC} value. Based on this measurement, we provide an updated estimate of the Gamow-Teller to Fermi mixing ratio and standard model values of the correlation coefficients.

The standard electroweak model (SM) assumes the vector and axial-vector character of the weak interaction with maximal parity violation. Precision measurements in nuclear β decay play a crucial role in establishing the limits of contributions from exotic scalar or tensor interactions as signatures for new physics. Within the SM, a result of the conserved vector current (CVC) hypothesis is that, for all superallowed pure Fermi transitions, the product of the corrected statistical rate function $\mathcal{F}$ and partial half-life t of the decay has the same value. A similar treatment is possible for β transitions between $T = 1/2$ isospin doublets in mirror nuclei provided that the Gamow-Teller to Fermi mixing ratio ρ is taken into account.

The relation between $\mathcal{F}t$, the statistical rate function for the vector part of the interaction f_V, and the small ($\sim$1%) correction terms is expressed as [1]

$$\mathcal{F}t^{\text{mirror}} = f_V t(1 + \delta'_R)(1 + \delta^V_{NS} - \delta^V_C), \quad (1)$$

where δ'_R is the nucleus-dependent radiative correction, δ^V_{NS} is the nuclear structure correction, and δ^V_C is the isospin symmetry breaking correction. For mixed transitions, $\mathcal{F}t$ is related to the V_{ud} element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the Gamow-Teller to Fermi mixing ratio by [1]

$$\mathcal{F}t^{\text{mirror}} = \frac{K}{G_F^2 V_{ud}^2 C_V^2 |M_F^0|^2 (1 + \Delta_R^V)(1 + \frac{f_A}{f_V}\rho^2)}, \quad (2)$$

where $K/(\hbar c)^6 = 2\pi^3 (\ln 2)\hbar/(m_e c^2)^5 = 8120.2787(11) \times 10^{-10}$ GeV^{-4} s, $G_F/(\hbar c)^3 = 1.16637(1) \times 10^{-5}$ GeV^{-3} is the Fermi constant, $C_V = 1$ is the vector coupling constant, $|M_F^0|^2$ is the isospin symmetry limit value of the Fermi matrix element squared, $\Delta_R^V = 0.02361(38)$ [2] is a transition-independent radiative correction, and f_A is the statistical rate function for the axial-vector part of the interaction. A precise determination of $\mathcal{F}t^{\text{mirror}}$ allows us to extract the Gamow-Teller to Fermi mixing ratio and standard model values of the correlation coefficients. Thus, constraints on scalar and tensor couplings can be obtained by comparing the measured and predicted values for the correlation coefficients.

Another stringent test of the SM is provided by the unitarity condition of the CKM matrix. For the first row of the CKM matrix, V_{ud} and V_{us} elements are the dominant contributors to the value and accuracy of the unitarity test. Traditionally, V_{ud} is extracted from superallowed $0^+ \rightarrow 0^+$ pure Fermi transitions [3], neutron [4] and pion β decays [5]. The most recent survey of the superallowed $0^+ \rightarrow 0^+$ nuclear decays reported a value of V_{ud} with a precision of 2.2×10^{-4} [3].

In addition to the traditional sources mentioned above to determine V_{ud}, a complementary, independent approach involving the β transitions between $T = 1/2$ isospin doublets in mirror nuclei has attracted increasing interest [6]. Such transitions are mixed and hence both vector and axial-vector interactions contribute. Although the value of V_{ud} obtained from mirror decays is not as precise as that from superallowed $0^+ \rightarrow 0^+$ transitions, an independent

determination of V_{ud} using data from mirror transitions in ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar, and ^{37}K reported a precision of 1.7×10^{-3} [6]. This uncertainty is dominated by the experimental precision in the Gamow-Teller to Fermi mixing ratio.

A precise determination of the $\mathcal{F}t$ value demands precision in three experimental quantities: the decay transition energy Q_{EC}, which enters in the calculation of the statistical rate function; the half-life of the parent state $t_{1/2}$, and the branching ratio for the particular transition, which are used to calculate the partial half-life. Of the nineteen $T = 1/2$ mirror β transitions surveyed in Ref. [1], there are only a few transitions for which the uncertainty on the Q_{EC} value is the dominant contributor to the uncertainty on the $\mathcal{F}t$ value. Of those few nuclei, ^{11}C stands out as the parent nucleus with the highest fractional uncertainty in $\mathcal{F}t$ due to the uncertainty in the Q_{EC} value.

In this Letter, we report a new determination of the Q_{EC} value of the mirror transition in ^{11}C β decay. We have measured the atomic masses of ^{11}C and ^{11}B using Penning trap mass spectrometry, which has proven to be the most precise and accurate method for determining atomic masses [7].

The measurements were performed at the Low Energy Beam and Ion Trap (LEBIT) facility [8] at the National Superconducting Cyclotron Laboratory (NSCL). The gas cell and the main components of the LEBIT facility are shown in Fig. 1. A ^{11}C rare isotope beam was produced by projectile fragmentation of a 150 MeV/nucleon ^{16}O primary beam on a beryllium target. The fragment beam was separated in flight by the A1900 separator [9] after which only ^{11}C (64%) and ^{10}B (36%) were observed in the beam composition. The beam was decelerated to an energy of less than 1 MeV/nucleon by passing through a system of aluminum degraders and a fused silica wedge before entering the gas cell filled with high-purity helium gas at a pressure of 93 mbar [10]. The thickness of the

FIG. 1. Schematic diagram of the gas stopper cell and the LEBIT facility.

degraders was selected to stop a large fraction of the ^{11}C beam in the gas cell. Ions from the gas cell were transported through a radiofrequency quadrupole ion guide and purified further by a magnetic dipole with resolving power of approximately 1500. The beta activity of the ^{11}C beam was verified by measuring its half-life using a detector located downstream of the magnetic dipole separator before entering the LEBIT facility.

The ^{11}C rare isotope beam was guided with an electrostatic beam transport system to a two-stage cooler and buncher [11], where the ions were cooled in a helium buffer gas at $\sim 2 \times 10^{-2}$ mbar and then bunched in a region with $\sim 10^{-3}$ mbar. The ions were then ejected as submicrosecond pulses towards the 9.4 T Penning trap mass spectrometer, where they were confined by a superposition of a strong homogeneous magnetic field and a weak electrostatic quadrupole field. In the Penning trap, the ions were excited by applying a quadrupolar rf electric field at a frequency ν_{rf} near the cyclotron frequency ν_c of the ion [12]. The ions were then ejected from the trap and directed to a multi-channel plate (MCP) detector in a Daly configuration with a measured efficiency of 63%. The time-of-flight ion cyclotron resonance (TOF-ICR) detection technique [13] was used to obtain TOF resonance curves. The resonance curves were fitted to the theoretical line shape [12] to determine the frequency with the minimum TOF, which occurs when $\nu_{rf} = \nu_c$. The relationship between the cyclotron frequency ν_c and mass of the ion m_{ion} is given by

$$\nu_c = \frac{1}{2\pi} \frac{qB}{m_{ion}}, \qquad (3)$$

where q is the charge of the ion and B is the magnetic field strength.

Four 50-ms and five 250-ms continuous excitation, and three 250-ms Ramsey-type resonances [14,15] were obtained with a total of 1906 ^{11}C$^+$ ions. For the Ramsey excitation scheme, two 50-ms long rf excitations separated by a 150-ms long wait time were used. One of the 250-ms Ramsey-type resonance curves is shown in Fig. 2. Cyclotron frequency measurements of ^{11}C$^+$ ions were bracketed by reference measurements of ^{14}N$^+$ ions that were produced in a plasma ion source located perpendicular to the LEBIT beam line and transported to the LEBIT cooler and buncher via an electrostatic bender. Each pair of cyclotron frequency measurements of ^{14}N$^+$, bracketing a ^{11}C$^+$ measurement, were linearly interpolated to account for any drift in the magnetic field. The ratio of the cyclotron frequencies of the measured and reference ions are related to the ratio of the ion masses by $\nu_c(^{11}\mathrm{C}^+)/\nu_c(^{14}\mathrm{N}^+) = m(^{14}\mathrm{N}^+)/m(^{11}\mathrm{C}^+)$. A 50-150-50-ms Ramsey excitation scheme was used for all ^{14}N$^+$ resonances with approximately 1250 ions in each measurement.

As the measured ^{11}C$^+$ ion and the ^{14}N$^+$ reference ion have different mass numbers, the measured frequency ratio

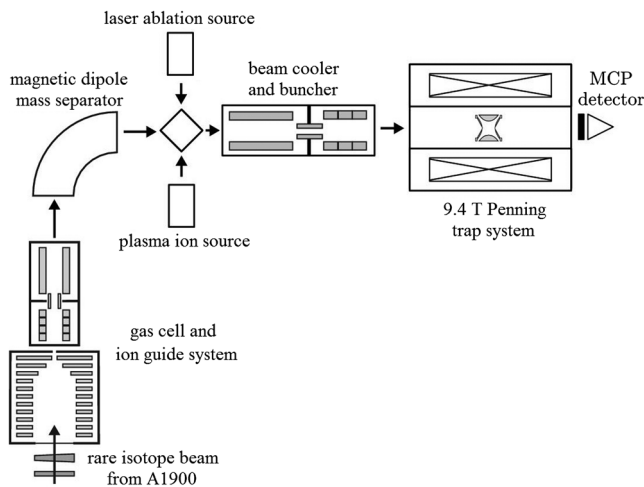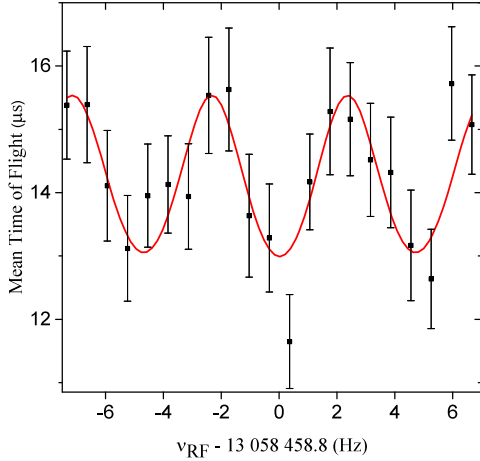FIG. 2.   One of the ^{11}C$^+$ time-of-flight ion cyclotron resonances used for the determination of $\nu_c(^{11}$C$^+)$. This Ramsey-type resonance contains 461 detected ^{11}C$^+$ ions. The solid line is the theoretical line shape [12] fitted to the data.

was corrected to account for the mass-dependent systematic effects arising from, for example, Penning trap imperfections such as deviations from a purely quadrupole electric potential and trap misalignment with respect to the magnetic field. The value of the mass-dependent systematic shift at LEBIT was previously determined to be 2.0×10^{-10}/u [16]. This shift was also added quadratically to the statistical uncertainty. Nonlinear fluctuations in the magnetic field strength were minimized by stabilizing the pressure in the liquid helium cryostat of the solenoid magnet to reduce their contribution to the uncertainty on the cyclotron frequency ratio to less than 1×10^{-10} [17]. Systematic frequency shifts that may arise from Coulomb interactions between ions of the same species were minimized by limiting the analysis to include only events with five or fewer detected ions, corresponding to eight or fewer ions in the trap. The relativistic frequency shift associated with the motional degrees of freedom in the trap was calculated to be more than an order of magnitude smaller than the statistical uncertainty. A near-unity Birge ratio [18] of 0.92(14) indicates that additional statistical effects are unlikely. The weighted average cyclotron frequency ratio was determined to be $R = \nu_c(^{11}$C$^+)/\nu_c(^{14}$N$^+) = 1.271\,698\,566\,8(74)$. Once the frequency ratio was determined, the atomic mass of ^{11}C was obtained using

$$m(^{11}\text{C}) = \left[m(^{14}\text{N}) - m_e + \frac{b_\text{N}}{c^2} \right] R^{-1} + m_e - \frac{b_\text{C}}{c^2}, \quad (4)$$

where $m(^{11}$C$)$ and $m(^{14}$N$)$ are the atomic masses of ^{11}C and ^{14}N, respectively, m_e is the mass of an electron, c is the speed of light, b_N and b_C are the first ionization energies of nitrogen and carbon [19], respectively.

Using the AME2012 value [20] for the atomic mass of ^{14}N, the atomic mass of ^{11}C is obtained as $m(^{11}$C$) = 11.011\,432\,598\,(64)$ u corresponding to a mass excess (ME) of 10 649.397(60) keV. The new value agrees with the AME2012 ME value, 10 650.3(9) keV, but is more than an order of magnitude more precise. In the AME2012, the mass of ^{11}C is influenced fully by the ^{11}C$(\beta^+)^{11}$B decay Q value [21]. The ^{11}B mass is related to ^{10}B via (n, γ) measurements (99.8% relative influence), and ^{10}B mass is related to ^{12}C via (α, d) reaction measurements (99.2% relative influence).

In order to determine the Q_EC value with the same level of improvement in precision, the mass of ^{11}B should also be known precisely. Since ^{11}B is stable, we have produced it using an offline ion source. A laser ablation ion source [22] located opposite to the plasma ion source was utilized in the production of ^{11}B and ^{12}C, which was used as the reference ion. Semicircular targets of natural boron and carbon were mounted on each half of a circular, rotatable holder inside the vacuum chamber. The desired ions were produced by selectively rotating the target via a computer-controlled stepper motor. The same procedure described above for the ^{11}C$^+$ vs ^{14}N$^+$ cyclotron frequency measurement was followed for the ^{11}B$^+$ vs ^{12}C$^+$ measurement.

A total of fifteen 250-ms Ramsey (50-150–50 ms) resonance curves were obtained with approximately 2000 ^{11}B$^+$ ions in each resonance. The ^{11}B$^+$ cyclotron frequency measurements were bracketed by ^{12}C$^+$ reference measurements with about 1400 ^{12}C$^+$ ions per resonance. After taking the mass dependent shift into account, the weighted average frequency ratio was found to be $\nu_c(^{11}$B$^+)/\nu_c(^{12}$C$^+) = 1.089\,991\,528\,8(13)$, resulting in an atomic mass value of $m(^{11}$B$) = 11.009\,305\,167(13)$ u and a ME of 8667.707(12) keV. This result agrees with the AME2012 ME value for ^{11}B with an improvement in precision by about a factor of 30.

Additionally, we have reduced the atomic mass uncertainty in the other stable isotope of boron, ^{10}B, by

TABLE I.   Measured frequency ratios, $R = \nu_c$(ion of interest)$/\nu_c$(reference ion), and calculated atomic mass and mass excess (ME) values of ^{11}C, ^{11}B, and ^{10}B and their comparison to the values from the 2012 Atomic Mass Evaluation [20].

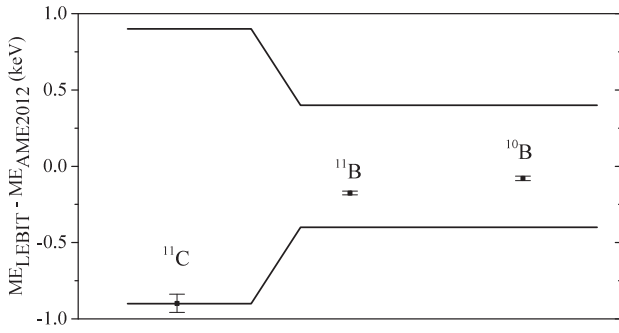| Isotope	Reference	Frequency ratio R	Mass (u)	ME (keV)	AME2012 (keV)
^{11}C	^{14}N	1.271 698 566 8(74)	11.011 432 598(64)	10 649.397(60)	10 650.3(9)
^{11}B	^{12}C	1.089 991 528 8(13)	11.009 305 167(13)	8667.707(12)	8667.9(4)
^{10}B	^{12}C	1.198 460 455 5(19)	10.012 936 862(16)	12 050.611(15)	12 050.7(4)

FIG. 3. Comparison of the mass excess values of the LEBIT measurements to the AME2012 values for ^{11}C, ^{11}B, and ^{10}B. The AME2012 uncertainties for each isotope are shown as horizontal lines centered on 0.
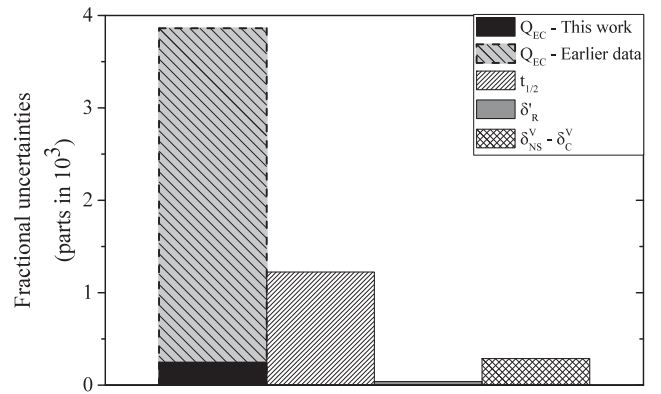

FIG. 4. Contribution of uncertainties in experimental and theoretical parameters to the final uncertainty in $\mathcal{F}t^{\mathrm{mirror}}$.

measuring the cyclotron frequency ratio $\nu_c(^{10}\mathrm{B}^+)/\nu_c(^{12}\mathrm{C}^+)$. The frequency ratio, our calculated atomic mass and ME values, and their comparison to AME2012 for ^{10}B are summarized in Table I along with the values for ^{11}B and ^{11}C. The comparison of the new ME values to the AME2012 values and the level of improvement in precision are shown graphically in Fig. 3. The new masses for ^{10}B and ^{11}B yield a Q value of 11 454.221(19) keV, in good agreement with $^{10}\mathrm{B}(n,\gamma)^{11}\mathrm{B}$ measurements, 11 454.12(16) keV [20]. The new mass for ^{10}B can also be combined with the JYFLTRAP ^{10}C β decay Q value measurement [23] to obtain a more precise value for the mass of ^{10}C. The new mass excess for ^{10}C was calculated to be 15 698.73(8) keV, in good agreement with the AME2012 value, 15 698.76(39) keV.

Using the new atomic masses for ^{11}C and ^{11}B, we calculate the Q_{EC} value of the mirror β transition to be 1981.690(61) keV, which is in good agreement with the AME2012 value, 1982.4(1.0) keV, with more than an order of magnitude smaller uncertainty.

Equation (1) was utilized to calculate the $\mathcal{F}t^{\mathrm{mirror}}$ value using the new Q_{EC} value. The vector part of the statistical rate function f_V was calculated with the parametrization presented in Ref. [24]. The accuracy of the method was probed by first calculating f_V using the parametrization with the Q_{EC} value from Ref. [1] and then comparing the result to f_V given in there. The difference was found to be at the 0.01% level, which is consistent with the desired level of accuracy in Ref. [24]. This discrepancy was added quadratically to the uncertainty in f_V. The partial half-life t was calculated using the decay half-life, electron capture probability, and branching ratio from Ref. [1]. The theoretical corrections δ_R', δ_{NS}^V, and δ_C^V were also taken from Ref. [1].

The precision of the new Q_{EC} value improves the precision of the $\mathcal{F}t^{\mathrm{mirror}}$ value by a factor of 3. As a result, the uncertainty in the $\mathcal{F}t^{\mathrm{mirror}}$ value is no longer dominated by the uncertainty in the Q_{EC} value. A summary of the contribution of the uncertainties in experimental and

theoretical parameters to the final uncertainty in $\mathcal{F}t^{\mathrm{mirror}}$ is shown in Fig. 4. The new $\mathcal{F}t^{\mathrm{mirror}}$ value and its comparison to the value from Ref. [1] are given in Table II.

A value for the Gamow-Teller to Fermi mixing ratio ρ is necessary to calculate V_{ud} using Eq. (1). The mixing ratio can be extracted from measurements of correlation coefficients such as the $\beta-\nu$ correlation coefficient a, the β asymmetry parameter A, or the neutrino asymmetry parameter B. However, there are currently no such measurements for ^{11}C that would allow for the extraction of V_{ud}. We have alternatively calculated ρ by comparing $\mathcal{F}t^{\mathrm{mirror}}$ to the world-average value, $\bar{\mathcal{F}}t^{0^+\to 0^+} = 3072.27(72)$ s, obtained from the recent survey of 14 superallowed $0^+ \to 0^+ T = 1$ β decays [3]. Using $|M_F^0|^2 = 1$ for $T = 1/2$ mirror transitions and $|M_F^0|^2 = 2$ for pure Fermi transitions in Eq. (2), we can write

$$\mathcal{F}t^{\mathrm{mirror}} = \frac{2\mathcal{F}t^{0^+\to 0^+}}{1 + \frac{f_A}{f_V}\rho^2}. \tag{5}$$

The axial-vector part of the statistical rate function f_A was also calculated with the parametrization presented in Ref. [24]. After solving for the Gamow-Teller to Fermi mixing ratio ρ we calculated the SM values for the correlation coefficients a_{SM}, A_{SM}, and B_{SM} [1] which are

TABLE II. Comparison between the values of parameters derived from the present measurement and the values calculated in the 2008 survey for ^{11}C [1].

Parameter	This work	Value from Ref. [1]
Q_{EC}	1981.690(61) keV	1982.4(9) keV
f_V	3.1829(8)	3.193(12)
$\mathcal{F}t^{\mathrm{mirror}}$	3920.4(5.0) s	3933(16) s
ρ	0.7493(15)	0.7456(43)
a_{SM}	0.5206(13)	0.5236(35)
A_{SM}	$-0.599\,59(5)$	$-0.599\,46(16)$
B_{SM}	$-0.8872(8)$	$-0.8853(23)$

summarized in Table II, for the $(3/2)^- \rightarrow (3/2)^- \beta$ decay of ^{11}C. The uncertainties on these coefficients are also reduced by a factor of 3.

In summary, we have improved the precision in the Q_{EC} value of the mirror transition between ^{11}C and ^{11}B by more than an order of magnitude. This resulted in a factor of 3 improvement in the precision of the corresponding $\mathcal{F}t$ value and of the SM values of the correlation coefficients. The new Q_{EC} value has a negligible impact on the uncertainty in $\mathcal{F}t$ leaving the uncertainty in half-life as the dominant contributor. However, to determine V_{ud}, measurements of the correlation coefficients are needed. A measurement of the $\beta - \nu$ correlation coefficient a, for example, with a relative precision of 0.5% would enable the determination of V_{ud} with a relative uncertainty of 1.7×10^{-3}, which is comparable to the uncertainty of some of the 14 most precisely known superallowed pure Fermi decays from which V_{ud} is extracted [3].

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