Towards More Precise Determinations of the Quark Mixing Phase β

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We derive a new flavor symmetry relation for the determination of the weak phase $\beta = \phi_1$ from time-dependent CP asymmetries and $B \to J/\psi P$ decay rates. In this relation, the contributions to $\sin 2\beta$ proportional to V_{ub} are parametrically suppressed compared to the contributions in the $B \to J/\psi K^0$ time-dependent CP asymmetry alone. This relation uses only $SU(3)$ flavor symmetry, and does not require further diagrammatic assumptions. The current data either fluctuate at the 2σ level from expectations, or may hint at effects of unexpected magnitude from contributions proportional to V_{ub} or from isospin breaking.

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Introduction.—CP violation in $B \to J/\psi K_S$ will be measured at the percent level at Belle II [1] and LHCb [2], a precision several times better than today [3–5], and crucial for improving the sensitivity to new physics in B mixing (see, e.g., Ref. [6]). This projected uncertainty is comparable to the characteristic size of the Cabbibo-Kobayashi-Maskawa (CKM) suppressed uncertainties, proportional to V_{ub} in the time-dependent CP asymmetry,

$$\frac{\Gamma[\bar{B}(t) \to f] - \Gamma[B(t) \to f]}{\Gamma[\bar{B}(t) \to f] + \Gamma[B(t) \to f]} = S_f \sin(\Delta m t) - C_f \cos(\Delta m t),$$
$$S_f = \frac{2\mathrm{Im}[(q/p)(\bar{A}_f/A_f)]}{1 + |\bar{A}_f/A_f|^2}, \qquad C_f = \frac{1 - |\bar{A}_f/A_f|^2}{1 + |\bar{A}_f/A_f|^2}. \tag{1}$$

(In the literature the terms proportional to V_{ub} are often referred to as "penguin pollution." Since we are not concerned with diagrammatic arguments, we identify such terms by CKM factors.) Here, f denotes final states composed of J/ψ and a pseudoscalar meson, P; $A_f = \langle f|\mathcal{H}|B^0\rangle$, $\bar{A}_f = \langle f|\mathcal{H}|\bar{B}^0\rangle$; Δm is the mass difference between the two neutral B mass eigenstates, $|B_{H,L}\rangle = p|B^0\rangle \mp q|\bar{B}^0\rangle$; and we neglect the small $\mathcal{O}(\Delta\Gamma/\Gamma, |q/p| - 1)$ effects in the B_d system, as well as $\mathcal{O}(\epsilon_K)$ effects, which are straightforward to include [7].

At the current level of precision, the relation

$$S_{K_S} = \sin(2\beta) + \mathcal{O}[V_{ub}^* V_{us}/(V_{cb}^* V_{cs})] + \cdots, \tag{2}$$

truncated at leading order, has been sufficient to extract the CKM phase $\beta \equiv \arg[-V_{cb}^* V_{cd}/(V_{tb}^* V_{td})]$. The theoretical uncertainty is limited by our ability to compute or bound the subleading contribution to the decay amplitude, proportional to V_{ub}. This is the A_u term in the decay amplitude,

$$A = \lambda_c^q A_c + \lambda_u^q A_u, \qquad \lambda_i^q \equiv V_{ib}^* V_{iq}, \tag{3}$$

($i = u, c$ and $q = d, s$), which has a different weak phase and possibly a different strong phase than the dominant A_c term.

The upcoming experimental precision has renewed interest in constraining the effects of this "V_{ub} contamination" in measurements of β and its analog in B_s decays, β_s. Comparisons between $B_d \to J/\psi\rho^0$ and $B_s \to J/\psi\phi$ [8,9] rely both on flavor symmetry and diagrammatic arguments. It has also been proposed to use $B_s \to J/\psi K_S$ to control the V_{ub} term in $B_d \to J/\psi K_S$ (see, e.g., Ref. [10]). Other approaches attempt to constrain the V_{ub} contribution from global fits to multiple observables using flavor $SU(3)$ [11–15], often with additional simplifying assumptions, or attempt to compute the corresponding hadronic matrix element using QCD factorization (see, e.g., Ref. [16]). Some of these works claim that the V_{ub} contamination can be enhanced to several percent, which is challenged by a lower estimate of rescattering effects using measured rates [17].

In this Letter we derive a flavor $SU(3)$ relation for β, involving the $B_d \to J/\psi K_S$, $B_d \to J/\psi\pi^0$, $B^+ \to J/\psi K^+$, and $B^+ \to J/\psi\pi^+$ branching ratios and CP asymmetries, in which, in the $SU(3)$ limit, the contributions linear in V_{ub} cancel. This permits extraction of β up to parametrically suppressed contributions, compared to the V_{ub} contamination in Eq. (2). Our results rely only on group theoretic relations among the decay amplitudes, and do not involve diagrammatic or factorization arguments. The same relations imply a lower bound for the presently unmeasured $B_s \to J/\psi\pi^0$ decay rate.

Amplitude relations.—We obtain $SU(3)$ relations for the $B \to J/\psi f$ decay amplitudes by application of a Wigner-Eckart expansion, after embedding the Hamiltonian and the in and out states into $SU(3)$ representations. The B in states furnish a flavor antitriplet, $[B_3]_i = (B^+, B_d, B_s)$. The charmless pseudoscalar out states furnish a singlet, $[P_1] = \eta_1$, and the usual octet,

$$[P_8]^i_j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}. \quad (4)$$

We allow an arbitrary η-η' mixing angle, such that the mass eigenstates are $\eta^{(\prime)} = \eta_8 \cos\theta \mp \eta_1 \sin\theta$.

The effective Hamiltonian for $B \to J/\psi P$ decay contains four-quark operators that mediate $\bar{b} \to \bar{q}^i q_j \bar{q}^k$ or $\bar{b} \to c\bar{c}\bar{q}^i$ transitions ($q = u, d, s$). Under $SU(3)$ flavor, this embeds into $\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{3} \oplus \mathbf{3}' \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$ irreducible representations. The nonzero independent components of the Hamiltonian are given in Eq. (53) of Ref. [18]. Finally, $SU(3)$ and isospin breaking is encoded by insertions of the usual octet spurions, $[\mathcal{M}]^i_j \equiv \varepsilon\,\mathrm{diag}\{1, 1, -2\}$ and $\delta\,\mathrm{diag}\{1, -1, 0\}$, respectively.

We work to first order in G_F and to all orders in α_s. In the $SU(3)$ limit, the A_c and A_u terms in Eq. (3) each depend on three reduced matrix elements, corresponding to the $\mathbf{3}$, $\bar{\mathbf{6}}$, and 15 pieces of the Hamiltonian. For A_c, the $\bar{\mathbf{6}}$ and $\mathbf{15}$ terms only arise from electroweak penguin contributions, suppressed by α_{em}. These are accounted together with other sources of isospin breaking in A_c, which are comparable in size. The electroweak penguin contributions to A_c transforming as the $\mathbf{3}$ (which probably dominate) are automatically absorbed in the leading A_c contributions.

The decay amplitudes are expanded to $\mathcal{O}(\varepsilon^p)$ via

$$A(B \to J/\psi f) = \sum_{w,p} X^p_w (C^p_w)_{B;f},$$
$$(C^p_w)_{B;f} \equiv \frac{\partial^2}{\partial f \partial B} [[P_{1,8}]^{i_1 \cdots}_{j_1 \cdots} \mathcal{H}^{p_1 \cdots}_{q_1 \cdots} ([\mathcal{M}]^{k_1}_{l_1} \cdots)[B_3]_r]_w. \quad (5)$$

Here, w labels a set of linearly independent $SU(3)$ tensor contractions, $\mathcal{H}$ is the Hamiltonian, and there are p insertions of $\mathcal{M}$. The X^p_w are reduced matrix elements, while C^p_w encode the weak physics, pth order $SU(3)$ breaking effects, and group theoretic factors. Finding $SU(3)$ sum rules at order ε^p is equivalent to computing kernels of $(C^p_w)_{B;f}$ [18,19].

It is useful to derive relations that hold independently for the A_c and A_u amplitudes in Eq. (3). In anticipation of the need to account for $SU(3)$ breaking effects, we further expand each reduced matrix element order by order in $SU(3)$ breaking, and write

$$A_c = A^{(0)}_c + \varepsilon A^{(1)}_c + \cdots,$$
$$A_u = A^{(0)}_u + \varepsilon A^{(1)}_u + \cdots. \quad (6)$$

In the $SU(3)$ limit, we have

$$0 = A^{(0)}_c(B_s \to J/\psi \pi^0), \quad (7a)$$
$$\mathcal{A}_c \equiv A^{(0)}_c(B_d \to J/\psi K^0) = A^{(0)}_c(B^+ \to J/\psi K^+)$$
$$= A^{(0)}_c(B^+ \to J/\psi \pi^+) = A^{(0)}_c(B_s \to J/\psi \bar{K}^0)$$
$$= -\sqrt{2} A^{(0)}_c(B_d \to J/\psi \pi^0). \quad (7b)$$

Hereafter, we write $\mathcal{A}_c$ instead of the $A^{(0)}_c$ amplitudes in Eq. (7b). Considering the first order $SU(3)$ breaking contributions to the amplitudes independently, we find

$$0 = A^{(1)}_c(B_s \to J/\psi \pi^0), \quad (8a)$$
$$0 = \sqrt{2} A^{(1)}_c(B_d \to J/\psi \pi^0) + A^{(1)}_c(B^+ \to J/\psi \pi^+), \quad (8b)$$
$$0 = A^{(1)}_c(B_d \to J/\psi K^0) - A^{(1)}_c(B^+ \to J/\psi K^+), \quad (8c)$$
$$0 = A^{(1)}_c(B^+ \to J/\psi K^+) + A^{(1)}_c(B^+ \to J/\psi \pi^+)$$
$$+ A^{(1)}_c(B_s \to J/\psi \bar{K}^0), \quad (8d)$$

Equations (8a)–(8c) are isospin relations, and hold to all orders in the $SU(3)$ breaking parameter ε. Finally, the A_u amplitudes in the $SU(3)$ limit satisfy [20–22]

$$0 = A^{(0)}_u(B^+ \to J/\psi \pi^+) - A^{(0)}_u(B^+ \to J/\psi K^+), \quad (9a)$$
$$0 = A^{(0)}_u(B_d \to J/\psi K^0) - A^{(0)}_u(B_s \to J/\psi \bar{K}^0), \quad (9b)$$
$$0 = \sqrt{2} A^{(0)}_u(B_d \to J/\psi \pi^0) - \sqrt{2} A^{(0)}_u(B_s \to J/\psi \pi^0)$$
$$+ A^{(0)}_u(B_d \to J/\psi K^0). \quad (9c)$$

Besides Eqs. (7)–(9), there are further relations involving $J/\psi \eta^{(\prime)}$ states, that are not needed for our analysis. Similar relations also hold for vector mesons, with obvious replacements.

It is often assumed based on diagrammatic arguments that the $A^{(0)}_u(B_s \to J/\psi \pi^0)$ contribution in Eq. (9c) can be neglected (see, e.g., Refs. [11–14]). We make no such assumption. The current limits on $A^{(0)}_u(B_s \to J/\psi \pi^0)$ are weak, in the sense that the data allow this contribution to be sizable. Below we use Eq. (9c) to set a lower bound on the branching ratio $\mathcal{B}(B_s \to J/\psi \pi^0)$.

Relation for $\sin(2\beta)$.—Given the flavor symmetry relations, we proceed to construct an $SU(3)$ relation among branching ratios and time-dependent CP asymmetries, that permits extraction of β without V_{ub} contamination in the $SU(3)$ limit. This relation will only involve B_d or B^+ decays, so hereafter we denote $A_f \equiv A(B \to J/\psi f)$, for $B = B_d$, B^+.

Besides the $SU(3)$ and isospin breaking parameters,

$$\varepsilon \sim \frac{f_K}{f_\pi} - 1 \sim 0.2, \qquad \delta \sim \frac{m_d - m_u}{\Lambda_{\chi\mathrm{SB}}} \lesssim 1\%, \quad (10)$$

we also expand certain observables in

$$\bar{\lambda}^2 \equiv -\frac{\lambda_u^s}{\lambda_c^s}\frac{\lambda_c^d}{\lambda_u^d} \simeq 0.05, \qquad R_u \equiv \sqrt{\bar{\rho}^2+\bar{\eta}^2} \simeq 0.37, \tag{11}$$

where $\bar{\rho}+i\bar{\eta} \equiv -\lambda_u^d/\lambda_c^d \simeq 0.15+0.34i$ is the apex of the unitarity triangle. Powers of R_u track powers of V_{ub}, and enter with corresponding powers of A_u/A_c. We make no assumptions concerning the size of $|A_u/A_c|$. While ε and R_u are not particularly small parameters, R_u^2, εR_u, and ε^2 can be treated as $\ll 1$. We therefore expand physical observables to this order, and seek relations without $\mathcal{O}(\varepsilon, R_u)$ terms.

Expanding to next-to-leading order in these small parameters, the CP-averaged rate is

$$\bar{\Gamma}(B\to J/\psi f) = [|\vec{p}_{B\to J/\psi f}|/(8\pi m_B^2)]|\lambda_c^q|^2|A_{c,f}^{(0)}|^2 \times\left[1+2\varepsilon\mathrm{Re}\frac{A_{c,f}^{(1)}}{A_{c,f}^{(0)}}+2\mathrm{Re}\frac{\lambda_u^q}{\lambda_c^q}\mathrm{Re}\frac{A_{u,f}^{(0)}}{A_{c,f}^{(0)}}+\cdots\right]. \tag{12}$$

Corrections are $\mathcal{O}(R_u^2,\varepsilon R_u,\varepsilon^2)$ and $\mathcal{O}(\varepsilon R_u\bar{\lambda}^2,\varepsilon^2)$ in $b\to d,s$ processes, respectively. The $\varepsilon A_c^{(1)}/A_c^{(0)}$ terms arise from first order $SU(3)$ breaking and must be kept, as they are parametrically larger than $\mathcal{O}(\bar{\lambda}^2)$. Note they do not satisfy the same relations as the $A_c^{(0)}$ terms.

Applying Eqs. (7b) and (8c) to Eq. (12) yields

$$\Delta_K \equiv \frac{\bar{\Gamma}(B_d\to J/\psi K^0)-\bar{\Gamma}(B^+\to J/\psi K^+)}{\bar{\Gamma}(B_d\to J/\psi K^0)+\bar{\Gamma}(B^+\to J/\psi K^+)} = \mathrm{Re}\frac{\lambda_u^s}{\lambda_c^s}\mathrm{Re}\frac{\sqrt{2}A_{u,K_S}^{(0)}-A_{u,K^+}^{(0)}}{\mathcal{A}_c}+\mathcal{O}(\varepsilon R_u\bar{\lambda}^2,\delta). \tag{13}$$

We emphasize that the $\varepsilon^n\mathrm{Re}[A_c^{(n)}/A_c^{(0)}]$ terms in Eq. (12) are canceled up to isospin breaking corrections. We have also made the replacement $A(B_d\to J/\psi K^0) = \sqrt{2}A(B_d\to J/\psi K_S)$. Analogously, we also obtain

$$\Delta_\pi \equiv \frac{2\bar{\Gamma}(B_d\to J/\psi\pi^0)-\bar{\Gamma}(B^+\to J/\psi\pi^+)}{2\bar{\Gamma}(B_d\to J/\psi\pi^0)+\bar{\Gamma}(B^+\to J/\psi\pi^+)} = -\mathrm{Re}\frac{\lambda_u^d}{\lambda_c^d}\mathrm{Re}\frac{\sqrt{2}A_{u,\pi^0}^{(0)}+A_{u,K^+}^{(0)}}{\mathcal{A}_c}+\mathcal{O}(R_u^2,\varepsilon R_u,\delta), \tag{14}$$

where we replaced $A_{u,\pi^+}^{(0)}$ with $A_{u,K^+}^{(0)}$ using Eq. (9a).

The CP asymmetry in $B_d\to J/\psi f$ can be written as

$$S_f = -\eta_f\left[\sin 2\beta+2\mathrm{Im}\frac{\lambda_u^q}{\lambda_c^q}\mathrm{Re}\frac{A_{u,f}^{(0)}}{A_{c,f}^{(0)}}\cos 2\beta+\cdots\right], \tag{15}$$

where $CP|J/\psi f\rangle = \eta_f|J/\psi f\rangle$, and corrections are $\mathcal{O}(R_u^2,\varepsilon R_u)$ and $\mathcal{O}(\varepsilon R_u\bar{\lambda}^2)$ for $b\to d,s$, respectively. The $\mathrm{Re}[A_u^{(0)}/A_c^{(0)}]$ term in Eq. (15) dominates the V_{ub} contamination in Eq. (2). From Eq. (15) the CP asymmetries for $B_d\to J/\psi K_S$ and $B_d\to J/\psi\pi^0$ are

$$S_{K_S}-\sin 2\beta = 2\mathrm{Im}\frac{\lambda_u^s}{\lambda_c^s}\mathrm{Re}\frac{\sqrt{2}A_{u,K_S}^{(0)}}{\mathcal{A}_c}\cos 2\beta+\cdots,$$

$$S_{\pi^0}+\sin 2\beta = 2\mathrm{Im}\frac{\lambda_u^d}{\lambda_c^d}\mathrm{Re}\frac{\sqrt{2}A_{u,\pi^0}^{(0)}}{\mathcal{A}_c}\cos 2\beta+\cdots. \tag{16}$$

Eliminating the V_{ub} contamination—the $A_u^{(0)}$ terms—in Eqs. (13), (14), and (16), one obtains the relation

$$(1+\bar{\lambda}^2)\sin 2\beta = S_{K_S}-\bar{\lambda}^2 S_{\pi^0} - 2(\Delta_K+\bar{\lambda}^2\Delta_\pi)\cos 2\beta\tan\gamma + \mathcal{O}(\varepsilon R_u\bar{\lambda}^2, R_u^2\bar{\lambda}^2,\delta), \tag{17}$$

where $\gamma \equiv \arg(-\lambda_u^d/\lambda_c^d)$. Equation (17) is the main result of this Letter. In the $SU(3)$ limit, the V_{ub} contamination in S_{K_S}, $\Delta S_{K_S} \equiv S_{K_S}-\sin 2\beta$, is canceled by contributions from Δ_K, Δ_π, and S_{π^0}. This leaves only corrections parametrically higher order in ε, δ, or R_u,

$$\varepsilon R_u\bar{\lambda}^2\mathrm{Re}\frac{A_u^{(0)}}{\mathcal{A}_c}, \qquad R_u^2\bar{\lambda}^2\left|\frac{A_{u,\pi}^{(0)}}{\mathcal{A}_c}\right|^2, \qquad \delta\mathrm{Re}\frac{A_{c,K}^{\delta}}{\mathcal{A}_c}, \tag{18}$$

where $\delta A_{c,K}^{\delta}$ is the isospin breaking difference of A_{c,K^0} and A_{c,K^+}, arising in Δ_K.

The $\mathcal{O}(\varepsilon R_u\bar{\lambda}^2)$ $SU(3)$-breaking correction in Eq. (17) is unambiguously smaller than the V_{ub} contamination in ΔS_{K_S}, of order $\mathcal{O}(R_u\bar{\lambda}^2)$.

The $\mathcal{O}(R_u^2\bar{\lambda}^2)$ terms in Eq. (17) are dominated by the V_{ub}^2 terms in Δ_π, which are numerically enhanced by $\tan\gamma \simeq 2.6$. If $A_u/A_c = \mathcal{O}(1)$, then these corrections are not numerically suppressed, since $R_u\tan\gamma \simeq 0.9$. However, in this case, future data should show an enhancement of Δ_π compared to its present value (see Table I), which will constrain this possibility. If $A_u/A_c \ll 1$ then this $\mathcal{O}(R_u^2\bar{\lambda}^2)$ correction is negligible.

Concerning the isospin breaking $\mathcal{O}(\delta)$ contribution to Eq. (17), if $A_u/A_c = \mathcal{O}(1)$ and $\delta\mathrm{Re}[A_c^{(\delta)}/\mathcal{A}_c] \sim 1\%$, then this term is subleading compared to ΔS_{K_S}. If $A_u/A_c \ll 1$ and $\delta\mathrm{Re}[A_c^{(\delta)}/\mathcal{A}_c] \sim 1\%$, then this term may be numerically larger than ΔS_{K_S}. However, in this case, the experimental upper bound on Δ_K should decrease. It may also be possible to obtain constraints on the isospin violating matrix element $A_{c,K}^{\delta}/\mathcal{A}_c$ using other methods, in order to extract β from Eq. (17) at subpercent precision.

Numerical results and predictions.—The four observables in Eqs. (13), (14), and (16) depend on β and the real parts of the three $A_{u,f}^{(0)}/\mathcal{A}_c$ amplitude ratios. We may therefore extract these matrix elements and β from a fit to these four observables, noting one may also extract β directly from Eq. (17). We use the standard model (SM) fit values

$\gamma = 67° \pm 2°$ and $\bar{\lambda}^2 \simeq 5.36 \times 10^{-2}$ [24] as inputs, and determine R_u from the identity $R_u \equiv \sin\beta/\sin(\gamma+\beta)$. The SM CKM fit results for R_u (or $\bar{\rho}$ and $\bar{\eta}$) are not used, as they depend strongly on the assumption of negligible V_{ub} contamination in β, whereas the SM fit result for γ has only a small dependence on the direct β measurement.

The experimental data for these observables are shown in Table I from HFAG [23]. The S_{K_S} value is the average of $S_{J/\psi K_S}$ from *BABAR*, Belle, and LHCb, with other charmonium states $\psi(2S)$, χ_c, etc., excluded, since those hadronic matrix elements are not related by $SU(3)$. One then finds from Eq. (17)

$$\beta = 27.8° \pm 2.9°, \tag{19}$$

and from Eqs. (13), (14), and (16), the matrix elements $\mathrm{Re}[A^{(0)}_{u,K^+}/\mathcal{A}_c] = -0.4 \pm 0.4$, $\mathrm{Re}[\sqrt{2}A^{(0)}_{u,\pi^0}/\mathcal{A}_c] = 0.2 \pm 0.3$, $\mathrm{Re}[\sqrt{2}A^{(0)}_{u,K_S}/\mathcal{A}_c] = -5.5 \pm 2.3$, and $R_u/R_u^{\mathrm{SM}} = 1.3 \pm 0.1$.

The π^0 matrix element is consistent with recent global fits or QCD factorization analyses (see, e.g., Refs. [10,16]). On the other hand, $\mathrm{Re}[\sqrt{2}A^{(0)}_{u,K_S}/\mathcal{A}_c]$ is larger than the expected size of V_{ub} contamination or isospin breaking. This arises from the large central value of the linear combination

$$\Delta_K + \bar{\lambda}^2\Delta_\pi = -0.052 \pm 0.028. \tag{20}$$

Assuming that the V_{ub} contamination and isospin violation are small, so that β takes its current SM fit value, $\beta = (21.9 \pm 0.8)°$ [24], then Eq. (17) and the S_{K_S} and S_{π^0} data predict

$$\Delta_K + \bar{\lambda}^2\Delta_\pi = 0.001 \pm 0.009. \tag{21}$$

The source of the 2σ tension between Eqs. (20) and (21) is the same as that between Eq. (19) and the SM fit for β. Future higher statistics data for the *CP* averaged $B \to J/\psi K$ and $B \to J/\psi\pi$ rates, together with the time dependent *CP* asymmetries in $B_d \to J/\psi K^0$ and $B_d \to J/\psi\pi^0$, is required to resolve this tension. (Future measurements of these rates may require combined analyses with other decays, to simultaneously constrain the isospin asymmetries and the B^+B^- versus $B_d\bar{B}_d$ production in $\Upsilon(4S)$ decay. Current analyses either assume isospin symmetry to measure the production rate difference, or assume equal production rates to measure the branching ratios entering $\Delta_{K,\pi}$ [23,25].)

TABLE I. The experimental data used, from Ref. [23].

Observable	Measurement
$\mathcal{B}(B_d \to J/\psi K^0)$	$(8.63 \pm 0.35) \times 10^{-4}$
$\mathcal{B}(B^+ \to J/\psi K^+)$	$(10.28 \pm 0.40) \times 10^{-4}$
Δ_K	$-(5.0 \pm 2.8) \times 10^{-2}$
$\mathcal{B}(B_d \to J/\psi\pi^0)$	$(1.74 \pm 0.15) \times 10^{-5}$
$\mathcal{B}(B^+ \to J/\psi\pi^+)$	$(4.04 \pm 0.17) \times 10^{-5}$
Δ_π	$-(3.7 \pm 4.8) \times 10^{-2}$
S_{K_S}	0.682 ± 0.021
S_{π^0}	-0.93 ± 0.15

Combining Eq. (9c) with Eqs. (13) and (14), one finds in the $SU(3)$ limit

$$\Delta_K + \bar{\lambda}^2\Delta_\pi = \mathrm{Re}\frac{\lambda_u^s}{\lambda_c^s}\mathrm{Re}\frac{\sqrt{2}A_u^{(0)}(B_s \to J/\psi\pi^0)}{\mathcal{A}_c}. \tag{22}$$

The sizable experimental central value for the left-hand side [cf. Eq. (20)] is therefore connected to the possibility of a sizable amplitude $A_u^{(0)}(B_s \to J/\psi\pi^0)$. According to Eq. (7a), $A_c^{(0)}(B_s \to J/\psi\pi^0)$ vanishes by isospin. Neglecting the possibility of cancellations between $A_u^{(0)}(B_s \to J/\psi\pi^0)$ and the isospin violating contribution to $A_c(B_s \to J/\psi\pi^0)$, Eq. (22) implies the lower bound

$$\frac{\bar{\Gamma}(B_s \to J/\psi\pi^0)}{\bar{\Gamma}(B \to J/\psi K)} \geq \frac{(\Delta_K + \bar{\lambda}^2\Delta_\pi)^2}{2\cos^2\gamma}, \tag{23}$$

where we neglected small phase space differences. From the current experimental data in Table I, we obtain

$$\mathcal{B}(B_s \to J/\psi\pi^0) \geq 4.4 \times 10^{-6}, \tag{24}$$

at the 1σ level, and $> 1.1 \times 10^{-6}$ at the 90% CL. This is to be compared to the SM expectation of $\mathcal{O}(10^{-7})$. The experimental uncertainties dominate this result, and are larger than the theoretical uncertainty in Eq. (23).

One can use Eq. (17) to derive an allowed region in the $(\bar{\rho}, \bar{\eta})$ plane. In Fig. 1 we show this constraint from the

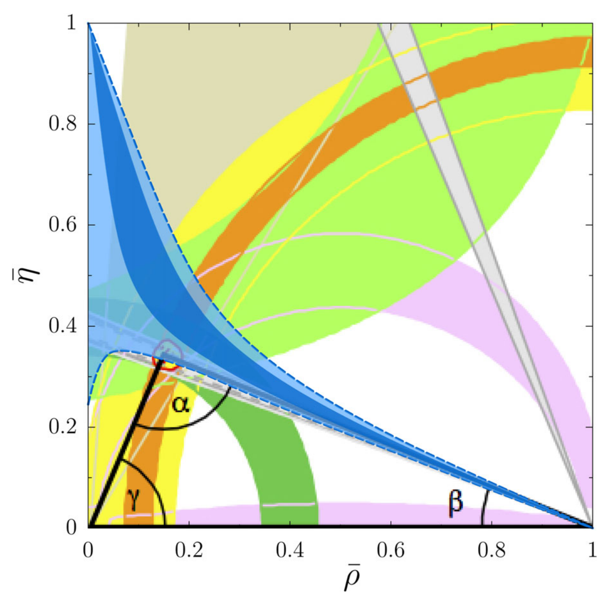

FIG. 1 (color online). Constraint from Eq. (17) at $\pm 1\sigma$ (transparent dark blue) and $\pm 2\sigma$ (transparent light blue) in the $(\bar{\rho}, \bar{\eta})$ plane, overlaid on the SM CKM fit [24].

current data, compared to other bounds. The sizable uncertainty of Δ_K leads to a somewhat loose constraint. The $\pm 1\sigma$ range at present tends to favor a slightly larger β, and is in better agreement with measurement of $|V_{ub}|$ from inclusive rather than exclusive semileptonic B decays. More precise measurements of S_{K_S}, S_{π^0}, Δ_K, and Δ_π are needed to improve the statistical significance of this constraint and to decide if there is an interesting tension with the SM CKM fit.

Future data will also give other means to explore whether the uncertainties in β are under control and to gain confidence about bounds on the V_{ub} contamination. For example, (i) the Δ_π observable in Eq. (14) only receives an $A_u^{(0)}$ contribution from the **15** representation, so more precise data can be used to constrain the size of this matrix element, which also contributes to ΔS_{K_S}, (ii) the direct CP asymmetries can be used to extract the imaginary parts of the $A_{u,f}^{(0)}/\mathcal{A}_c$ amplitude ratios, which provide a lower bound on the $|A_u/A_c|^2$ terms in Δ_π, and (iii) when $|V_{ub}|$ measurements improve, comparison of the SM CKM fit excluding S_{K_S} with Eq. (17) will provide independent information on possible origins of the tension in Fig. 1.

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