Cascading Multicriticality in Nonrelativistic Spontaneous Symmetry Breaking

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Without Lorentz invariance, spontaneous global symmetry breaking can lead to multicritical Nambu-Goldstone modes with a higher-order low-energy dispersion $\omega \sim k^n$ ($n = 2, 3, \ldots$), whose naturalness is protected by polynomial shift symmetries. Here, we investigate the role of infrared divergences and the nonrelativistic generalization of the Coleman-Hohenberg-Mermin-Wagner (CHMW) theorem. We find novel cascading phenomena with large hierarchies between the scales at which the value of n changes, leading to an evasion of the "no-go" consequences of the relativistic CHMW theorem.

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Some of the most pressing questions about the fundamental laws of the Universe (such as the cosmological constant problem or the hierarchy between the Higgs mass and the Planck scale) can be viewed as puzzles of technical naturalness [1]. In this Letter, we study the interplay of technical naturalness with spontaneous symmetry breaking (SSB) in nonrelativistic systems.

SSB is ubiquitous in nature. For relativistic systems and global continuous internal symmetries, the universal features of SSB are controlled by the Goldstone theorem. Much progress in SSB has also been achieved in the nonrelativistic cases, where the reduced spacetime symmetries allow a much richer behavior, still very much the subject of active research (see, e.g., Refs. [2–8] and the references therein). Important novelties emerge already in the simplest case of theories in the flat nonrelativistic spacetime $\mathbf{R}^{D+1}$ [covered with Cartesian coordinates $(t, \mathbf{x})$, $\mathbf{x} \equiv (x^i, i = 1, \ldots, D)$] and with the Lifshitz symmetries of spatial rotations and spacetime translations. In such theories, the Nambu-Goldstone (NG) modes can be either of two distinct types: type A, effectively described by a single real scalar $\phi(t, \mathbf{x})$ with a kinetic term quadratic in the time derivatives; or type B, described by *two* scalar fields $\phi_{1,2}(t, \mathbf{x})$ which have a first-order kinetic term and thus form a canonical pair.

In Refs. [7,8], we showed that this type A-B dichotomy is further refined into two discrete families, labeled by a positive integer n: type A_n NG modes are described by a single scalar with dispersion $\omega \sim k^n$ (and dynamical critical exponent $z = n$), while type B_{2n} modes are described by a canonical pair and exhibit the dispersion relation $\omega \sim k^{2n}$ (and dynamical exponent $z = 2n$). These two families are technically natural, and therefore stable under renormalization in the presence of interactions [7]. As usual, such naturalness is explained by a new symmetry. For $n = 1$, the NG modes are protected by the well-known constant shift symmetry $\delta\phi(t, \mathbf{x}) = b$. The $n > 1$ theories enjoy shift symmetries by a degree-P polynomial in the spatial coordinates [7]

$$\delta\phi(t, \mathbf{x}) = b + b_i x^i + \cdots + b_{i_1 \cdots i_P} x^{i_1} \cdots x^{i_P}, \qquad (1)$$

with a suitable P. Away from the type A_n and B_{2n} Gaussian fixed points, the polynomial shift symmetry is generally broken by most interactions. The lowest, least irrelevant interaction terms invariant under the polynomial shift were systematically discussed in Ref. [8] (see also Ref. [9]). Such terms are often highly irrelevant compared to all of the other possible interactions that break the symmetry.

Having established the existence of the multicritical type A and B families of NG fixed points, in this Letter we study the dynamics of flows between such fixed points in interacting theories. We uncover a host of novel phenomena involving large, technically natural hierarchies of scales, protected again by the polynomial shift symmetries. As a given theory flows between the short-distance and the long-distance regime, it can experience a natural cascade of hierarchies, sampling various values of the dynamical critical exponent z in the process. Such cascades represent an intriguing mechanism for evading some of the consequences of the relativistic Coleman-Hohenberg-Mermin-Wagner (CHMW) theorem.

Recall that in relativistic systems, all NG bosons are of type A_1, assuming that they exist as well-defined quantum objects. Whether or not they exist, and whether or not the corresponding symmetry can be spontaneously broken, depends on the spacetime dimension. This phenomenon is controlled by a celebrated theorem, discovered independently in condensed matter by Mermin and Wagner [10] and by Hohenberg [11], and in high-energy physics by Coleman [12]. We therefore refer to this theorem, in alphabetical order, as the CHMW theorem.

The relativistic CHMW theorem states that the spontaneous breaking of global continuous internal symmetries is not possible in $1 + 1$ spacetime dimensions. The proof is beautifully simple. $1+1$ is the "lower critical dimension," where ϕ is formally dimensionless at the Gaussian fixed point. Quantum mechanically, this means that its propagator is logarithmically divergent, and we must regulate it by an infrared (IR) regulator μ_{IR}:

$$\langle\phi(x)\phi(0)\rangle = \int \frac{d^2k}{(2\pi)^2}\frac{e^{ik\cdot x}}{k^2+\mu_{\mathrm{IR}}^2} \approx -\frac{1}{2\pi}\log(\mu_{\mathrm{IR}}|x|) + \mathrm{const} + O(\mu_{\mathrm{IR}}|x|). \tag{2}$$

The asymptotic expansion in Eq. (2), valid for $\mu_{\mathrm{IR}}|x| \ll 1$, shows that as we take $\mu_{\mathrm{IR}} \to 0$, the propagator stays sensitive at long length scales to the IR regulator. We can still construct various composite operators from the derivatives and exponentials of ϕ, with consistent and finite renormalized correlation functions in the $\mu_{\mathrm{IR}} \to 0$ limit, but the field ϕ itself does not exist as a quantum object. Since the candidate NG mode ϕ does not exist, the corresponding symmetry could never have been broken in the first place, which concludes the proof.

For nonrelativistic systems with type A_n NG modes, we find an intriguing nonrelativistic analog of the CHMW theorem. The scaling dimension of $\phi(t,\mathbf{x})$ at the A_n Gaussian fixed point, measured in the units of spatial momentum, is

$$[\phi(t,\mathbf{x})]_{A_n} = (D-n)/2. \tag{3}$$

The type A_n field ϕ is at its lower critical dimension when $D=n$. Its propagator then requires an IR regulator. There are many ways how to introduce μ_{IR}—for example, by modifying the dispersion relation of ϕ, as in

$$\langle\phi(t,\mathbf{x})\phi(0)\rangle = \int \frac{d\omega d^D\mathbf{k}}{(2\pi)^{D+1}}\frac{e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}}{\omega^2+|\mathbf{k}|^{2D}+\mu_{\mathrm{IR}}^{2D}}, \tag{4}$$

or

$$\langle\phi(t,\mathbf{x})\phi(0)\rangle = \int \frac{d\omega d^D\mathbf{k}}{(2\pi)^{D+1}}\frac{e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}}{\omega^2+(|\mathbf{k}|^2+\mu_{\mathrm{IR}}^2)^D}. \tag{5}$$

Regardless of how μ_{IR} is implemented, as we take $\mu_{\mathrm{IR}} \to 0$, the propagator again behaves logarithmically, both in space

$$\langle\phi(t,\mathbf{x})\phi(0)\rangle \approx -\frac{1}{(4\pi)^{D/2}\Gamma(D/2)}\log(\mu_{\mathrm{IR}}|\mathbf{x}|) + \cdots \tag{6}$$

for $|\mathbf{x}|^D \gg t$ and, in time,

$$\langle\phi(t,\mathbf{x})\phi(0)\rangle \approx -\frac{1}{(4\pi)^{D/2}D\Gamma(D/2)}\log(\mu_{\mathrm{IR}}^D t) + \cdots \tag{7}$$

for $|\mathbf{x}|^D \ll t$. The propagator remains sensitive to the IR regulator μ_{IR}. Consequently, we obtain the nonrelativistic *multicritical CHMW theorem* for type A modes:

The propagator of the type A_n would-be NG mode $\phi(t,\mathbf{x})$ at its lower critical dimension $D=n$ is logarithmically sensitive to μ_{IR}, and therefore $\phi(t,\mathbf{x})$ does not exist as a quantum mechanical object. Consequently, no spontaneous symmetry breaking with type A_n NG modes is possible in $D=n$ spatial dimensions.

By extension, this invalidates all type A_n would-be NG modes with $n>D$, whose propagator would also be pathological at long distances.

In contrast, the scaling dimension of the type B_{2n} fields is [13]

$$[\phi_{1,2}(t,\mathbf{x})]_{B_{2n}} = D/2, \tag{8}$$

and the lower critical dimension is $D=0$. Hence, in all spatial dimensions $D>0$, the type B_{2n} NG modes are free of IR divergences and are well-defined quantum mechanically for all $n=1,2,\ldots$. The nonrelativistic multicritical CHMW theorem for type B modes then simply states that the type B_{2n} symmetry breaking is possible in any $D>0$ and for any $n=1,2,\ldots$.

Whereas in the relativistic case all NG modes must always be of type A_1, in nonrelativistic systems the existence of the type A_n and B_{2n} families allows a much richer dynamical behavior.

For example, with the changing momentum or energy scales, a given NG mode can change from type A_n (or B_{2n}) to type $A_{n'}$ (or $B_{2n'}$), with $n \neq n'$, or it could change from type A to type B. The hierarchies of scales that open up in this process are naturally protected by the corresponding polynomial symmetries. One of the simplest cases is the type A_n scalar with $n>1$, whose polynomial shift symmetry of degree P is broken at some momentum scale μ to the polynomial shift symmetry of degree $P-2$, by some small amount $\varepsilon \ll 1$. This breaking modifies the dispersion relation to $\omega^2 \approx k^{2n} + \zeta_{n-1}^2 k^{2n-2}$, with $\zeta_{n-1}^2 \approx \varepsilon\mu^2$. Here, as in Ref. [1], we identify μ as the scale of naturalness. At a hierarchically much smaller scale, $\mu_\times \equiv \mu\sqrt{\varepsilon}$, the system exhibits a crossover, from type A_n above $\mu_\times$ to type A_{n-1} below $\mu_\times$. The technical naturalness of the large hierarchy $\mu_\times \ll \mu$ is protected by the restoration of the polynomial shift symmetry of degree P as $\varepsilon \to 0$.

In the special case of $n=D$, this crossover from type A_D to type A_{D-1} yields an intriguing mechanism for evading the naive conclusion of our CHMW theorem. For a large range of scales close to μ, the would-be NG mode can exhibit a logarithmic propagator. The hierarchically smaller scale $\mu_\times \ll \mu$ then serves as a natural IR regulator, allowing the NG mode to cross over to type A_{D-1} at very long distances. Therefore, the mode is well defined as a quantum mechanical object, despite the large hierarchy across which it behaves effectively logarithmically.

An interesting refinement of this scenario comes from breaking the polynomial symmetries hierarchically, in a sequence of partial breakings, from a higher polynomial symmetry of degree P to symmetries with degrees $P' < P$,

$P'' < P'$, …, all the way to constant shift. This gives rise to a cascading phenomenon, with a hierarchy of crossover scales $\mu \gg \mu' \gg \mu'' \gg \ldots$, separating plateaus governed by the fixed points with the dynamical exponent taking the corresponding different integer values. Again, such cascading hierarchies are technically natural and are protected by the underlying breaking pattern of the polynomial symmetries.

Before we illustrate this behavior in a series of examples, it is worth pointing out one very simple yet important feature of large hierarchies in nonrelativistic theories. Consider a theory dominated over some range of scales by the dispersion relation $\omega \approx k^n$, with $n > 1$. If we open up a large hierarchy of momentum scales $\mu \gg \mu'$ (say, by N orders of magnitude), this hierarchy of momentum scales gets magnified into an even larger hierarchy (by nN orders of magnitude) in energy scales.

The first model that we use to illustrate the hierarchy is a relatively well-known system in $2+1$ dimensions: the $z = 2$ Gaussian model of a single Lifshitz scalar field $\phi(t, \mathbf{x})$, with a derivative four-point self-interaction turned on [14,15]:

$$S_2 = \frac{1}{2}\int dt d^2\mathbf{x}\{\dot{\phi}^2 - (\partial^2\phi)^2 - c^2\partial_i\phi\partial_i\phi - g(\partial_i\phi\partial_i\phi)^2\}.$$

This action contains all the marginal and relevant terms of the $z = 2$ fixed point consistent with the constant shift symmetry and the reflection symmetry $\phi \to -\phi$. At the $z = 2$ Gaussian fixed point, g is classically marginal and breaks the polynomial shift symmetry of this fixed point to constant shift. Quantum corrections at one loop turn g marginally irrelevant [14].

This system allows a natural hierarchy of scales that is stable under quantum corrections. At the naturalness scale μ, we can break the polynomial shift symmetry of the $z = 2$ fixed point to constant shifts by a small amount, $\varepsilon_0 \ll 1$. This implies $g \sim \varepsilon_0$ and $c^2 \sim \varepsilon_0\mu^2$, relations which can be shown to be respected by the loop corrections. In particular, c^2 can stay naturally small, much less than μ^2. The dispersion relation changes from $z = 2$ close to the high scale μ to $z = 1$ around the much lower scale $\mu_1 \equiv \mu\sqrt{\varepsilon_0} \ll \mu$.

Our next example is a new model, which not only illustrates the cascading hierarchy with multiple crossovers but also exhibits additional intriguing renormalization properties of independent interest.

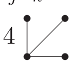
We start with the Gaussian $z = 3$ fixed point of a single scalar $\Phi(t, \mathbf{x})$ in $3+1$ dimensions, and we turn on derivative self-interactions and relevant terms as in our previous example. The free theory is

$$\begin{aligned} S_3 = \frac{1}{2}\int dt d^3\mathbf{x}\{&\dot{\Phi}^2 - \zeta_3^2(\partial_i\partial_j\partial_k\Phi)(\partial_i\partial_j\partial_k\Phi) \\ &-\zeta_2^2(\partial^2\Phi)^2 - c^2\partial_i\Phi\partial_i\Phi\}. \end{aligned} \tag{9}$$

At the classical level we can set $\zeta_3^2 = 1$ by the rescaling of space. The terms on the second line represent relevant Gaussian deformations away from the $z = 3$ fixed point. The spectrum of available self-interaction operators that are classically marginal or relevant at the $z = 3$ Gaussian fixed point is much richer than in our $2+1$ dimensional example. We shall again restrict our attention only to the operators even under $\Phi \to -\Phi$, and invariant at least under the constant shift. Up to total derivatives, which we ignore, there are three independent marginal four-point operators $O_4^{(a)}$, $a = 1, 2, 3$, one marginal six-point operator $O_6 = (\partial_i\Phi\partial_i\Phi)^3$, and one relevant four-point operator

$$\mathcal{W} = (\partial_i\Phi\partial_i\Phi)^2. \tag{10}$$

Among them, there is one unique operator O invariant under the linear shift symmetry up to a total derivative:

$$\begin{aligned} O &= 4\partial_i\partial_j\partial_k\Phi\partial_i\Phi\partial_j\Phi\partial_k\Phi + 12\partial_i\Phi\partial_i\partial_j\Phi\partial_j\partial_k\Phi\partial_k\Phi \\ &= 4 \end{aligned}$$

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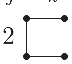
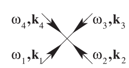

$$; \tag{11}$$

cf. Fig. 1. This operator is classically marginal.

To construct our model, we start with the free theory S_3 and turn on just the unique linear-shift invariant self-interaction O, with coupling λ. The Feynman rules of this model involve one four-vertex, which can be simplified using the momentum conservation $\mathbf{k}_4 = -\sum_{I=1,2,3}\mathbf{k}_I$ to



$$\begin{aligned} &= -i\lambda\,[k_1^2k_2^2k_3^2 + 2(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1) \\ &\quad - k_1^2(\mathbf{k}_2\cdot\mathbf{k}_3)^2 - k_2^2(\mathbf{k}_3\cdot\mathbf{k}_1)^2 - k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)^2]\,. \end{aligned} \tag{12}$$

Note that in this vertex, each momentum appears quadratically, with no subleading terms. We can write it even more compactly with the use of the fully antisymmetric ϵ_{ijk} tensor: If for any three momenta $\mathbf{k}$, $\mathbf{p}$, $\mathbf{q}$ we define $[\mathbf{kpq}] \equiv \epsilon_{ijk}k_ip_jq_k$, our vertex becomes simply

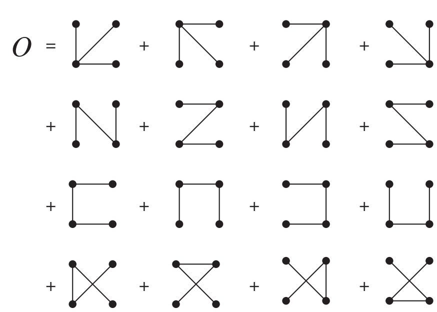
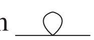


FIG. 1. Graphical representation of the unique four-point invariant O of the linear shift symmetry, as an equal-weight sum of all trees with distinguishable vertices (see Ref. [8]). Each dot represents one copy of Φ. Each link represents a contracted pair of derivatives $\partial_i \cdots \partial_i$ acting at the ends of the link.

$$-i\lambda[\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3]^2. \tag{13}$$

This simple vertex structure is intimately related to the underlying symmetries: When translated into momentum space, the linear shift symmetry $\delta\phi(t,\mathbf{x}) = b_i x^i + b$ becomes a shift of the Fourier modes $\phi(t,\mathbf{k})$ by $b_i(\partial/\partial\mathbf{k}_i)\delta(\mathbf{k}) + b\delta(\mathbf{k})$. Acting with this symmetry on any of the legs of the vertex produces zero, as the vertex is purely quadratic in each of the outside momenta.

Quantum properties.—This model has intriguing renormalization group properties, which are discussed in detail elsewhere [16]. First, note that λ satisfies a nonrenormalization theorem: Consider the $2N$-point function of Φ, with $N > 1$, and with external momenta $\mathbf{k}_1, \ldots, \mathbf{k}_{2N}$. Any one-particle irreducible diagram will be of the form $k_1^{i_1}k_1^{j_1}\cdots k_{2N}^{i_{2N}}k_{2N}^{j_{2N}} \times \mathcal{I}_{i_1 j_1\cdots i_{2N} j_{2N}}(\mathbf{k}_1,\ldots,\mathbf{k}_{2N})$. The factor $\mathcal{I}$ has no ultraviolet divergences, and with $c^2 \neq 0$ or $\zeta_2^2 \neq 0$ it approaches a finite value at $\mathbf{k}_1 = \cdots = \mathbf{k}_{2N} = 0$. The special case of $N = 2$ implies that λ does not get renormalized at any loop order. Note also that none of the operators $\mathcal{W}$, O_6, or $O_4^{(a)}$ that would break the linear shift symmetry are generated by the loop corrections.

The remarkable nonrenormalization of λ does not imply that the effective self-interaction strength would be independent of scale: There is a nontrivial renormalization of the two-point function. While the one-loop diagram  is identically zero, the two-loop diagram  gives the generic behavior which persists at higher loops: There is no wave-function renormalization, no renormalization of c^2, the loop corrections to ζ_2^2 are quadratically divergent, and those to ζ_3^2 diverge logarithmically [16]. This log divergence effectively corrects the dynamical exponent of the ultraviolet fixed point away from the classical value $z = 3$. The modified scaling in turn implies that the theory becomes effectively strongly coupled at some finite scale μ_s.

Having understood the quantum corrections, we can now study cascading hierarchies of symmetry breaking in this model, and we can confirm their technical naturalness. At some high scale μ, which will be our naturalness scale and which we will keep below μ_s, consider the following hierarchical breaking of polynomial symmetries: First, break the $P = 4$ symmetry of the $z = 3$ Gaussian fixed point to the $P = 2$ symmetry of the $z = 2$ fixed point by some small amount $\varepsilon_2 \ll 1$. Then, break $P = 2$ to $P = 1$ by an even smaller amount, $\varepsilon_1 \ll \varepsilon_2$. This pattern corresponds to

$$\zeta_3^2 \approx 1, \qquad \zeta_2^2 \approx \mu^2\varepsilon_2, \qquad c^2 \approx \mu^4\varepsilon_1, \qquad \lambda \approx \varepsilon_1. \tag{14}$$

The dispersion relation cascades from $z = 3$ at high-energy scales, to $z = 2$ at intermediate scales, to $z = 1$ at low scales [17]. Both the large hierarchies in Eq. (14) and the cascading behavior of the dispersion relation are respected by all loop corrections, and therefore are technically natural. This follows by inspection from the properties of the quantum corrections discussed above.

So far, we have focused on the cascading mechanism in the type A case. Type B systems can form their own hierarchies, in the obvious generalization of the type A cascades exemplified above. There is no analog of the lower critical dimension and the CHMW limit on n. Type A NG modes can also exhibit a flow to type B. This behavior, albeit not new (see, e.g., Ref. [18]), can be embedded as one step into the more general technically natural hierarchies of type A or B, as discussed above. In particular, the crossover to type B can provide a new IR regulator of the type A cascade at the lower critical dimension.

We shall illustrate this with the simplest type A_1 example, although the full story is, of course, more general. Consider two would-be type A NG fields, $\phi_{1,2}(t,\mathbf{x})$, in the vicinity of the $z = 1$ Gaussian fixed point

$$S_1 = \frac{1}{2}\int dt d^D\mathbf{x}\{\dot{\phi}_1^2 + \dot{\phi}_2^2 - c_1^2(\partial_i\phi_1)^2 - c_2^2(\partial_i\phi_2)^2\}.$$

For simplicity, we will set $c_1 = c_2 = 1$, although this is not necessary for our argument. Besides the rotations and translations of the two scalars, note two independent $\mathbf{Z}_2$ symmetries—the field reflection R: $(\phi_1,\phi_2) \to (\phi_1,-\phi_2)$, and the time reversal $\mathcal{T}$: $t \to -t$, $\phi_{1,2}(t,\mathbf{x}) \to \phi_{1,2}(-t,\mathbf{x})$. We can now turn on the type B kinetic term,

$$S' = S_1 + \Omega\int dt d^D\mathbf{x}(\phi_1\dot{\phi}_2 - \phi_2\dot{\phi}_1). \tag{15}$$

Ω now provides an IR regulator for the propagator. At that scale, the field reversal R and the time reversal $\mathcal{T}$ are broken to their diagonal subgroup. At energy scales below Ω, one of the would-be type A NG modes survives and turns into the type B NG mode, while the other would-be type A mode develops a gap set by Ω. Note that in $1+1$ dimensions, the "no-go" consequences of the relativistic CHMW theorem are again naturally evaded by this hierarchy: A NG mode exists quantum mechanically, after all, and symmetry breaking is possible, despite the fact that above the scale Ω, the two would-be type A modes exhibit the logarithmic two-point function, suggesting that symmetry breaking may not be possible.

The hierarchy between the type A and type B behavior is also protected by symmetries. In fact, the system has multiple symmetries that can do this job. One can rely on the breaking pattern of the discrete symmetries R and $\mathcal{T}$ mentioned above. If the type A system is Lorentz invariant, one can use Lorentz symmetry breaking to protect small Ω's. More interestingly, without relying on the discrete or Lorentz symmetries, one can introduce a shift symmetry linear in time, $\delta\phi_{1,2} = b_{1,2}t$. While the type A kinetic term is invariant under this symmetry, the type B kinetic term is not. Breaking the linear shift symmetry to constant shifts allows the type A-type B crossover scale to be hierarchically smaller than the naturalness scale.

Our original motivation for this study of technical naturalness and hierarchies in SSB came from quantum gravity and high-energy physics [7,8], especially in the context of nonrelativistic gravity [19,20]. Besides extending our understanding of the general "landscape of naturalness," we expect that our results could find their most immediate applications in two other areas: in condensed matter physics and in effective field theory of inflationary cosmology [21–23]. Both areas treat systems with nonrelativistic, Lifshitz-like symmetries similar to ours. In condensed matter, the multicriticality of NG modes will affect their thermodynamic and transport properties; for example, the type A_D modes at the lower critical dimension will exhibit specific heat linear in temperature T over the range of T dominated by the $z = D$ dispersion (up to $\log T$ corrections due to self-interactions). In the context of inflation, our self-interacting scalar field theories represent a new nonrelativistic variation on the theme of the Galileon [24], an extension of the $z = 2$ ghost condensate [25,26], and of the $z = 3$ cosmological scalar theory of Mukohyama [27,28].

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