

### Comment on “Surface Plasmons and Nonlocality: A Simple Model”

In Ref. [1] the authors show how one can approximate the effects of nonlocal optical response at a flat interface between a dielectric and a conductor by considering instead the fully local optical response of a layered system in which a thin layer, characterized by an effective permittivity, is imagined to lie between the dielectric and the conductor. However, a numerical illustration of the accuracy of their approximation is incorrect and does not fairly show its limitations. At issue is the dispersion curve of their LAM in Fig. 2c. We have attempted to do the same calculation but obtain very different results.

To simplify the presentation we make several extra approximations, which still allow a direct comparison with their numerical results. Specifically, the interface is the boundary between vacuum with  $\epsilon_1 = 1$  and a metal with  $\epsilon_3 \rightarrow \epsilon = 1 - \omega_p^2/\omega^2$ , where  $\omega_p$  is the bulk plasma frequency and no damping is allowed. The focus is on the  $p$ -wave reflection amplitude at frequency  $\omega$  and surface parallel wave vector  $Q$  with retardation neglected. Using the simplest hydrodynamic model for the metal’s nonlocal response then yields [2]

$$r = \frac{\epsilon - 1 + Q(1 - \epsilon)/\bar{Q}_L}{\epsilon + 1 - Q(1 - \epsilon)/\bar{Q}_L}, \quad (1)$$

where  $\bar{Q}_L = [Q^2 + (\omega_p^2 - \omega^2)/\beta^2]^{1/2}$  is the wave vector that controls the decay of the induced longitudinal wave into the bulk metal. The goal is to mimic this result by that due to a system with purely local permittivities, plus the addition of a thin sheet replacing a top layer of the metal. To first order in the thickness  $w$  of this layer, one has

$$r_{123} = \frac{\epsilon - 1 + wQ[(\epsilon_{2,\parallel} - \epsilon) + (1 - \epsilon/\epsilon_{2,\perp})]}{\epsilon + 1 + wQ[(\epsilon_{2,\parallel} - \epsilon) - (1 - \epsilon/\epsilon_{2,\perp})]}, \quad (2)$$

where we have allowed for an anisotropic (but still spatially local) response within the inserted layer. Comparing the two equations, we let  $w \rightarrow 0$  and require  $\epsilon_{2,\perp} \propto w$ , so one term dominates in the square brackets in Eq. (2). In detail, we choose  $\epsilon_{2,\parallel} = \epsilon$  and set  $w\epsilon/\epsilon_{2,\perp} = (\epsilon - 1)/q_L$ , where  $q_L$  is the  $Q = 0$  value of  $\bar{Q}_L$ . Equation (2) becomes

$$r_{123} \rightarrow \frac{\epsilon - 1 + Q[w + (1 - \epsilon)/q_L]}{\epsilon + 1 - Q[w + (1 - \epsilon)/q_L]}. \quad (3)$$

For Eq. (3) to match Eq. (1) one needs both small  $w$  and  $Q$  not large, and/or  $\omega$  not near  $\omega_p$ , in order to have  $\bar{Q}_L \approx q_L$ . To illustrate these constraints, we present in Fig. 1 a comparison of the predictions of Eqs. (1) and (3) for the mode dispersions in a metal-insulator-metal (MIM) system. Following Ref. [1] we set the velocity parameter to

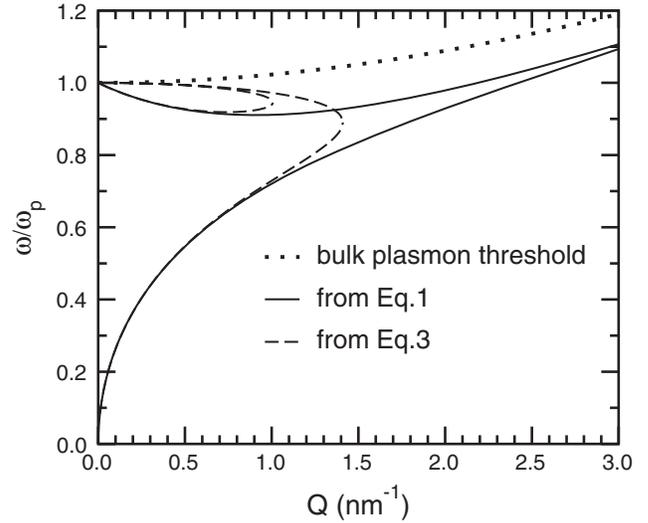


FIG. 1. Mode dispersions for a MIM system with a gap size of  $d = 1$  nm. See text for other parameter choices.

$\beta/c = 0.0036$ , choose  $\hbar\omega_p = 3.3$  eV, and calculate for an insulator gap of 1 nm. Our figure should be compared to Fig. 2c and Fig. S1 of Ref. [1]. Rather than “perfect agreement” we find significant differences for  $Q > 1/\text{nm}$ , where a “back bending” of the modes develops. Our calculation in Fig. 1 is done with  $w = 0$  but since at  $\omega/\omega_p = 1/\sqrt{2}$ , one has  $(1 - \epsilon)/q_L = 0.68$  nm, we find that using  $w = 0.1$  nm in Eq. (3) produces changes of order 10%, but does not suppress the erroneous back bending. This dependence on  $w$  is also inconsistent with the claims of Ref. [1].

Since retardation effects are only important for  $Q \leq \omega_p/c = 0.017/\text{nm}$ , their inclusion does not remove the back bending. However, we do find that the mimicking of Eq. (1) by Eq. (3) does work very well in the small  $Q$  region. Unfortunately, a primary aim of Ref. [1] was to devise a scheme that would ease computational burdens for a nanostructured interface, which would necessarily involve large  $Q$  values and, hence, the failure of the mimicking process.

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