

Experimental Observation of Two-Dimensional Anderson Localization with the Atomic Kicked Rotor

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Dimension 2 is expected to be the lower critical dimension for Anderson localization in a time-reversal-invariant disordered quantum system. Using an atomic quasiperiodic kicked rotor—equivalent to a two-dimensional Anderson-like model—we experimentally study Anderson localization in dimension 2 and we observe localized wave function dynamics. We also show that the localization length depends exponentially on the disorder strength and anisotropy and is in quantitative agreement with the predictions of the self-consistent theory for the 2D Anderson localization.

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The metal-insulator Anderson transition plays a central role in the study of quantum disordered systems. Using a tight-binding description of an electron in a lattice, Anderson [1] postulated in 1958 that the dominant effect of impurities in a crystal is to randomize the diagonal term of the Hamiltonian, and showed that this may lead to a localization of the wave function, in sharp contrast with the Bloch-wave solution for a perfect crystal. In a weakly disordered (3D) crystal, the eigenstates are delocalized, leading to a diffusive (metallic) transport, while strong disorder produces an insulator with localized eigenstates. From its original solid-state physics scope [1–4] this approach has been applied to a large class of systems in which waves propagate in disorder. This includes quantum-chaotic systems [5,6] and electromagnetic radiation [7–9]. Important theoretical progress was obtained in Ref. [10], which postulated that Anderson localization can be described by a one-parameter scaling law, leading to the prediction that, for $d \leq 2$, the dynamics is generically localized, even if the disorder is very weak. For $d > 2$, it predicted the existence of the Anderson transition between a diffusive dynamics at weak disorder and a localized dynamics at strong disorder.

There is no fully quantitative theory of Anderson localization, and analytic results are scarce. Supersymmetry techniques [11] allow derivation of expansions in powers of $d - 2$ of the various quantities of interest, but reaching even $d = 3$ is difficult. A useful, simplified theoretical approach is the so-called self-consistent theory of localization. In few words, it can be thought as a mean field theory where large fluctuations are neglected, but where weak localization corrections to transport, due to interference between time reversed multiply scattered paths, are included self-consistently. For spinless time-invariant systems, belonging to the orthogonal symmetry class [11], this approach correctly predicts the existence of the

metal-insulator Anderson transition for $d > 2$, although it fails to predict the correct critical exponent. For $d = 1$, it quantitatively predicts the localization length in a weak disorder. Other approaches lead to approximate values for the critical exponent not far from the numerical prediction [12].

Dimension $d = 2$ —the lower critical dimension—is very special, the localization properties depending on the symmetry class. In the orthogonal symmetry class, the dynamics is always localized, but the localization length is predicted to scale *exponentially* with the inverse of the disorder strength, i.e., $\xi \propto \ell \exp(\pi k \ell / 2)$ [13] where k is the wave vector and ℓ the mean-free path for propagation in the disordered medium. As discussed in the Supplemental Material [14], such an exponential dependence is a signature of the fact that $d = 2$ is the lower critical dimension for Anderson localization. The 2D case has been previously studied experimentally in optical and ultracold atom systems [8,18], but no quantitative indication of the exponential scaling has been demonstrated yet. In the present Letter, we use the well-known correspondence between the d -dimension Anderson model and the d -frequency quasiperiodic kicked rotor [6,19,20] to test experimentally these predictions.

The quasiperiodic kicked rotor (QPKR) [5,6,19–21] is a spatially one-dimensional system with an engineered time dependence such that its dynamics is similar to the dynamics of a time-independent multidimensional system. The QPKR can be simply realized experimentally by exposing laser-cooled atoms (Cesium in the present work) to a delta-pulsed (kicked) laser standing wave of wave number k_L and time period T_1 . The *amplitude* of the kicks is quasiperiodically time modulated with a frequency ω_2 . The dynamics is effectively one dimensional along the axis of the laser beam, as transverse directions are uncoupled. The corresponding Hamiltonian is

$$H = \frac{p^2}{2} + K \cos x [1 + \varepsilon \cos(\omega_2 t)] \sum_{n=0}^{N-1} \delta(t-n), \quad (1)$$

where x is the particle position, p its momentum, K the kick intensity, and ε the amplitude of the modulation. We have chosen conveniently scaled variables such that distances along the x axis are measured in units of $(2k_L)^{-1}$, the particle's mass is unity and time is measured in units of the pulse period T_1 . In the quantum case, a crucial parameter is $\tilde{\hbar} \equiv 4\hbar k_L^2 T_1 / M$, the reduced Planck constant.

For $\varepsilon = 0$, one obtains the periodic kicked rotor, which can be mapped onto a one-dimensional Anderson-like model [6], and displays “dynamical” localization [5,22], that is, Anderson localization in momentum space instead of configuration space. For nonzero ε , the temporal dynamics of the QPKR is exactly that of a two-dimensional periodic kicked system [19,20,23], which itself can be mapped—provided $2\pi/T_1$, ω_2 and $\tilde{\hbar}$ are incommensurable numbers—onto a two-dimensional Anderson-like anisotropic-hopping model, where anisotropy is controlled by ε and the ratio of hopping to diagonal disorder is controlled by $K/\tilde{\hbar}$ [24].

The experimental observation of dynamical localization in the atomic kicked rotor in 1995 [21] can thus be interpreted as the first experimental observation of 1D Anderson localization with atomic matter waves. The two-frequency modulation of the QPKR—which can be mapped on a 3D Anderson model [20,23]—was used to experimentally observe 3D Anderson localization and the metal-insulator Anderson transition [25], accurately measure the critical exponent, and demonstrate its universality [26].

The experimental study of the 2D case is more challenging than the 3D one, because the observation of the exponential behavior of the localization length p_{loc} requires p_{loc} to vary over about 1 order of magnitude. The localization time increasing with p_{loc} , this also requires the ability to preserve coherence over several hundreds of kicks. This needed major evolutions of our experimental setup [27].

Experimentally, an atomic sample consisting of few million atoms is prepared in a thermal state (3.2 μK) whose momentum distribution is much narrower than the expected localization length. The atomic cloud is then “kicked” by a far-detuned ($\Delta \approx 13$ GHz) pulsed standing wave (SW). Pulse duration is typically $\tau = 300$ ns, while the typical pulse period $T_1 = 27.778$ μs corresponds to an effective Planck constant $\tilde{\hbar} = 2.89$. According to Eq. (1), an adjustable amplitude modulation with $\omega_2/2\pi = \sqrt{5}$ is superimposed to the kick sequence. In our previous experiments, to minimize coupling with gravity, the SW was horizontal. However, for 1000 kicks the atoms fall down by 3.8 mm, compared to the 1.5 mm SW waist, limiting the maximum number of kicks to 200. In order to overcome this limit we used in the present experiment a *vertical* SW, and the atoms fall freely between kicks. The new SW is

formed by two beams that can be independently controlled, both in amplitude and phase, through a radio frequency driving two acousto-optic modulators. This allows us to accurately cancel gravity effects, by imposing a linear frequency chirp to one arm of the SW with respect to the other, so that the SW itself “falls” with acceleration g . A kicked rotor is thus realized in the free-falling reference frame. These technical improvements are discussed in more detail in the Supplemental Material [27]. At the end of the sequence, the velocity distribution is measured by a standard time-of-flight technique.

Figure 1(a) shows experimental momentum distributions $\Pi(p)$ recorded after 0 to 1000 kicks for $K = 5.34$, $\tilde{\hbar} = 2.89$, $\varepsilon = 0.36$. If the dynamics were classical, the momentum distribution would keep its initial Gaussian shape and the average kinetic energy would increase linearly with time, $E_{\text{kin}} = E_{\text{kin}}(t=0) + Dt$, where D is the classical diffusion constant in momentum space. In contrast, the experimental result displays a distribution which diffusively broadens at short times, but tends to freeze, i.e., to *localize* at long times. This clear-cut proof of localization is confirmed by the shape of the momentum distribution, shown in Fig. 1(b) after 200 kicks. It very clearly displays an exponential shape [28] (a straight line in the logarithmic plot) $\exp(-|p|/p_{\text{loc}})/2p_{\text{loc}}$ characteristic of localization with a localization length p_{loc} [29]. Figure 1(c) shows the momentum distributions after 1000 kicks for $K = 5.34$, $\tilde{\hbar} = 2.89$, and increasing values of ε . It demonstrates that the localization length varies very rapidly with ε , indicating the evolution from a 1D localization at $\varepsilon = 0$ to a truly 2D localization with a much longer localization length at $\varepsilon = 0.6$. In order to prevent trivial localization on KAM tori [30], we always used $K > 4$, ensuring that the classical dynamics is ergodic.

Instead of measuring the full momentum distribution, it is sufficient to measure the population $\Pi_0(t)$ of the zero velocity class as

$$E_{\text{kin}} \propto \frac{1}{4\Pi_0^2(t)} \quad (2)$$

is proportional to $\langle p^2 \rangle(t)$ (as the total number of atoms is constant) [31].

Figure 2 displays E_{kin} (at 1000 kicks) vs ε for various values of K and $\tilde{\hbar}$, showing that the exponential dependence in ε is a general feature, with a rate that decreases with $\tilde{\hbar}$ and increases with K . Note the overall E_{kin} dynamics of a factor of 60 (corresponding to an eightfold increase in the localization length), a key feature of the present experiment.

The scaling theory of localization [10] predicts that dimension $d = 2$ is the lower critical dimension for the Anderson transition. For a time-reversal invariant spinless system (thus belonging to the *orthogonal* universality class), all states are localized with an exponentially large localization length. For a usual 2D time-independent

system, the relevant parameter is the dimensionless conductance at short scale, equal to the product $k\ell$ of the wave vector by the mean free path, so that the logarithm of the localization length is proportional to $k\ell$ [14].

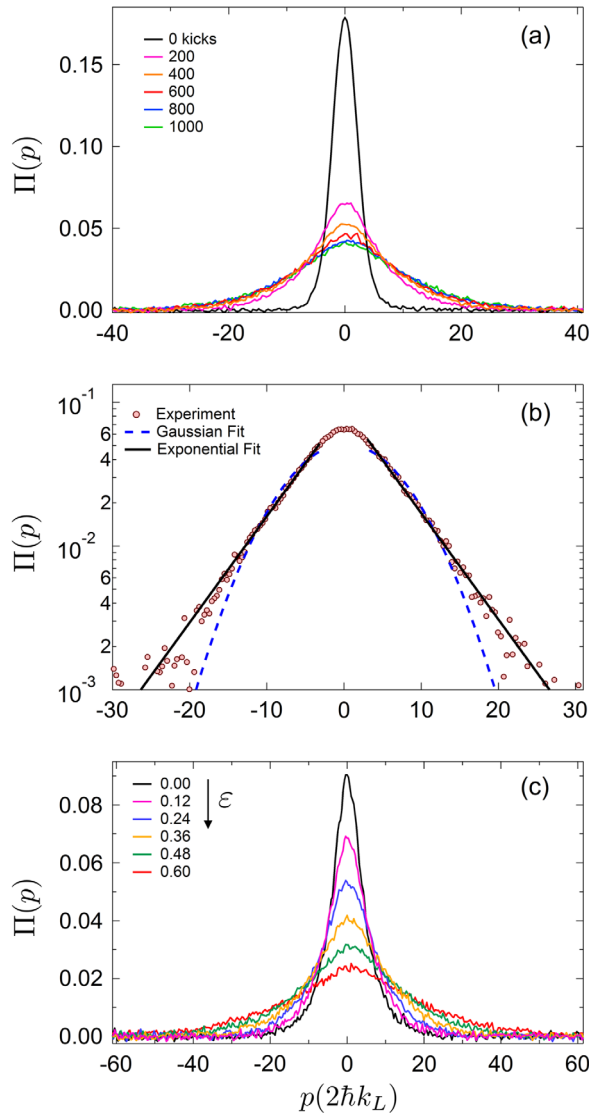


FIG. 1 (color online). Experimentally recorded momentum distributions for the kicked rotor exposed to a quasiperiodic driving, Eq. (1). (a) $K = 5.34$, $\hbar k = 2.89$, $\epsilon = 0.36$, 0 to 1000 kicks (step 200). The momentum distribution diffusively broadens at short times and freezes at longer times, proving the existence of 2D Anderson localization. Time increases from top to bottom curves. (b) Momentum distribution at 200 kicks in log scale, showing the exponential shape characteristic of localization. The circles are experimental points, the blue dashed line is a Gaussian fit and the black solid line an exponential fit for $|p| > 3(2\hbar k_L)$. (c) Localized momentum distributions after 1000 kicks, as a function of the anisotropy parameter ϵ , for $K = 5.34$, $\hbar k = 2.89$ as in (a) and (b). The modulation amplitude ϵ increases from top to bottom curves. The rapid increase of the localization length shows the evolution from the 1D localization at $\epsilon = 0$ to the truly 2D Anderson localization. Note the different horizontal scales in the various plots.

The scaling theory cannot be directly transposed to the case of the kicked rotor for two reasons. (i) There is no wave vector playing the role of k . Instead, one must consider the diffusion constant (in momentum space), which is, for a periodic kicked rotor, approximately equal to $K^2/4$. (ii) The diffusion process for the 2D quasiperiodic kicked rotor is not isotropic. As shown in Ref. [32] and discussed in the Supplemental Material [14], the quasiperiodic kicked rotor can be mapped on a 2D Anderson-like model, whose dynamics at short time is indeed diffusive, but anisotropic. Along the “physical” direction (which coincides with the atom momentum component along the standing wave), the diffusion constant is—for small ϵ —almost equal to the one of the periodic kicked rotor, $D_{11} \approx K^2/4$; along the other (virtual) direction, the diffusion constant is $D_{22} \approx K^2\epsilon^2/8$, so that it vanishes in the limit $\epsilon \rightarrow 0$, where one must recover the usual 1D periodic kicked rotor.

Altogether, the relevant parameter is the geometric average of the diffusion constant along the two directions $\sqrt{D_{11}D_{22}} \propto \epsilon K^2$. The scaling theory predicts that the logarithm of the localization length should be proportional to $\sqrt{D_{11}D_{22}}/\hbar k^2$. A similar prediction was made in Ref. [19] using a slightly different method.

The self-consistent theory of localization is an attempt towards more quantitative predictions, based on the same ideas as the scaling theory. It has been successfully used to predict properties of the Anderson transition [33,34], and was transposed to the periodic kicked rotor in Refs. [22,35] and to the quasiperiodic kicked rotor with two additional driving frequencies in Ref. [36]. It consists in computing perturbatively the weak localization correction to the (anisotropic) diffusion constant and to extrapolate to the strong localization regime. It, however, depends on

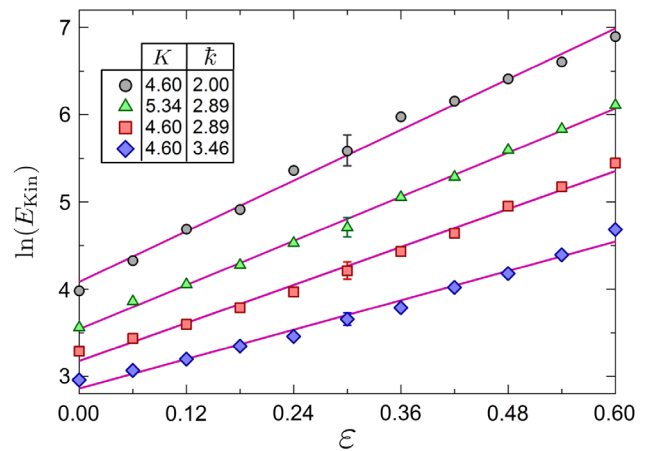


FIG. 2 (color online). Kinetic energy E_{kin} of the quasiperiodic kicked rotor vs the modulation amplitude ϵ , for various values of the kicking strength K and effective Planck constant $\hbar k$. The error bars indicate the typical experimental uncertainty. The four curves are straight lines in this logarithmic scale, with a slope that decreases with $\hbar k$ and increases with K .

the cutoffs used. For our quasiperiodic kicked rotor [14] it confirms the prediction of the scaling theory, namely,

$$p_{\text{loc}} = \frac{K^2}{4\bar{k}} \exp\left(\frac{\alpha \varepsilon K^2}{\bar{k}^2}\right) \quad (3)$$

where α is a number of the order unity, which may however depend smoothly on the parameters. In the limit $\varepsilon \rightarrow 0$, it is $\alpha = \pi/\sqrt{32}$.

In Fig. 3, we display the results of 275 measurements, corresponding to 12 values of the ratio $K/\bar{k} \in [1.3, 2.5]$, with $K \in [4.33, 7.26]$ and $\bar{k} = \{2.89, 3.2, 3.46\}$, and to ε values from 0 to 0.6 (step 0.06). Dividing $E_{\text{kin}}(\varepsilon)$ by $E_{\text{kin}}(\varepsilon = 0)$ makes it possible to probe the exponential term in Eq. (3). The exponential dependence (straight line in logarithmic scale) is visible for $\varepsilon \lesssim 1$, materialized by the red dashed line, corresponding to the prediction $\alpha = \pi/\sqrt{32}$ of the self-consistent theory. Despite the spreading of the experimental results around the average trend, the overall agreement is rather good. This proves the exponential dependence of the localization length in 2D, and

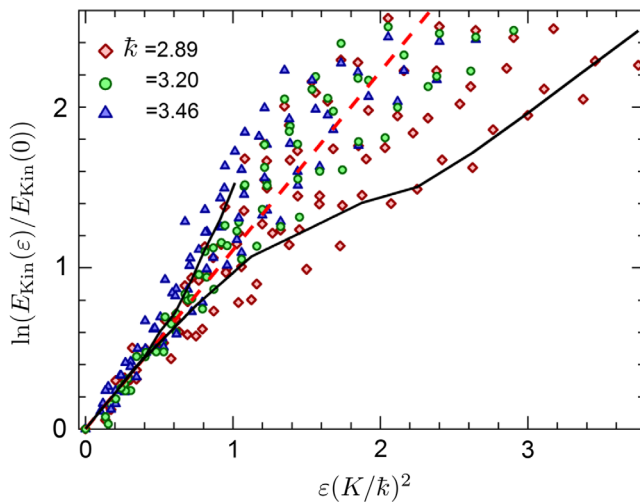


FIG. 3 (color online). Increase in the kinetic energy at $t = 1000$ ($\propto p_{\text{loc}}^2$) of the quasiperiodic kicked rotor with respect to the purely one-dimensional situation $\varepsilon = 0$ vs the scaling parameter $\varepsilon(K/\bar{k})^2$. The cloud of experimental points—collected at various values of K , ε and \bar{k} —is distributed around an average linear dependence in this semi-logarithmic plot, which shows the exponential dependence of the localization length, characteristic of 2D Anderson localization. The red dashed line is the prediction of Eq. (3). The spread is due in part to experimental imperfections [at large $\varepsilon(K/\bar{k})^2$, the localization time is not much shorter than the duration of the experiment] and in part to fundamental reasons: The linear dependence on $\varepsilon K^2/\bar{k}^2$ in the argument of the exponential, Eq. (3), is valid only at small ε , and the formula assumes that the classical diffusion constant is proportional to K^2 , while the actual diffusion constant has oscillatory corrections. The black curves are numerical simulations corresponding to the two “extreme” values of $K/\bar{k} = 1.3$, $\bar{k} = 3.46$ (higher curve) and $K/\bar{k} = 2.5$, $\bar{k} = 2.89$ (lower curve); they display the same spreading phenomenon.

thus that $d = 2$ is the lower critical dimension for the metal-insulator Anderson transition. Some deviations are nevertheless visible. They arise from different phenomena. First, for large ε , the localization time can be only slightly shorter than the duration of the experiment (1000 kicks), meaning that the measured momentum distribution is not the asymptotic one for infinite time and underestimates the eventual saturation of E_{kin} at long time. This explains why the experimental points at large ε tend to lie below the theoretical prediction. This is confirmed by numerical calculations in the experimental conditions for the largest value of $K/\bar{k} = 2.5$ (longest localization time), see the solid lower curve in Fig. 3. A second, more fundamental, phenomenon is that Eq. (3) assumes that the classical diffusion constant is simply $K^2/4$, which is valid only for $K \gg 1$, whereas oscillatory corrections at moderate K are known to exist for the 1D kicked rotor [37] and to persist even for the 3D QPKR [32]. This dependence is thus not eliminated by the normalization to $E_{\text{kin}}(\varepsilon = 0)$. This explains a significant part of the spreading of the data. Finally, Eq. (3) is expected to be valid in the $\varepsilon \rightarrow 0$ limit, see Supplemental Material [14]. At larger ε values, higher order terms must come into play and are responsible for significant deviations. This is visible in Fig. 3, where both experimental (points) and numerical (solid lines) data are well predicted at small $\varepsilon K^2/\bar{k}^2$, but are more widely spread as $\varepsilon K^2/\bar{k}^2$ increases. A thorough analysis of all these deviations is beyond the scope of this Letter.

To summarize, we presented the first experimental evidence of two-dimensional Anderson localization with atomic matter waves. We studied the variation of the localization length with the system parameters and showed that it displays an exponential dependence characteristic of time-reversal spinless systems. To the best of our knowledge, such experimental evidence has not been observed previously. It demonstrates experimentally that $d = 2$ is the lower critical dimension of the Anderson transition. The observed localization length varies as predicted by the scaling and the self-consistent theories of localization.

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