

Spacetime Curvature and Higgs Stability after Inflation

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We investigate the dynamics of the Higgs field at the end of inflation in the minimal scenario consisting of an inflaton field coupled to the standard model only through the nonminimal gravitational coupling ξ of the Higgs field. Such a coupling is required by renormalization of the standard model in curved space, and in the current scenario also by vacuum stability during high-scale inflation. We find that for $\xi \gtrsim 1$, rapidly changing spacetime curvature at the end of inflation leads to significant production of Higgs particles, potentially triggering a transition to a negative-energy Planck scale vacuum state and causing an immediate collapse of the Universe.

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The standard model (SM) of particle physics can be consistently extrapolated to the Planck scale without any new physics, but the current measurements of the Higgs boson and top quark masses suggest that the current vacuum state of the Universe would not be stable [1]. This instability depends sensitively on the top mass m_t , which is subject to significant experimental and theoretical uncertainty [2], but for the best fit values, the Higgs potential turns negative above the instability scale $\Lambda_I \sim 10^{11}$ GeV [3]. This implies that the current vacuum would eventually decay into a negative-energy Planck scale true vacuum, but its lifetime exceeds the age of the Universe by a wide margin [1].

Whether such a metastable universe could have survived the cosmological evolution, especially inflation, has recently attracted significant interest [5–7]. In most of the simplest models of inflation, the Hubble rate during inflation is comparable to the current upper bound $H \lesssim 9 \times 10^{13}$ GeV [8]. It may therefore well be above the instability scale, in which case production of Higgs fluctuations could push the field over the potential barrier into the true Planck-scale vacuum [5]. This instability problem is exacerbated by spacetime curvature-induced running of the couplings, which makes the Higgs self-coupling negative even at low field values [6,7,9].

Notably, vacuum stability can still be maintained even during inflation without any new physics coupled to the SM fields [6], thanks to the Higgs-curvature coupling $\xi R \hat{H}^\dagger \hat{H}$. This coupling is inevitably generated by radiative corrections and when assuming the SM to be valid up to the Planck scale it is the only relevant new term when probing sub-Planckian scales. The current experimental constraints are extremely weak, $|\xi| \lesssim 2.6 \times 10^{15}$ [10]. With a positive coupling, this term increases the height of the potential

barrier between the vacua, thereby increasing the lifetime of the metastable vacuum. Vacuum stability is maintained for all inflationary scales compatible with the tensor bound [8], provided the electroweak scale value of the running coupling $\xi(\mu)$ lies above $\xi_{EW} \gtrsim 0.1$ [6].

In this Letter, we investigate the instability problem at the end of inflation, again assuming no new physics or higher-dimensional operators but taking the gravitational coupling ξ into account. In contrast with the Higgs inflation scenario [11], we assume that the Higgs field is subdominant, and for simplicity we ignore a possible direct coupling to the inflation. We focus on the parametric region $\xi > \xi_c = 1/6$ in which case the Higgs is effectively massive during inflation and does not get displaced from the SM vacuum. When inflation ends, the curvature scalar R drops rapidly, reducing the height of the potential barrier. Eventually, a new barrier is generated by the thermalized degrees of freedom restabilizing the vacuum. During the intermediate period rapid changes in the scalar curvature R can lead to nonadiabatic evolution, giving rise to significant excitations of the Higgs field, and potentially triggering the instability. Two major sources of excitations are a sudden drop in R when inflation ends [12], and parametric resonance from the oscillating curvature term sourced by the inflaton [13,14].

We can write the standard model Higgs Lagrangian as

$$\mathcal{L} = |D_\mu \hat{H}|^2 - (-M^2 + \xi R)|\hat{H}|^2 - \lambda|\hat{H}|^4, \quad (1)$$

where D_μ is the standard $SU(2) \times U(1)$ covariant derivative generalized to curved space, M the Higgs field mass parameter, and ξ the nonminimal coupling. We will parametrize the complex Higgs field doublet \hat{H} as

$$\hat{H} = \frac{1}{\sqrt{2}} \left(\hat{h}_1 + i\hat{h}_2 \right). \quad (2)$$

Assuming the Friedmann-Robertson-Walker metric $ds^2 = dt^2 - a^2 d\mathbf{x}^2 = a^2(d\eta^2 - d\mathbf{x}^2)$, and using the conformal time coordinate η , we write each of the four components \hat{h}_i as

$$\hat{h} = \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 a^2}} [\hat{a}_{\mathbf{k}} f(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger f^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}], \quad (3)$$

where \mathbf{k} is the comoving momentum and $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{k}'}^\dagger] = 0$. Ignoring interactions and the Minkowski space mass term M^2 , we obtain the mode equation

$$f''(\eta) + \left[\mathbf{k}^2 + \left(\xi - \frac{1}{6} \right) a^2 R \right] f(\eta) = 0, \quad (4)$$

where the prime denotes $d/d\eta$ and $R = 6a''/a^3$.

We can see that the mode has an effective curvature-induced mass term

$$m_{\text{curv}}^2 = \left(\xi - \frac{1}{6} \right) a^2 R. \quad (5)$$

If the frequency of the mode $\omega^2 = \mathbf{k}^2 + m_{\text{curv}}^2$ satisfies the adiabaticity condition $|\omega'^2/\omega^4| \sim |\omega''/\omega^3| \ll 1$, the mode equation may be solved adiabatically via a Wentzel-Kramers-Brillouin (WKB)-type ansatz

$$f(\eta) = \frac{e^{-i \int^\eta d\eta' \omega}}{\sqrt{2\omega}}. \quad (6)$$

The occupation number of a mode is an adiabatic invariant [15], so if a mode starts in the vacuum and evolves adiabatically, it will stay unexcited. Only for a fast change in R do we expect to find sizable excitations.

For simplicity, we will first assume that the equation of state $w = p/\rho$ changes discontinuously from $w = w_{\text{in}} = -1$ to a constant value $w = w_{\text{out}} \geq 0$ at the end of inflation. For constant w , the scale factor behaves as $a = (\eta/\eta_0)^{2/(3w+1)}$, and the scalar curvature expressed in terms of the Hubble rate $H = \dot{a}/a = a'/a^2$ is $R = 3(1 - 3w)H^2$. We assume that $\xi \gtrsim 1$, so that the adiabaticity condition is satisfied on both sides of the transition, but at the transition itself the curvature mass term drops instantaneously by a factor

$$\frac{m_{\text{out}}^2}{m_{\text{in}}^2} = \frac{1 - 3w_{\text{out}}}{4}. \quad (7)$$

This nonadiabatic change produces excitations of the Higgs field.

Following [16] we can write two sets of solutions: $f_{\text{in}}(\eta)$ that reduces to the incoming adiabatic mode in the past and $f_{\text{out}}(\eta)$ that reduces to the outgoing mode in the future with $\omega_{\text{in}}^2 = \mathbf{k}^2 + m_{\text{in}}^2$ and $\omega_{\text{out}}^2 = \mathbf{k}^2 + m_{\text{out}}^2$, respectively. The two vacua can be connected with a Bogoliubov transformation $f_{\text{in}}(\eta) = \alpha_{\mathbf{k}} f_{\text{out}}(\eta) + \beta_{\mathbf{k}} f_{\text{out}}^*(\eta)$, with Bogoliubov coefficients

$$\alpha_{\mathbf{k}} = \frac{1}{2} \left[\frac{\sqrt{\omega_{\text{out}}}}{\sqrt{\omega_{\text{in}}}} + \frac{\sqrt{\omega_{\text{in}}}}{\sqrt{\omega_{\text{out}}}} \right], \quad \beta_{\mathbf{k}} = \frac{1}{2} \left[\frac{\sqrt{\omega_{\text{out}}}}{\sqrt{\omega_{\text{in}}}} - \frac{\sqrt{\omega_{\text{in}}}}{\sqrt{\omega_{\text{out}}}} \right]. \quad (8)$$

For $w \neq 1/3$, we can approximate $\omega_{\text{in/out}} = m_{\text{in/out}}$ in the long-wavelength limit $\mathbf{k}^2 \ll m_{\text{out}}^2$. In that case the Bogoliubov coefficients are independent of momentum and we find the occupation number

$$n_{\mathbf{k}} \equiv \langle \text{in}, 0 | \hat{n}_{\mathbf{k}}^{\text{out}} | 0, \text{in} \rangle = |\beta_{\mathbf{k}}|^2 = \frac{(2 - \sqrt{1 - 3w})^2}{8\sqrt{1 - 3w}}. \quad (9)$$

In the matter-dominated case ($w = 0$) this reduces to $n_{\mathbf{k}} = 1/8$.

In the radiation-dominated case ($w = 1/3$), $\omega_{\text{out}} = |\mathbf{k}|$. At long wavelengths $\mathbf{k}^2 \ll m_{\text{in}}^2$ we find a momentum-dependent occupation number

$$n_{\mathbf{k}} = |\beta_{\mathbf{k}}|^2 = \frac{\sqrt{3}\xi a_* H_*}{2|\mathbf{k}|}, \quad (10)$$

where the star $*$ refers to values at the time of the transition and we have assumed $\xi \gg 1$.

In reality, the change in the equation of state is not instantaneous, and the slower the transition, the fewer modes are excited. To quantify this, we make use of Ref. [16] (see also Ref. [17]), and model the time dependence of the curvature mass term by

$$m_{\text{curv}}^2(\eta) = \frac{m_{\text{in}}^2 + m_{\text{out}}^2}{2} - \frac{m_{\text{in}}^2 - m_{\text{out}}^2}{2} \tanh \nu \eta. \quad (11)$$

Here, ν controls the speed of the transition, with $\nu \rightarrow \infty$ and $\nu \rightarrow 0$ corresponding to infinitely fast and infinitely slow transitions, respectively. This case can also be solved analytically, and gives the occupation number

$$n_{\mathbf{k}} = \frac{\sinh^2 [\pi(\omega_{\text{out}} - \omega_{\text{in}})/(2\nu)]}{\sinh(\pi\omega_{\text{in}}/\nu) \sinh(\pi\omega_{\text{out}}/\nu)}. \quad (12)$$

From the various limits of (12) follow a number of physical implications: In the limit of large momentum, large mass or slow transition, i.e., $\mathbf{k} \rightarrow \infty$, $\xi \rightarrow \infty$, or $\nu \rightarrow 0$ the occupation number approaches zero exponentially. For a mode to receive excitations one must satisfy (assuming $m_{\text{in}} > m_{\text{out}}$)

$$|\mathbf{k}| \lesssim \min \{m_{\text{in}}, \sqrt{(\nu/2\pi)^2 - m_{\text{out}}^2}\}. \quad (13)$$

The excitation of the Higgs field due to the nonadiabatic behavior at the end of inflation leads to a rapid growth of the fluctuations of the field. To describe this we calculate the variance of the field in the “in” vacuum state, at time η after the transition

$$\begin{aligned} \langle \text{in}, 0 | \hat{h}_{\text{out}}^2 | 0, \text{in} \rangle \\ = \int \frac{d^3|\mathbf{k}|}{2\omega(2\pi)^3 a^2} \left[1 + 2|\beta_{\mathbf{k}}|^2 + 2\alpha_{\mathbf{k}}\beta_{\mathbf{k}} \cos\left(\int^\eta dr' \omega\right) \right]. \end{aligned} \quad (14)$$

These fluctuations are uncorrelated on superhorizon scales, and therefore they correspond to a stochastic background as opposed to the coherent Higgs condensate generated during inflation [18]. This will lead to vacuum decay, if the field value averaged over a sufficiently large volume, which we conservatively consider to be the whole Hubble volume, exceeds the position of the potential barrier. Therefore, we only include modes up to $\Lambda = aH$. We also neglect the vacuum part and coherently oscillating terms in (14),

$$\langle \hat{h}^2 \rangle_\Lambda \equiv \int_0^\Lambda \frac{d|\mathbf{k}| \mathbf{k}^2 |\beta_{\mathbf{k}}|^2}{2\pi^2 a^2 \omega}. \quad (15)$$

Substituting Eqs. (9) and (10), we get for $\xi \gg 1$

$$\langle \hat{h}^2 \rangle_{aH} \approx \left(\frac{H}{2\pi}\right)^2 \begin{cases} (12\sqrt{3\xi})^{-1} \left(1 - \frac{m_{\text{in}}}{m_{\text{out}}}\right)^2, & w \neq \frac{1}{3} \\ \sqrt{3\xi} a/a_*, & w = \frac{1}{3} \end{cases}. \quad (16)$$

From (16) and (7) we see that for $w \neq 1/3$ the superhorizon variance is suppressed by $\xi^{-1/2}$. This is, however, not the case for conformal equation of state $w = 1/3$ due to inverse scaling $\xi^{1/2}$, whereby a large variance may be generated for $\xi \gg 1$ for a sufficiently rapid drop in m_{curv} [19].

As an example, we assume a model where inflation ends within a time scale $\nu^{-1} \sim (400H)^{-1}$ with an effective reheating equation of state $w = 1/3$. Using (16) with $\xi \gtrsim 500$ we get for the magnitude of the generated variance: $\langle \hat{h}^2 \rangle_{aH} \gtrsim H^2$. Therefore, we may conclude that if inflation ends abruptly to a state with $w_{\text{out}} \sim 1/3$, a variance larger than Λ_I can be generated for sufficiently high inflationary scale $H \gtrsim \Lambda_I$, such that the fatal transition to the negative-energy vacuum is likely triggered.

Above, we assumed that the equation of state changes monotonically to its final value when inflation ends. However, this is not what happens in most models of inflation. Instead, the inflaton field oscillates coherently about its minimum, and this leads to a much stronger effect. For an inflationary model with a single inflaton field ϕ in potential $V(\phi)$, the scalar curvature R is given by

$$R = \frac{1}{M_{\text{pl}}^2} \left[4V(\phi) - \left(\frac{d\phi}{dt}\right)^2 \right]. \quad (17)$$

It is clear from this expression that R , and therefore also the curvature mass term m_{curv}^2 , oscillates between positive and negative values. As a result, the Higgs field grows exponentially in a way that can be interpreted either as tachyonic or resonant growth.

Reheating via the curvature coupling ξ was first studied in Ref. [13] where it was named “geometric reheating” (see also [14]). A similar tachyonic resonance effect present in reheating models with trilinear interactions was studied in [20]. Assuming that the inflaton potential can be well approximated by the quadratic term

$$V(\phi) = \frac{m^2}{2} \phi^2, \quad (18)$$

we can write the solution as $\phi(t) = \Phi \cos(mt)$, where $\Phi \approx \sqrt{6}HM_{\text{pl}}/m$ is a slowly varying amplitude and $H \equiv 2/(3t)$ as given by the equation of state for matter. Substituting this background solution into Eq. (17) we obtain the Mathieu form of the mode equation (4):

$$\begin{aligned} \frac{d^2 f(z)}{dz^2} + [A_k - 2q \cos(2z)]f(z) = 0, \\ z = mt, A_k = \frac{\mathbf{k}^2}{a^2 m^2} + \xi \frac{\Phi^2}{2M_{\text{pl}}^2}, \quad q = \frac{3\Phi^2}{4M_{\text{pl}}^2} \left(\frac{1}{4} - \xi\right). \end{aligned} \quad (19)$$

Following the analysis of [20] we get for the occupation number after the first oscillation

$$n_{\mathbf{k}}^1 = e^{2X_{\mathbf{k}}}, \quad X_{\mathbf{k}} = \int_{\Delta z} \Omega_{\mathbf{k}} dz \approx \sqrt{\xi} \frac{\Phi}{M_{\text{pl}}}, \quad (20)$$

where $\Omega^2 \equiv -\omega^2$ with ω^2 being the term in the square brackets in (19) and Δz covers the time when $\omega^2 < 0$ during first oscillation. In the last form of (20) we have assumed that $|\mathbf{k}| \leq aH$ and that the amplitude Φ is roughly constant ($a \sim 1$) during the oscillation. It is clear from (20) that for large ξ the occupation number may become very large already during the first oscillation. Using (20) we get an estimate for the superhorizon variance after the first oscillation ($|\beta_{\mathbf{k}}|^2 = n_{\mathbf{k}}^1$)

$$\langle \hat{h}^2 \rangle_{aH} \approx \int_0^{aH} \frac{d|\mathbf{k}| \mathbf{k}^2 |\beta_{\mathbf{k}}|^2}{2\pi^2 \omega_{\text{out}}} \sim \left(\frac{H}{2\pi}\right)^2 \frac{\exp\{\sqrt{\xi} \frac{2\Phi}{M_{\text{pl}}}\}}{3\sqrt{3\xi}}, \quad (21)$$

which is accurate for $\xi \geq 1/6 + 3/8 \sim 0.5$. Below this region only superhorizon modes with $\mathbf{k}^2 \leq k_{\text{cut}}^2 \equiv a^2 H^2 6(\xi - 3/8)$ receive tachyonic amplification. In this case, an accurate approximation for the superhorizon variance can be obtained by an expansion near the threshold point $\xi = 3/8$

$$\langle \hat{h}^2 \rangle_{aH} \approx 16 \sqrt{\frac{3}{5}} \left(\frac{H}{2\pi} \right)^2 (\xi - 3/8)^{3/2}, \quad \frac{3}{8} < \xi \lesssim \frac{1}{2}. \quad (22)$$

The exponential growth of the particle number and the variance is constrained by backreaction which eventually shuts off the tachyonic resonance and makes the dynamics nonlinear. We consider two backreaction effects: Higgs self-interaction and gravity.

When considering the Higgs self-interaction, we adopt the convention that the RG-running parameters are evaluated at the scale $\mu = H$, which corresponds to the optimal choice in terms of RG-improved effective potential [6] in case H is the highest scale of the problem [21]. The RG-running self-coupling $\lambda(\mu)$ becomes negative at the scale of instability Λ_I and therefore we may approximate ($\mu = H$):

$$\lambda(H) \approx \lambda_0 \text{sgn}(\Lambda_I - H), \quad \text{with } \lambda_0 \approx 0.01. \quad (23)$$

Hence, the backreaction from self-interactions is limited to the region $H < \Lambda_I$, while for $H > \Lambda_I$ the negative self-coupling enhances the tachyonic resonance and accelerates particle production. However, here we will not consider this enhancement but use the conservative estimate of (21).

For $H < \Lambda_I$, the positive Higgs self-interaction generates an effective mass

$$m_{\text{eff}}^2 = m_{\text{curv}}^2 + 6\lambda_0 \langle \hat{h}^2 \rangle. \quad (24)$$

The resonance is shut off once (see also [13,22]) $A_k + \delta A > 2|q|$ with $\delta A = 6\lambda \langle \hat{h}^2 \rangle / m^2$. From this and Eqs. (23) we then obtain a condition for maximal variance in the region $H < \Lambda_I$: $\langle \hat{h}^2 \rangle \lesssim \xi H^2 / \lambda_0$, including contributions from all tachyonic modes $|\mathbf{k}| \leq k_{\text{cut}}$. The corresponding superhorizon variance with $|\mathbf{k}| \leq aH$ is related to the total variance by $\langle \hat{h}^2 \rangle_{aH} \sim \langle \hat{h}^2 \rangle / [2(3\xi)^{3/2}]$ and is hence constrained by

$$\langle \hat{h}^2 \rangle_{aH} \lesssim \frac{H^2}{6\lambda_0 \sqrt{3\xi}} \quad \text{for } H < \Lambda_I, \quad (25)$$

where we have used $R \approx 3H^2$ for $w \approx 0$ assumed to be valid after oscillations (or in average during oscillations).

For gravity backreaction we need to compare the energy density contained in produced Higgs particles to total energy density $\rho \sim 3H^2 M_{\text{pl}}^2$. In Gaussian (Hartree) approximation with $\langle \hat{h}^4 \rangle = 3\langle \hat{h}^2 \rangle^2$ we get for the Higgs energy density an estimate

$$\rho_{\text{Higgs}} \approx 24\xi H^2 \langle \hat{h}^2 \rangle + 6\lambda \langle \hat{h}^2 \rangle^2, \quad (26)$$

whereby we find the following constraints for the superhorizon variance for the Higgs energy density to remain subdominant during oscillations [23]:

$$\langle \hat{h}^2 \rangle_{aH} \lesssim \frac{2M_{\text{pl}}^2}{11(3\xi)^{5/2}} \quad \text{and} \quad \langle \hat{h}^2 \rangle_{aH} \lesssim \frac{HM_{\text{pl}}}{2(2\lambda_0)^{1/2}(3\xi)^{3/2}}. \quad (27)$$

We estimate that the probability for a potentially catastrophic transition to the negative-energy vacuum is significant once the superhorizon variance has exceeded the instability scale $\Lambda_I \sim 10^{11}$ GeV of the Higgs effective potential:

$$\Delta h \equiv \sqrt{\langle |\hat{H}|^2 \rangle_{aH}} = \sqrt{4\langle \hat{h}^2 \rangle_{aH}} > \Lambda_I. \quad (28)$$

We show the corresponding region after the first oscillation as blue in Fig. 1. On the left, at $\xi \lesssim 10^2$, the increase of the variance is limited by the weakness of the resonance (21), although the nonshaded area above the dashed blue line would be mostly filled by the subsequent 3–4 oscillations. Further to the left, the effect becomes even weaker and disappears completely at $\xi = 3/8$. The bottom half of the plot is excluded by backreaction from the self-interactions. On the right, the backreaction from the gravity effects makes the linear approximation invalid, and more sophisticated methods are needed. Backreaction from the other SM degrees of freedom is not likely to change the qualitative picture as resonant amplification takes at least a few inflaton oscillations. A direct Higgs-inflaton coupling can enhance or weaken the effect depending on the model but would generally not eliminate it as long as the Higgs has a tachyonic phase during the first inflaton oscillation.

In conclusion, we have shown that for a sufficiently high inflationary scale $H \gtrsim \Lambda_I$, changing spacetime curvature can trigger a fatal transition to a negative-energy vacuum, for a wide range of the curvature coupling $1 \lesssim \xi \lesssim 10^5$. This conclusion applies to both rapid reheating and oscillating inflaton. Combining this with the lower bound

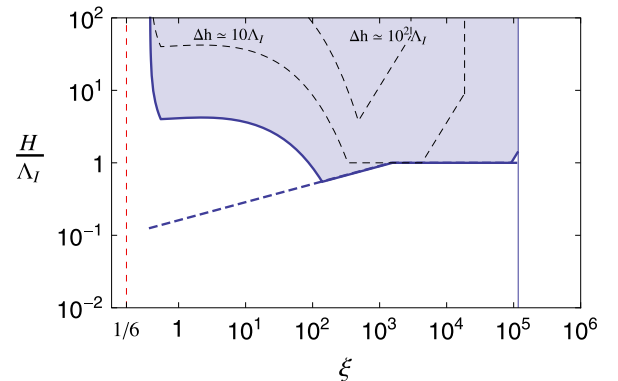


FIG. 1 (color online). The estimated instability region (shaded, blue) where $\Delta h \gtrsim \Lambda_I$ after the first oscillation, given by Eqs. (21), (22), (25), (27), and (28). The dashed lines correspond to $\Delta h \gtrsim 10\Lambda_I$ and $\Delta h \gtrsim 10^2\Lambda_I$. For the parameters we have used $\Phi = 0.3M_{\text{pl}}$ and $\Lambda_I = 10^{-7}M_{\text{pl}}$.

$\xi_{EW} \gtrsim 0.1$ imposed by stability during inflation [6], we find that the minimal scenario of standard model in a background of conventional high-scale inflation constrains the coupling ξ to be close to its conformal value $\xi = 1/6$.

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