Transient Dynamics of d-Wave Superconductors after a Sudden Excitation

Francesco Peronaci,[1] Marco Schiró,[2] and Massimo Capone[1]
[1]International School for Advanced Studies (SISSA/ISAS) and CNR-IOM Democritos, Via Bonomea 265, 34136 Trieste, Italy
[2]Institut de Physique Théorique, Université Paris Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France
(Received 3 June 2015; revised manuscript received 10 November 2015; published 15 December 2015)

Motivated by recent ultrafast pump-probe experiments on high-temperature superconductors, we discuss the transient dynamics of a d-wave BCS model after a quantum quench of the interaction parameter. We find that the existence of gap nodes, with the associated nodal quasiparticles, introduces a decay channel which makes the dynamics much faster than in the conventional s-wave model. For every value of the quench parameter, the superconducting gap rapidly converges to a stationary value smaller than the one at equilibrium. Using a sudden approximation for the gap dynamics, we find an analytical expression for the reduction of spectral weight close to the nodes, which is in qualitative agreement with recent experiments.

DOI: 10.1103/PhysRevLett.115.257001 PACS numbers: 74.40.Gh, 05.70.Ln, 74.20.Rp, 74.25.Gz

Introduction.—Recent advances in time-resolved spectroscopies have triggered a growing interest in the transient dynamical behavior of high-temperature superconductors optically excited far from equilibrium. By shining the sample with intense ultrafast pulses (pump) one can trigger nonequilibrium transient states, whose physical properties are then recorded by a second pulse (probe) which hits the sample with a given time delay. This approach opens up a wealth of information unavailable to conventional time-averaged spectroscopies [1–9]. When used in combination with angle-resolved photoemission spectroscopy (ARPES), these ultrafast methods allow us to track and follow in real time the evolution of quasiparticle modes in different momentum sectors, such as those close to the gap nodes and antinodes of d-wave cuprate superconductors [10–12]. In addition, when irradiation is sufficiently strong and optically tailored in such a way to selectively excite specific modes, one can even stabilize transient states with fundamentally different physical properties [13–15]. A spectacular example is given by recent experiments reporting signatures of light-induced metastable superconductivity in cuprates at much higher temperature than at equilibrium [16,17].

These experimental breakthroughs raise a number of intriguing questions. From one side, a nontrivial and rich transient dynamical behavior is expected in correlated materials, which feature complex phase diagrams characterized by competing phases [18]. On a more fundamental level, pump-probe experiments suggest the possibility to explore novel metastable phases that can only be accessed along nonthermal pathways, e.g., by means of photoexcitation. Theoretical investigations along these directions have appeared in the literature in recent years [19–22] addressing questions like the thermalization of pump-excited Mott or Kondo insulators [23,24] and the role of lattice vibrations [25], orbital degrees of freedom [26], and competing orders [27,28] in the relaxation dynamics. The dynamics of conventional superconductors has also been studied with reference to pump-probe experiments [29–31] or in the presence of electron-phonon interactions [32–34] and Coulomb repulsion [35]. In the context of ultracold atomic systems, the research has focused on the s-wave weak coupling BCS regime [36–39], with recent attempts to extend the analysis to the crossover into the BEC regime [40,41] and to exotic order-parameter symmetries [42,43].

Yet, despite this recent activity, many fundamental questions concerning specifically the transient response of superconducting materials remain wide open. In particular, a characteristic feature of high-T_c cuprates is the momentum anisotropy of the superconducting gap, which has a $d_{x^2-y^2}$ symmetry (which we indicate as d wave in the following). This leads to the existence of nodal lines along which the superconducting gap vanishes, a characteristic feature which is sometimes used as a definition of exotic superconductivity. These gapless excitations significantly affect the thermodynamics and spectral properties as compared to the conventional s-wave case, where the gap is uniform over the whole Fermi surface, and they can possibly play an even bigger role in the nonequilibrium dynamics.

In this Letter we consider a minimal model of a d-wave superconductor and the simplest nonequilibrium protocol of a quantum quench of the interaction parameter. Within a BCS-like mean-field approximation, we calculate the gap dynamics and compare the results with those for the conventional s-wave superconductor, identifying the distinctive features descending from the existence of the nodes. Then, we derive an approximate formula for the spectral weight which is directly relevant to ARPES experiments in d-wave superconductors. Of course a mean-field description, while reasonable for the deeply overdoped region of the phase diagram of cuprates, is unable to capture the strong correlations which dominate the low-doping region, leading to a complex interplay between different phases. However, we believe it is useful to highlight the physics of a simple model with d-wave symmetry, as this will help to disentangle the effect of the anisotropic gap in more involved calculations in which competing orders, strong-coupling effects, and any other realistic feature are included.

Model and gap dynamics.—As a starting point of our analysis we consider the two-dimensional BCS Hamiltonian with a momentum-dependent separable interaction in the d-wave channel

$$H = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - J \sum_{\mathbf{kp}} \gamma_{\mathbf{k}} \gamma_{\mathbf{p}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow}, \qquad (1)$$

with $\varepsilon_{\mathbf{k}} = |\mathbf{k}|^2 - \mu$ and $\gamma_{\mathbf{k}} = \cos 2\theta \sim k_x^2 - k_y^2$ where θ is the polar angle in k space. The superconducting gap, or order parameter, is defined as

$$\Delta_{\mathbf{k}} = \Delta \gamma_{\mathbf{k}}, \qquad \Delta = J \sum_{\mathbf{k}} \gamma_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle, \qquad (2)$$

and it vanishes along the nodal lines $k_x = \pm k_y$. This leads to modes at arbitrary low energies $E_{\mathbf{k}} = \sqrt{\varepsilon^2_{\mathbf{k}} + |\Delta^2_{\mathbf{k}}|}$ which, as we show below, have important consequences on the relaxation dynamics.

The existence of the nodal lines marks a fundamental difference not only with the s-wave symmetry which has $\gamma_{\mathbf{k}} = 1$ and a fully gapped spectrum [37,39], but also with unconventional symmetries such as $p + ip$ ($\gamma_{\mathbf{k}} \sim e^{i\theta}$) [42] and $d + id$ ($\gamma_{\mathbf{k}} \sim e^{2i\theta}$) [44] where the gap vanishes at most for one point in momentum space. Notably, in all the above nodeless cases the θ dependence of the gap function can be gauged away leaving us with an effective one-dimensional problem of the Richardson-Gaudin form [45,46] which can be solved exactly.

In the absence of such a full solution, we resort to a time-dependent BCS variational ansatz $|\Psi(t)\rangle = \Pi_{\mathbf{k}}(u_{\mathbf{k}}(t) + v_{\mathbf{k}}(t) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|0\rangle$. This is equivalent to introducing a quasiparticle Hamiltonian

$$H_{\mathrm{qp}}(t) = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} (\gamma_{\mathbf{k}} \Delta(t) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \mathrm{H.c.}) \qquad (3)$$

and solving the equations of motion for the expectation values $\langle c^\dagger_{\mathbf{k}\sigma}(t) c_{\mathbf{k}\sigma}(t) \rangle$ and $\langle c_{-\mathbf{k}\downarrow}(t) c_{\mathbf{k}\uparrow}(t) \rangle$ with a time-dependent order parameter $\Delta(t) = J \sum_{\mathbf{k}} \gamma_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow}(t) c_{\mathbf{k}\uparrow}(t) \rangle$ which is calculated self-consistently at each time and couples the different points in k space [47].

To drive the system out of equilibrium we consider the simplest and very popular quantum quench protocol. We take at $t = 0$ the ground state of the Hamiltonian [Eq. (3)] with a given value of the interaction parameter $J = J_i$ and calculate the time evolution according to the same Hamiltonian but with a different interaction $J = J_f$. It is convenient to discuss the results in terms of Δ_i and Δ_f, the gaps which the system would have at equilibrium for J_i and J_f, respectively.

In Fig. 1 we plot the time-dependent order parameter $\Delta(t)$ for s- and d-wave symmetry and for four different values of the quench parameter Δ_i/Δ_f, ranging from very small to very large ratios. In the s-wave case we recover three different dynamical regimes, in accordance with previous studies [38]: For small Δ_i/Δ_f the order parameter exhibits persistent oscillations between two limiting values; for intermediate Δ_i/Δ_f it has damped oscillations towards a nonequilibrium asymptotic value $\Delta_{\mathrm{st}} \neq 0$; and at large Δ_i/Δ_f it has an overdamped exponential decay to zero.

In the d-wave case we find damped oscillations for every Δ_i/Δ_f except for very large values. On the qualitative level, we remark the disappearance of the regime with persistent oscillations [panel (a) of Fig. 1] and, more importantly, the much faster decay of the gap oscillations as compared to the s-wave case [panels (b) and (c) of Fig. 1]. On the contrary, for very large Δ_i/Δ_f the behavior is similar, with both the s- and d-wave gap decaying exponentially to zero. Indeed, as pointed out in Ref. [38], this regime is similar to a complete switch-off of the interaction, a case in which the structure factor $\gamma_{\mathbf{k}}$ has clearly little influence.

The increased damping of the d-wave gap dynamics is a signature of the existence of low-energy excitations, as it can be understood at least in the case of small quenches $\Delta_i/\Delta_f \simeq 1$ for which we can calculate the linear response theory variation $\delta\Delta(t) = \Delta(t) - \Delta_i$ [47]

$$\delta\Delta(t)/\Delta_i \propto \sum_{\mathbf{k}} \frac{\gamma^2_{\mathbf{k}} \varepsilon^2_{\mathbf{k}}}{E^3_{\mathbf{k}i}} (1 - \cos 2E_{\mathbf{k}i} t) \qquad (4)$$

where $E_{\mathbf{k}i} = \sqrt{\varepsilon^2_{\mathbf{k}} + \gamma^2_{\mathbf{k}} \Delta^2_i}$. The time-dependent contribution to Eq. (4) is dominated at long times by the low-energy

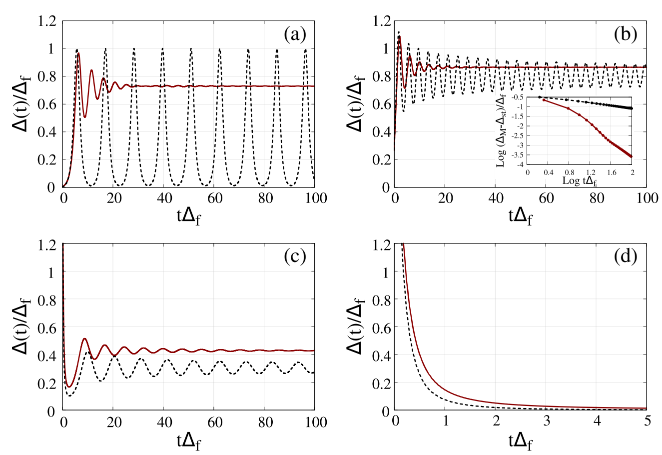

FIG. 1 (color online). Plot of the gap dynamics for the d-wave (full red line) and s-wave (dashed black line) symmetries and for different quench parameters Δ_i/Δ_f: (a) 0.001, (b) 0.2, (c) 4.0, and (d) 5.2. Inset of panel (b): log-log plot of the local maxima of the gap.

modes, as it is evident if we replace the sum over momenta with an energy integral and we change variables in order to introduce the superconducting density of states $\rho(E) = \sum_{\mathbf{k}} \delta(E - E_{\mathbf{k}})$. For the s-wave superconductor the density of states has a sharp edge at Δ_i where it has a squared-root divergence. This leads to power-law damped oscillations with frequency $2\Delta_i$. The d-wave symmetry introduces a qualitative difference in the density of states, which diverges only logarithmically at Δ_i and has finite value for energies down to zero. This results in oscillations which damp much faster.

Steady state.—The different dynamical regimes and the long-time gap values for s- and d-wave symmetries are summarized in Fig. 2 where, following Ref. [38], we plot $\Delta_{\rm st}$ as a function of the quench parameter Δ_i/Δ_f. For any value of Δ_i/Δ_f we find that at long times the quench leads to a reduction of the gap with respect to the zero temperature equilibrium value Δ_f. The difference between the s- and d-wave cases occurs for $\Delta_i/\Delta_f \lesssim 0.2$, where the d-wave gap goes to a stationary value while the s-wave gap exhibits undamped oscillations [38]. On the other hand, for $\Delta_i/\Delta_f \gtrsim 0.2$ the gap reaches essentially the same asymptotic value, despite the much faster decay of the d-wave gap.

It is important to emphasize that in the absence of pair-breaking scattering terms and of any real dissipation mechanism, the system persists in a nonequilibrium state. In particular, the expectation values $\langle c^\dagger_{\mathbf{k}\sigma}(t) c_{\mathbf{k}\sigma}(t)\rangle$ and $\langle c_{-\mathbf{k}\downarrow}(t) c_{\mathbf{k}\uparrow}(t)\rangle$ do not come to a steady state and a stationary value of the gap is eventually reached only as a result of destructive interference between different momenta, a phenomenon which can also be interpreted in terms of a quench-induced decoherence [48].

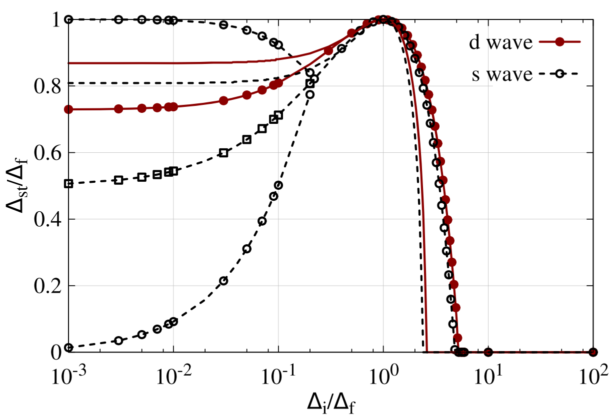

FIG. 2 (color online). Plot of the stationary gap $\Delta_{\rm st}/\Delta_f$ as a function of the quench parameter Δ_i/Δ_f for the d-wave (full red line with full circles) and s-wave (dashed black line with empty circles) symmetries. For $\Delta_i/\Delta_f \lesssim 0.2$ the s-wave gap exhibits undamped oscillations around the value indicated with a square, in this case the circles indicate the extrema of the oscillations. Curves without symbols: equilibrium gap $\Delta(T^*)$ corresponding to an effective temperature T^*.

For completeness, in Fig. 2 we also plot $\Delta(T^*)$, the gap for a system in equilibrium at the temperature T^* corresponding to the energy pumped into the system through the quench (lines without symbols). The system could eventually reach this thermal value if we include scattering processes not contained in the Hamiltonian [Eq. (3)]. While the overall behavior of $\Delta(T^*)$ is qualitatively similar to $\Delta_{\rm st}$, the quantitative difference is substantial confirming the nonthermal character of the asymptotic stationary state.

Spectral features.—We now focus on the spectral properties of the transient state in the d-wave case, thus moving a first step towards a comparison with recent time-resolved ARPES experiments on high-T_c cuprates [10–12]. These have tracked the evolution of the quasiparticle energy $E_{\mathbf{k}}$ and weight $Z_{\mathbf{k}}$, which are directly related to the lesser Green's function $G^<_{\mathbf{k}}(t,t') = i\langle c^\dagger_{\mathbf{k}\sigma}(t) c_{\mathbf{k}\sigma}(t')\rangle$, whose Fourier transform at equilibrium and zero temperature has a peak at negative energy

$$-\frac{i}{\pi} G^<_{\mathbf{k}\mathrm{eq}}(\omega) = Z_{\mathbf{k}\mathrm{eq}} \delta(\omega + E_{\mathbf{k}}) \tag{5}$$

corresponding to the quasiparticle energy $E_{\mathbf{k}}$ with a weight

$$Z_{\mathbf{k}\mathrm{eq}} = \left(1 - \frac{\varepsilon_{\mathbf{k}}}{E_{\mathbf{k}}}\right). \tag{6}$$

In principle, from the quasiparticle Hamiltonian [Eq. (3)] we can also calculate the out-of-equilibrium lesser Green's function. However, the presence of an arbitrary time-dependent order parameter $\Delta(t)$ and the fact that out of equilibrium the Green's function depends on both time arguments, makes this a rather challenging task that we leave for further studies. Here, in order to proceed analytically and obtain some physical insight on the main effect of the quench, we exploit the observation of the extremely fast dynamics of the d-wave gap. Hence, we assume that the quasiparticle modes do not have enough time to rearrange and approximate the actual dynamics with a sudden change of the gap $\Delta(t) = \Delta_i \theta(-t) + \Delta_{\rm st}\theta(t)$.

This approximation is also based on some experimental results. In particular Refs. [11,12] have highlighted two clearly distinct time scales for the gap dynamics following the pump pulse: on a short interval of approximately 0.3 ps the gap reaches a value smaller than at equilibrium ($\Delta_{\rm st}$ in our model) and on a longer time it relaxes to the equilibrium value, typically attained after 10–20 ps. Our sudden approximation for the gap dynamics should therefore be reasonable for times of a few picoseconds after the pump pulse. It is interesting to notice that on these time scales an effective temperature picture of the ARPES spectra is not adequate, as firmly pointed out in Ref. [12]. Finally, if we consider the maximum gap value at equilibrium for Bi2212 $\Delta_f = 60$ meV we can estimate a time scale of about 0.01 ps for Fig. 1.

Within this sudden approximation for the gap dynamics, we derive an analytical expression for $G^<_{\mathbf{k}}(t,t')$ which is convenient to Fourier transform with respect to the time difference $t-t'$ and average over the waiting time t' to finally obtain [47]

$$-i/\pi G^<_{\mathbf{k}\mathrm{neq}}(\omega) = Z^-_{\mathbf{k}\mathrm{neq}}\delta(\omega+E_{\mathbf{k}}) + Z^+_{\mathbf{k}\mathrm{neq}}\delta(\omega-E_{\mathbf{k}}), \quad (7)$$

where $E_{\mathbf{k}} = \sqrt{\varepsilon^2_{\mathbf{k}} + \gamma^2_{\mathbf{k}}\Delta^2_{\mathrm{st}}}$ and $Z^\pm_{\mathbf{k}\mathrm{neq}}$ are the out of equilibrium positive and negative energy weights. We notice that as a result of the sudden excitation also, the positive energy peak has a finite occupation. In the following we will focus on the negative energy peak, which is the one observed in ARPES, and compare its weight $Z^-_{\mathbf{k}\mathrm{neq}}$ to the equilibrium case. To this extent it is important to discuss first the interpretation of the quench protocol in the framework of pump-probe experiments. In the standard picture, the quench is used to describe the change of a Hamiltonian parameter, in this case $J(t) = J_i\theta(-t) + J_f\theta(t)$, as a result of an external perturbation. However, this scenario—which can be realized in cold-atom systems—is not directly relevant to solid-state experiments. In this context the Hamiltonian parameters can be considered largely independent of the excitation process and the quench is merely a theoretical tool to study the evolution of an out-of-equilibrium state. In this approach J_i and Δ_i are just used to parameterize the initial state which results from the impulsive excitation, whereas the interaction parameter J_f which controls the time evolution has to coincide with the actual interaction that characterizes the material.

In this light, it is appropriate to compare the out-of-equilibrium spectral weight $Z^-_{\mathbf{k}\mathrm{neq}}$ with the one at equilibrium [Eq. (6)] with Δ_f. The result of this calculation reads [47]

$$\frac{Z^-_{\mathbf{k}\mathrm{neq}}}{Z_{\mathbf{k}\mathrm{eq}}} = \frac{1-\varepsilon_{\mathbf{k}}/E_{\mathbf{k}}}{1-\varepsilon_{\mathbf{k}}/E_{\mathbf{k}\mathbf{f}}}\left(\frac{1}{2} + \frac{\varepsilon^2_{\mathbf{k}} + \Delta_{\mathrm{st}}\Delta_{\mathrm{i}}\gamma^2_{\mathbf{k}}}{2E_{\mathbf{k}}E_{\mathbf{k}\mathbf{i}}}\right) \quad (8)$$

which takes a particularly clear and interesting form if we expand in the neighborhood of the nodal lines, i.e., for $\gamma_{\mathbf{k}} \to 0$. In this case we obtain $Z^-_{\mathbf{k}\mathrm{neq}}/Z_{\mathbf{k}\mathrm{eq}} = (\Delta_{\mathrm{st}}/\Delta_f)^2 < 1$ for $\varepsilon_{\mathbf{k}} > 0$ while $Z^-_{\mathbf{k}\mathrm{neq}}/Z_{\mathbf{k}\mathrm{eq}} = 1 - \alpha\gamma^2_{\mathbf{k}}/4\varepsilon^2_{\mathbf{k}}$ for $\varepsilon_{\mathbf{k}} < 0$, with $\alpha = \Delta^2_i - \Delta^2_f + 2\Delta^2_{\mathrm{st}} - 2\Delta_{\mathrm{st}}\Delta_i > 0$. In other words the nodal spectral weight is always reduced with respect to the equilibrium value, except possibly in a small region close to the Fermi surface. This reduction is clearly a nonthermal effect since finite temperature excitations of a quasiparticle would lead to a reduction proportional to the Fermi function.

Conclusions.—We have studied the real-time dynamics of a simple model of d-wave superconductor excited by a sudden perturbation. For every value of the quench parameter Δ_i/Δ_f the system relaxes to a nonthermal stationary state with a gap parameter smaller than at equilibrium. The presence of nodal gapless excitations results in a much faster dynamics compared to the uniform s-wave case and to the disappearance of the regime of undamped oscillations which characterizes an isotropic superconductor when the quench parameter is smaller than 0.2.

We have derived an analytical expression for the momentum-dependent photoemission spectral weight, which demonstrates a strong dependence on the momentum. In particular, we have found a strong suppression of the weight at the gap nodes which is clearly of nonthermal nature, a result which is consistent with recent time-resolved ARPES experiments.

M. C. and F. P. acknowledge financial support from The European Research Council under FP7/ERC Starting Independent Research Grant “SUPERBAD” (Grant Agreement No. 240524). We acknowledge discussions with A. Amaricci, D. Fausti, and C. Giannetti.

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