Work Fluctuation-Dissipation Trade-Off in Heat Engines

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Reducing work fluctuation and dissipation in heat engines or, more generally, information heat engines that perform feedback control, is vital to maximize their efficiency. The same problem arises when we attempt to maximize the efficiency of a given thermodynamic task that undergoes nonequilibrium processes for arbitrary initial and final states. We find that the most general trade-off relation between work fluctuation and dissipation applicable to arbitrary nonequilibrium processes is bounded from below by the information distance characterizing how far the system is from thermal equilibrium. The minimum amount of dissipation is found to be given in terms of the relative entropy and the Renyi divergence, both of which quantify the information distance between the state of the system and the canonical distribution. We give an explicit protocol that achieves the fundamental lower bound of the trade-off relation.

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Recent developments in nonequilibrium statistical mechanics enable us to assign physical meanings to nonequilibrium entropies such as Shannon and von Neumann entropies in certain situations [1–3]. The information-theoretic analysis of thermodynamics starting from and ending at arbitrary nonequilibrium states has been carried out, as in encoding and erasure of information [3–6]. An important subset of this category is the information heat engines [1,7–15], since the measurement projects the state of the system into the postmeasurement state, which is usually out of equilibrium. They play a pivotal role in controlling small thermodynamic systems that operate at the level of thermodynamic fluctuations. Viewing biological processes as information processing requires us to quantify thermodynamic costs of biological sensory adaptation in terms of information-theoretic quantities [16]. Suppressing both work fluctuation and dissipation as much as possible is vital to heat engines and thermodynamic tasks since reducing dissipation allows us to increase the efficiency and reducing work fluctuation makes it possible to supply an exact amount of work needed to complete a given task or to extract a definite amount of work from the system.

Considerable effort has been devoted in the search for a protocol that minimizes work fluctuation and dissipation under nonequilibrium situations. Previous studies have explored the regime around vanishing work fluctuation by using techniques known as single-shot statistical mechanics [17–21], and the regime around vanishing dissipation on the basis of the second law of thermodynamics [3,4]. However, as we prove in the present work, these two aims (vanishing work fluctuation and vanishing dissipation) are incompatible. We find the trade-off relation between work fluctuation and dissipation with its fundamental lower bound set by the information distance characterizing the nonequilibriumness of the system. We also show that the bounds on dissipation in the single-shot (vanishing work fluctuation) and reversible (vanishing dissipation) regimes can be smoothly connected via the relative entropy [22] and the Renyi divergence [23], both of which quantify the information distance between the nonequilibrium distribution and the canonical distribution. We apply the trade-off relation to information heat engines, where the fundamental lower bound of the trade-off relation is characterized by the obtained information. Numerical simulations on an information heat engine based on a single-electron box [14,15] are performed to verify the trade-off relation. We propose a method to construct explicit protocols that achieve the lower bound of the trade-off relation.

Main results.—We define the extractable work from the system as a change of the internal energy that is not absorbed by the heat bath: $W[\Gamma]=E_{\lambda_0}(x)-E_{\lambda_1}(y)+Q[\Gamma]$, where Γ denotes the trajectory of the process, $Q[\Gamma]$ is the heat absorbed by the system, and $E_{\lambda_0}(x)$ and $E_{\lambda_1}(y)$ are the initial and final energies of the eigenstates, respectively. For nonequilibrium initial and final states, the maximum extractable work from the system is quantified by the nonequilibrium free-energy difference [1,3] $-\Delta\mathcal{F}(x,y)=\mathcal{F}_{\lambda_0}(x)-\mathcal{F}_{\lambda_1}(y)$, where $\mathcal{F}_{\lambda_0}(x)=E_{\lambda_0}(x)-\beta^{-1}S[p_{\mathrm{ini}}(x)]$, $S[q(x)]=-\ln q(x)$ is the Shannon entropy, and β is the inverse temperature of the heat bath. We define dissipation as the difference between the maximum extractable work and the actually extracted work:

$$\sigma[\Gamma]=-\beta[W[\Gamma]+\Delta\mathcal{F}(x,y)]. \tag{1}$$

The first main result of our work is the trade-off relation between work fluctuation and fluctuation in dissipation (see the Supplemental Material [24] for the proof):

$$\sqrt{\langle W^2\rangle-\langle W\rangle^2}+\beta^{-1}\sqrt{\langle\sigma^2\rangle-\langle\sigma\rangle^2}\geq\sqrt{\langle(\Delta\mathcal{F})^2\rangle-\langle\Delta\mathcal{F}\rangle^2}. \tag{2}$$

This result implies that the sum of the work fluctuation and the fluctuation in dissipation is bounded from below by the fluctuation of the nonequilibrium free-energy difference $\Delta\mathcal{F}$ [See Fig. 1(a)]. If the initial and final states are far from equilibrium, the lower bound of Eq. (2) becomes very large. The trade-off relation (2) indicates that work and dissipation cannot simultaneously take definite values; if we reduce work fluctuation, the fluctuation in entropy production inevitably increases, and vice versa.

The second main result is the trade-off relation between work fluctuation and dissipation. From Eq. (2), there is a nontrivial relation between σ and W if $\langle W^2\rangle - \langle W\rangle^2 \leq \langle(\Delta\mathcal{F})^2\rangle - \langle\Delta\mathcal{F}\rangle^2$. Then, let

$$\alpha = \frac{\sqrt{\langle W^2\rangle - \langle W\rangle^2}}{\sqrt{\langle(\Delta\mathcal{F})^2\rangle - \langle\Delta\mathcal{F}\rangle^2}}, \tag{3}$$

where $\alpha \in [0, 1]$. In this case, dissipation and work satisfy the following inequalities (see the Supplemental Material [24] for the proof):

$$\langle\sigma\rangle \geq (1-\alpha)[\Delta\mathcal{D}_\alpha(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) + \Delta d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1})], \tag{4}$$

$$\langle W\rangle \leq -\alpha\langle\Delta\mathcal{F}\rangle - (1-\alpha)[f_\alpha(p_{\rm fin}) - \mathcal{F}_\alpha(p_{\rm ini})]. \tag{5}$$

Here, $\Delta\mathcal{D}_\alpha = D - D_\alpha$ ($\Delta d_\alpha = d_\alpha - D$) gives the distance between the initial (final) distribution and the canonical distribution, $D(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) = \sum_x p_{\rm ini}(x)\ln[p_{\rm ini}(x)/p^{\rm can}_{\lambda_0}(x)]$ is the Kullback-Leibler divergence (relative entropy) [22], and

$$D_\alpha(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) = \frac{1}{\alpha-1}\ln\left[\sum_x [p_{\rm ini}(x)]^\alpha[p^{\rm can}_{\lambda_0}(x)]^{1-\alpha}\right] \tag{6}$$

is the Renyi divergence [23]. Here, d_α is defined by

$$d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1}) = \frac{1}{\alpha-1}\ln\left[\sum_{y\in Y}[p_{\rm fin}(y)]^\alpha[p^{\rm can}_{\lambda_1}(y)]^{1-\alpha}\right], \tag{7}$$

where the support Y is defined such that d_α takes the smallest value that satisfies

$$d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1}) \geq D_\infty(p_{\rm fin}\|p^{\rm can}_{\lambda_1}) = \ln\max_y \frac{p_{\rm fin}(y)}{p^{\rm can}_{\lambda_1}(y)}. \tag{8}$$

The lower bound of Eq. (4) is given by the black solid curve in Fig. 1(b). The asymmetry between D_α and d_α is due to the absence of the time-reversed protocol of the thermalization process as discussed later. In Eq. (5), $\langle\mathcal{F}_{\lambda_0}\rangle = \beta^{-1}D(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) + F^{\rm eq}(p^{\rm can}_{\lambda_0})$ is the averaged nonequilibrium free energy and $\mathcal{F}_\alpha(p_{\rm ini}) = \beta^{-1}D_\alpha(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) + F^{\rm eq}(p^{\rm can}_{\lambda_0})$ is the α generalization of the free energy, where we denote by $F^{\rm eq}(q)$ the equilibrium free energy whose corresponding canonical distribution is equal to the distribution q. We also define the free energy $f_\alpha(p_{\rm fin}) = \beta^{-1}d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1}) + F^{\rm eq}(p^{\rm can}_{\lambda_1})$ by using d_α. We note that the ordering of the Renyi divergence [32] $D_\infty \geq D \geq D_\alpha$ for $1 \geq \alpha$ with Eq. (8) implies $\Delta\mathcal{D}_\alpha \geq 0$ and $\Delta d_\alpha \geq 0$.

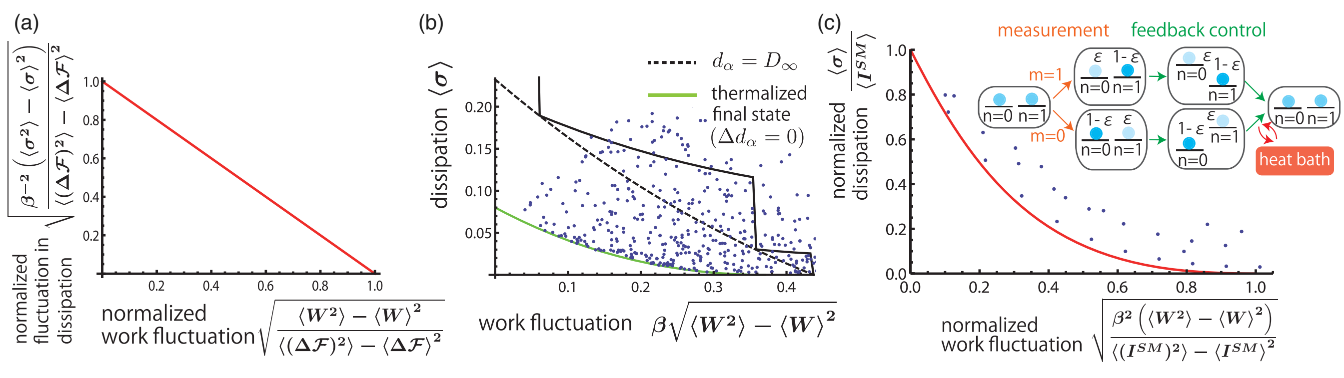

FIG. 1 (color online). Trade-off relations. (a) Normalized standard deviation of dissipation σ versus that of work W. The solid line shows the lower bound of the trade-off relation (2). (b) Average dissipation versus the standard deviation of work. The black solid curve shows the lower bound of the trade-off relation (3) and (4) for arbitrary initial and final states. If d_α takes the minimum value D_∞, the lower bound is given by the dashed curve. For a thermalized final state, $\Delta d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1}) = 0$ and the lower bound is given by the green solid curve. Each blue dot is obtained by a numerical simulation of a random quench of a five-level system followed by thermalization and isothermal expansion (see the Supplemental Material [24] for details). (c) The abscissa shows the standard deviation of work normalized by that of the fluctuation of the obtained information, and the ordinate shows the dissipation normalized by the mutual information between the system and the measuring apparatus. The solid curve shows the lower bound of the trade-off relation (10) and (12). Blue dots are obtained by a numerical simulation of a Szilard engine in a single-electron box [15] as illustrated in the inset (see the Supplemental Material [24] for details). Here, n denotes the excess number of electrons in the quantum dot, m denotes the outcome of the measurement of n, and ϵ is the measurement error rate, which is set to be $\epsilon = 0.02$ in the numerical simulation. The relevant two states $n = 0$ and $n = 1$ are assumed to be degenerate and initially populated with equal probability. Depending on the outcome m of the state measurement, the feedback control is performed by lowering the energy level of the m state relative to the other. Finally, the two energy levels are relaxed to their initial (equal-energy) state through thermal contact with a heat bath.

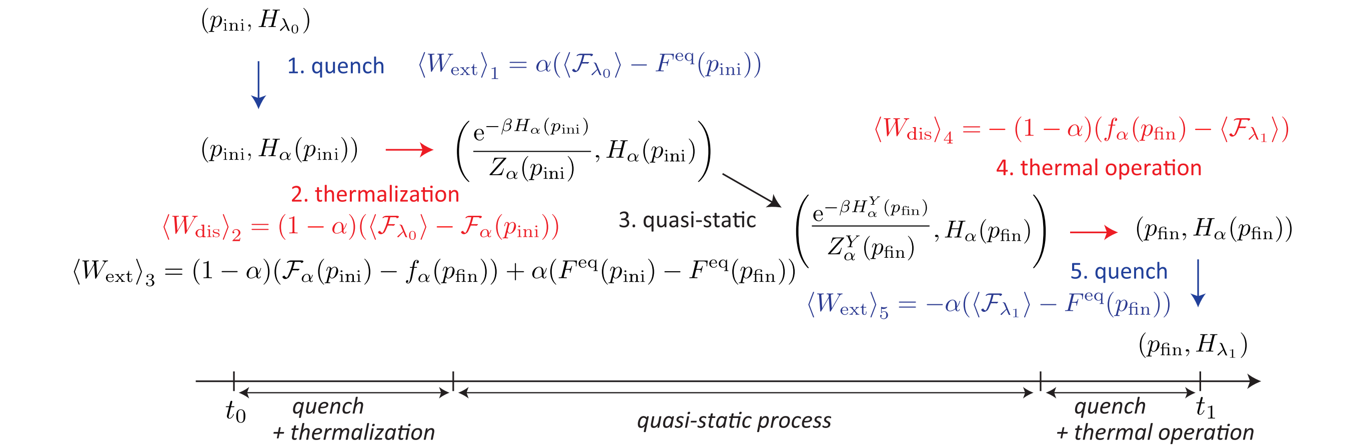

FIG. 2 (color online). Protocol achieving the lower bounds of the trade-off relations. We denote by (p, H) a pair of the probability distribution p and the Hamiltonian H. The transformation $(p_{\rm ini}, H_{\lambda_0}) \to (p_{\rm fin}, H_{\lambda_1})$ that achieves minimum work fluctuation and dissipation is illustrated, where a change in the Hamiltonian is shown in a vertical direction and a change in the state is shown in a horizontal direction. The explicit protocol consists of five steps, where the extractable work $\langle W_{\rm ext}\rangle$ and the dissipated work $\langle W_{\rm dis}\rangle \coloneqq \beta^{-1}\langle\sigma\rangle$ for each process are shown. Here, $H_\alpha(p_{\rm fin})$ is a Hamiltonian that satisfies $e^{-\beta H_\alpha(p_{\rm fin})}/Z_\alpha(p_{\rm fin}) \coloneqq (p_{\rm fin})^\alpha (p^{\rm can}_{\lambda_1})^{1-\alpha} e^{(1-\alpha)D_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1})}$.

Explicit protocol and the trade-off relation.—For given $(p_{\rm ini}, H_{\lambda_0})$ and $(p_{\rm fin}, H_{\lambda_1})$, we want to find a protocol that connects them by reducing both work fluctuation and dissipation as much as possible. Although a quasistatic process makes both work fluctuation and dissipation vanish, we cannot directly connect $(p_{\rm ini}, H_{\lambda_0}) \to (p_{\rm fin}, H_{\lambda_1})$ by the quasistatic process alone because the initial and final distributions are out of equilibrium. Instead, we prepare two canonical distributions as auxiliary intermediate states, and connect them by the quasistatic process. Then, we connect $(p_{\rm ini}, H_{\lambda_0})$ with one of the canonical distributions by combining a quench process followed by thermalization, and the other canonical distribution is connected with $(p_{\rm fin}, H_{\lambda_1})$ by a thermal operation and a quench process. The entire protocol is illustrated in Fig. 2, and as we show in the Supplemental Material [24], this protocol is necessary and sufficient to achieve the lower bound of Eqs. (2) and (4).

Now let us discuss the explicit protocol in more detail and consider the physical meanings of the quantities that appear in Eqs. (4) and (5). We change the initial distribution to the canonical distribution $e^{-\beta H_\alpha(p_{\rm ini})}/Z_\alpha(p_{\rm ini}) \coloneqq (p_{\rm ini})^\alpha (p^{\rm can}_{\lambda_0})^{1-\alpha} e^{(1-\alpha)D_\alpha(p_{\rm ini}\|p^{\rm can}_{\lambda_0})}$, which is an intermediate distribution between the initial state and the canonical distribution for the initial Hamiltonian. This is done by quenching the Hamiltonian from H_{λ_0} to $H_\alpha(p_{\rm ini})$ and extracting the work given by $\langle W_{\rm ext}\rangle_1 = \alpha[\langle\mathcal{F}_{\lambda_0}\rangle - F^{\rm eq}(p_{\rm ini})]$. Note that the maximum extractable work from the initial state is quantified by the nonequilibrium free energy $\langle\mathcal{F}_{\lambda_0}\rangle$. The unexpended free energy $(1-\alpha)\langle\mathcal{F}_{\lambda_0}\rangle$ is partly lost during the thermalization, and the remaining free energy, which can be extracted by the quasistatic process, is given by $(1-\alpha)\mathcal{F}_\alpha(p_{\rm ini})$, as can be seen by noting the dissipated work due to the measurement: $\langle W_{\rm dis}\rangle_2 = (1-\alpha)[\langle\mathcal{F}_{\lambda_0}\rangle - \mathcal{F}_\alpha(p_{\rm ini})]$. This dissipation $\beta\langle W_{\rm dis}\rangle_2 = (1-\alpha)\Delta D_\alpha(p_{\rm ini}\|p^{\rm can}_{\lambda_0}) = D(p_{\rm ini}\|e^{-\beta H_\alpha(p_{\rm ini})}/Z_\alpha(p_{\rm ini}))$ appears on the right-hand side of Eq. (4), which gives the information distance between the initial state and the canonical distribution, which we connect during the thermalization process. Thus, the right-hand side of Eq. (5) is composed of a part of the nonequilibrium free energy $\alpha\langle\mathcal{F}_{\lambda_0}\rangle$, which can be extracted by the quench process and the free energy $(1-\alpha)\mathcal{F}_\alpha(p_{\rm ini})$, which remains in the system after the thermalization.

The rest of the protocol is the transformation of the canonical distribution to the final state. Because we cannot perform time reversal of thermalization, we invoke a thermal operation [18] that transforms the state of the system by exchanging energy with the heat bath. This operation always changes the system closer to the thermal equilibrium, and we need to prepare a distribution whose "nonequilibriumness" is larger than that of the target final state. For this purpose, we prepare a localized distribution $e^{-\beta H^Y_\alpha(p_{\rm fin})}/Z^Y_\alpha(p_{\rm fin}) \coloneqq (p_{\rm fin})^\alpha (p^{\rm can}_{\lambda_1})^{1-\alpha} e^{(1-\alpha)d_\alpha(p_{\rm fin}\|p^{\rm can}_{\lambda_1})}$, whose support is restricted to Y. The term $(1-\alpha)f_\alpha(p_{\rm fin})$ is the free energy that is needed to prepare this localized thermal state and $\alpha\langle\mathcal{F}_{\lambda_1}\rangle$ is the free energy needed to quench the Hamiltonian back to the final one [see Eq. (5) and Fig. 2]. The asymmetry between the transformation of a nonequilibrium state into a thermalized state and its opposite transformation (i.e., from a thermalized state to a nonequilibrium state) gives rise to the difference between D_α and d_α [see Eq. (4)].

As shown in Ref. [18], the minimum work cost to create $p_{\rm fin}$ from a canonical distribution via the thermal operation with the fixed Hamiltonian $H_\alpha(p_{\rm fin})$ is given by $\beta^{-1}D_\infty(p_{\rm fin}\|e^{-\beta H_\alpha(p_{\rm fin})}/Z_\alpha(p_{\rm fin}))$, with the help of a two-level auxiliary system. If we can introduce this auxiliary system, the dissipated work for the thermal operation is found to be $\langle W_{\mathrm{dis}}\rangle_4 = (1-\alpha)[\mathcal{F}_\infty(p_{\mathrm{fin}}) - \langle \mathcal{F}_{\lambda_1}\rangle]$, and the equality condition in Eq. (8) is achieved (see the Supplemental Material [24] for details). This condition is also achieved if the energy level of the system is dense. The lower bound of Eq. (4) with $d_\alpha = D_\infty$ is shown by the dashed curve in Fig. 1(b). Note that the solid curve jumps (i.e., the support Y changes) wherever the line touches the dashed curve because we take discrete energy levels.

Comparison with previous studies.—For $\alpha = 1$, Eqs. (4) and (5) are equivalent to the second law of thermodynamics for arbitrary initial and final states: $\langle\sigma\rangle \geq 0$ and $\langle W\rangle \leq -\langle\Delta\mathcal{F}\rangle$. Since the canonical distribution $e^{-\beta H_\alpha(p_{\mathrm{ini}})}/Z_\alpha(p_{\mathrm{ini}})$ is equal to the initial state for $\alpha = 1$ (the same relation holds for the final state), we do not need thermalization and thermal operations to achieve the lower bound of the trade-off relations. Then, dissipation does not occur and we can extract the maximum average work from the system (see also Fig. 2). For $\alpha = 0$, Eq. (5) takes the form $\langle W\rangle \leq \mathcal{F}_0(p_{\mathrm{ini}}) - f_0(p_{\mathrm{fin}})$, which reproduces the single-shot results given in Refs. [17,18]. Here, $\mathcal{F}_0(p_{\mathrm{ini}}) = -\beta^{-1}\ln[\sum_{x\in X}\exp(-\beta E_{\lambda_0}(x))]$ is equal to the equilibrium local free energy whose support X is the same as the initial state. By raising the initially unoccupied energy levels, this amount of free energy remains after thermalization.

For a general α, the trade-off relation gives the minimum amount of work fluctuation and dissipation in the intermediate regime. Comparing Eqs. (3) and (5), we find that the distribution of the extractable work is broadened (meaning larger work fluctuation) if we want to increase the average value of work, and vice versa. Thus, the trade-off relation gives the best combinations of the "quality of work" and the average amount of extractable work. For equilibrium initial and final states, we can directly connect them by the quasistatic process and the lower bound of the trade-off relation (solid curve) in Fig. 1(b) shrinks to a single point at the origin; i.e., work fluctuation and dissipation can both vanish.

Applications to information heat engines.—The information heat engines utilize the information obtained by the measurement to extract work from the system. For simplicity, we consider a classical system and assume that the premeasurement state $p^S(x)$ is given by a canonical distribution. Then, dissipation is defined as the difference between the maximum amount of extractable work [13] $-\Delta F^S + \beta^{-1} I^{\mathrm{SM}}$ and the actually extracted work $W^S[\Gamma]$:

$$\sigma[\Gamma] = -\beta(W^S[\Gamma] + \Delta F^S) + I^{SM}(x,a) \tag{9}$$

and $I^{SM}(x,a) = \ln p^{SM}(x,a) - \ln[p^S(x)p^M(a)]$ is the (unaveraged) classical mutual information between the system (S) and the measurement apparatus (M) [22]. Here, $p^{SM}(x,a)$ is the joint probability distribution of SM for the postmeasurement state, $p^S(x) = \sum_a p^{SM}(x,a)$, and $p^M(a) = \sum_x p^{SM}(x,a)$. The trade-off relation (4) takes the following form [see also Fig. 1(c)]:

$$\langle\sigma\rangle \geq (1-\alpha)(I^{SM} - I^{SM}_\alpha), \tag{10}$$

$$\beta\langle W\rangle \leq \alpha\langle I^{SM}\rangle + (1-\alpha)I^{SM}_\alpha - \beta\Delta F^S, \tag{11}$$

where α is defined by

$$\alpha = \frac{\beta\sqrt{\langle W^2\rangle - \langle W\rangle^2}}{\sqrt{\langle (I^{SM})^2\rangle - \langle I^{SM}\rangle^2}} \tag{12}$$

and $I^{\mathrm{SM}}_\alpha = [1/(\alpha-1)]\ln\sum_{x,a}[p^{\mathrm{SM}}(x,a)]^\alpha[p^S(x)p^M(a)]^{1-\alpha}$ is the Renyi generalization of the mutual information. If we extract the maximum amount of work from the system for each measurement outcome, we can extract $W^S[\Gamma] = -\Delta F^S + \beta^{-1}I(x,a)$ from the system, with finite work fluctuations. On the other hand, if we discard the measurement outcome, we can extract a definite amount of work $W^S[\Gamma] = -\Delta F^S$ from the system with large dissipation. This means that Eqs. (10) and (12) show a trade-off relation between work fluctuation and dissipation due to the fluctuation in the obtained information.

Possible experimental test of the trade-off relations.—The proposed trade-off relations can be tested by using a single-electron box, which was used to realize a Szilard engine [14,15]. Suppose that we prepare degenerate states of a two-level system and perform a measurement to distinguish the state of the system, which is initially distributed with equal probabilities $P(n=0) = P(n=1) = 1/2$, where n labels the state of the system. Let the measurement error rate be ϵ and the joint probability distribution of the system being n and the measurement outcome being m be given by $P(n,m) = (1-\epsilon)/2$ for $m = n$ and $P(n,m) = \epsilon/2$ for $m \neq n$. A feedback control is implemented by lowering the energy level of the state $n = m$ and letting the energy level return to the degeneracy point [see the inset of Fig. 1(c)]. By tracking the state of the system during this feedback, we can measure the extracted work for each run of the experiment, and calculate work fluctuation and dissipation. If we change the feedback protocol, e.g., by changing the degree of the energy-level shift, we obtain a different experimental data set of work fluctuation and dissipation. By plotting $\beta\sqrt{\langle W^2\rangle - \langle W\rangle^2}/\sqrt{\langle (I^{SM})^2\rangle - \langle I^{SM}\rangle^2}$ against $\langle\sigma\rangle/\langle I^{SM}\rangle$ as shown in Fig. 1(c), we can test the trade-off relation between work fluctuation and dissipation in information heat engines. The results of numerical simulations of a Szilard engine in a single-electron box using a master equation described in Ref. [15] are shown as dots in Fig. 1(c).

Summary.—We have found a set of fundamental trade-off relations between work fluctuation and dissipation for nonequilibrium initial and final states. We can reproduce single-shot results in the limit of vanishing work fluctuation and thermodynamically reversible results (the lower bound of the conventional second law) in the limit of vanishing dissipation. These two limits are smoothly connected and the minimum dissipation along this boundary is characterized by the information distance between the state of the system and the canonical distribution. This result gives the fundamental bound on both work fluctuation and dissipation starting from and/or ending at nonequilibrium states. An application of the trade-off relation to information heat engines is discussed, including numerical simulations that vindicate the obtained trade-off relation.

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