Gravitation-Wave Emission in Shift-Symmetric Horndeski Theories

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Gravity theories beyond general relativity typically predict dipolar gravitational emission by compact-star binaries. This emission is sourced by "sensitivity" parameters depending on the stellar compactness. We introduce a general formalism to calculate these parameters, and show that in shift-symmetric Horndeski theories stellar sensitivities and dipolar radiation vanish, provided that the binary's dynamics is perturbative (i.e., the post-Newtonian formalism is applicable) and cosmological-expansion effects can be neglected. This allows one to reproduce the binary-pulsar-observed orbital decay.

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General relativity (GR) is very successful at interpreting gravity on a huge range of scales, field strengths, and velocities. Nevertheless, evidence for dark matter and dark energy may be interpreted as a breakdown of GR on cosmological scales. Also, GR is intrinsically incompatible with quantum field theory, and should be replaced, at high energies, by a (still unknown) quantum theory of gravity. Given this situation, guidance may come from experiments, namely, those testing gravity in regimes involving strong fields and/or relativistic speeds. These experiments include measurements of the orbital decay of binary pulsars driven by the emission of gravitational waves (GWs) [1] and upcoming GW interferometers [2–4]. It is therefore crucial to analyze gravitational emission in theories alternative to GR.

Modified gravity theories typically generalize GR by introducing extra gravitational fields nonminimally coupled to the metric. An example is Fierz-Jordan-Brans-Dicke (FJBD) gravity [5–7]; see, e.g., Ref. [8] for a recent review of more theories. Often, even theories where no extra fields are explicitly added [e.g., $f(R)$ gravity] can be recast as GR plus extra fields by a suitable change of variables. The extra fields generally introduce "fifth forces," and the motion of a body free falling in a gravitational field will generally depend on the body's nature. This effect can be suppressed for weakly gravitating bodies by assuming that the extra gravitational fields do not couple to matter directly; i.e., the equivalence principle (EP) can be restored for weakly gravitating bodies ("weak EP"). However, for bodies with strong self-gravity, the extra fields will still effectively couple to matter (because they are nonminimally coupled to the metric, which in turn is coupled to matter via gravity). This coupling will be increasingly important as the body's gravitational binding energy—which measures the "strength" of the body's self-gravity—increases. Indeed, the extra gravitational fields will generally affect the body's binding energy, and since in relativistic theories all forms of energy gravitate, the body's gravitational mass will depend on the extra fields via the binding energy. As such, the inertial mass may differ from the gravitational mass if the binding energy's contribution to the latter is important. Indeed, possible deviations from the "strong" EP (i.e., the universality of free fall for strongly gravitating bodies) in modified gravity theories are typically parametrized by the "sensitivities" [9]

$$s_{Q_A} = \frac{1}{M}\frac{\partial M}{\partial Q_A}\bigg|_{N,\Sigma}, \tag{1}$$

i.e., the derivatives of the gravitational mass M relative to the theory's extra fields Q_A, while keeping the body's total baryon number N and entropy Σ fixed. (The sensitivities thus measure the body's response to changes in the local value of Q_A.) For weakly gravitating bodies to obey the weak EP, it must be $s_{Q_A} \approx 0$. Indeed, if the extra fields Q_A do not couple to matter directly, they only enter M via the binding energy, whose contribution is negligible if the body's self-gravity is weak.

The sensitivities enter both the conservative dynamics of binary systems (e.g., the periastron precession) and the dissipative one (i.e., GW emission). Their leading-order dissipative effect is the emission of dipolar gravitational radiation; i.e., binaries will produce GWs $\phi \sim G/c^3 \times \mathcal{O}[(s_\phi^{(1)} - s_\phi^{(2)})^2]$, where $s_\phi^{(1)}, s_\phi^{(2)}$ are the bodies' sensitivities. This is a -1 post-Newtonian (PN) effect, i.e., it is enhanced by $(v/c)^{-2}$ compared to GR (v being the binary's relative velocity), if $s_\phi^{(1)}$, $s_\phi^{(2)} \sim 1$. Therefore, knowledge of the sensitivities is crucial to verify the agreement between a theory and GW observations (binary-pulsar data or direct detections). For example, binary pulsars have already placed strong constraints on Lorentz-symmetry violations in gravity [10,11] and on certain scalar-tensor theories [8,12,13], and even stronger bounds will be possible with direct GW detections [14–16].

Here, we generalize previous work in scalar-tensor theories [17,18] and Lorentz-violating gravity [10,11] by introducing a formalism to calculate the sensitivities of stars (including pulsars) in *generic* theories. As an application, we calculate them in the most general scalar-tensor theories ("Horndeski theories" or "generalized galileons") [19–21] that have second-order field equations and are invariant under a shift of the scalar field ϕ, i.e., $\phi \to \phi + \mathrm{const}$. These theories have received much attention because some of them provide a screening mechanism (the "Vainshtein mechanism" [22,23]; see also Refs. [24,25]), which may permit modifying gravity on cosmological scales (possibly reproducing cosmological data without dark energy) while recovering GR on small scales, where gravitational modifications would be "screened." Another reason for interest in these theories is that galileon interactions arise in the decoupling limit of massive gravity [26,27].

We show that in shift-symmetric Horndeski theories (SSHTs) stellar sensitivities vanish (this agrees with previous results for a specific theory of this class, i.e., Einstein-dilaton Gauss-Bonnet gravity in the decoupling limit [28]), and the leading-order GW emission matches that of GR. We conclude that SSHTs reproduce existing binary-pulsar data (provided that a PN expansion over Minkowski space is adequate—i.e., the binary's dynamics is perturbative—and that cosmological-expansion effects can be neglected [29–32]), but deviations might still appear for the sources targeted by upcoming GW interferometers.

We use Latin (Greek) lower-case letters for space (spacetime) indices, and capital Latin letters for indices running on fields. Repeated indices denote summations, and we assume $c=1$ and signature $(-,+,+,+)$.

A general expression for the sensitivities.—Consider an action

$$S = \int \mathcal{L}(Q_A, \partial_\mu Q_A) d^4x, \tag{2}$$

where Q_A are fields, and the corresponding field equations

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu Q_A)}\right) - \frac{\partial\mathcal{L}}{\partial Q_A} = 0. \tag{3}$$

A solution's canonical mass-energy is

$$M = \int d^3x(\pi_A \partial_t Q_A - \mathcal{L}), \tag{4}$$

where $\pi_A = \partial\mathcal{L}/\partial(\partial_t Q_A)$ for brevity. For a solution with $\pi_A \partial_t Q_A = 0$ [this is the case for, e.g., stationary solutions, where $\partial_t Q_A = 0$, or solutions where some variables Q_A depend on time, but $\pi_A = 0$; see, e.g., Eq. (8)], the mass is then simply

$$M = -\int d^3x \mathcal{L}(Q_A, \partial_\mu Q_A). \tag{5}$$

(We have checked that this mass matches the ADM mass, both in GR and SSHTs; see also Ref. [33].) Consider a neighboring solution, with $\pi_A \partial_t Q_A = 0$ and the same total baryon number and entropy [cf. Eq. (1)]. The two solutions represent the same star with different local values of the fields Q_A, and their mass difference is

$$\delta M = -\int d^3x \partial_t(\pi_A \delta Q_A) - \int d^2S_i \frac{\partial\mathcal{L}}{\partial(\partial_i Q_A)}\delta Q_A, \tag{6}$$

where δQ_A is the difference between the fields, d^2S_i is a coordinate surface element, and we have used Gauss's theorem, as well as Eq. (3) to show that the bulk terms vanish.

Because this result assumes an action with no derivatives higher than first order, it would not seem to apply to GR, since the Einstein-Hilbert action depends on second metric derivatives. However, the latter enter the Einstein-Hilbert action only through a total divergence; i.e., GR can be described by a first-order "Einstein Lagrangian" $\mathcal{L}_g = \sqrt{-g}g^{\mu\nu}(\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\alpha} - \Gamma^\lambda_{\mu\nu}\Gamma^\alpha_{\lambda\alpha})/(16\pi G)$ (see, e.g., Ref. [34]). Besides the metric, in theories different from GR there are other gravitational degrees of freedom, which we denote by ϕ_A, and which we assume to be coupled to the metric and its derivatives, i.e., with Lagrangian $\mathcal{L}_\phi(\phi_A, \partial_\mu\phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$. Moreover, the matter fields ψ_B must couple minimally to the metric [i.e., with Lagrangian $\mathcal{L}_m(\psi_B, \partial_\mu\psi_B, g_{\mu\nu})$] to satisfy the weak EP. (The fields ϕ_A and ψ_B are not necessarily scalars; they could represent, e.g., the components of a vector or tensor.) Consider now a stationary (i.e., time-independent) star in one such theory. The mass difference between neighboring solutions is

$$\begin{aligned}\delta M = &-\int d^2S_i \frac{\partial\mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})}\delta g_{\mu\nu} \\ &-\int d^3x\partial_t(\pi_{\psi_B}\delta\psi_B) - \int d^2S_i \frac{\partial\mathcal{L}_m}{\partial(\partial_i\psi_B)}\delta\psi_B \\ &-\int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi_A)}\delta\phi_A - \int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})}\delta g_{\mu\nu},\end{aligned} \tag{7}$$

where we have used $\partial_t(\pi_{g_{\mu\nu}}\delta g_{\mu\nu}) = \partial_t(\pi_{\phi_A}\delta\phi_A) = 0$ because of stationarity. Note that we have *not* assumed $\partial_t(\pi_{\psi_B}\delta\psi_B) = 0$ [but only $\pi_{\psi_B}\partial_t\psi_B = 0$, so that Eqs. (5) and (6) hold], which will allow us to use this expression for perfect-fluid stars.

The Lagrangian for a perfect fluid with equation of state $\rho = \rho(n, \sigma)$ (ρ, n, and σ being the energy density, baryon-number density, and entropy per particle, respectively) is [35,36]

$$\mathcal{L}_m = -\sqrt{-g}\rho - \varphi\partial_\mu J^\mu - \theta\partial_\mu(\sigma J^\mu) - \alpha_A\partial_\mu(\beta_A J^\mu), \tag{8}$$

where α_A, β_A, φ, and θ are scalars (α_A can be interpreted as Lagrangian coordinates). The fluid four-velocity is defined as $U^\mu = J^\mu/|J|$ (with $|J| = \sqrt{-g_{\mu\nu}J^\mu J^\nu}$) and $n = |J|/\sqrt{-g}$; i.e., J^μ is the baryon-number density current $J^\mu = \sqrt{-g}nU^\mu$. Variation with respect to $g_{\mu\nu}$ (keeping $J^\mu, \varphi, \theta, \sigma, \alpha_A$, and β_A fixed) gives the perfect-fluid stress-energy tensor, by using the first law of thermodynamics $n\partial\rho/\partial n|_\sigma = p+\rho$ (p being the pressure). Variations with respect to $J^\mu, \varphi, \theta, \sigma, \alpha_A$, and β_A yield

$$\partial_\mu J^\mu = \partial_\mu(\sigma J^\mu) = J^\mu \partial_\mu \beta_A = J^\mu \partial_\mu \alpha_A = 0,$$
$$hU_\mu = -\partial_\mu\varphi - \sigma\partial_\mu\theta - \beta^A\partial_\mu\alpha_A, \qquad U^\mu\partial_\mu\theta = T, \tag{9}$$

where $h=(p+\rho)/n$ is the specific enthalpy, and we have used $\partial\rho/\partial\sigma|_n = nT$ (T being the temperature) from the first law of thermodynamics. From these equations, it is clear that φ and θ are Lagrange multipliers enforcing the local baryon-number and entropy conservation, while β_A and α_A are constant along the fluid lines. In addition, these equations imply the conservation of the fluid stress-energy tensor [35,36].

For a stationary fluid, we can adopt comoving coordinates where $U^i = J^i = 0$. Therefore, the third term in Eq. (7) vanishes (since $\delta J^i = 0$), while the second becomes

$$\begin{aligned} -\int d^3x \partial_t(\pi_{\psi_B}\delta\psi_B) &= \int d^3x\partial_t[\varphi\delta J^t + \theta\delta(\sigma J^t) + \alpha_A\delta(\beta^A J^t)] \\ &= -\int d^3x[(h-\sigma T)U_t\delta J^t + TU_t\delta(\sigma J^t)], \end{aligned} \tag{10}$$

where we have used $\partial_t\alpha_A = \partial_t\beta^A = \partial_t J^t = \partial_t\sigma = 0$, $-\partial_t\varphi = (h-\sigma T)U_t$, and $-\partial_t\theta = TU_t$, obtained from Eq. (9) (with $J^i = 0$). For a fluid in hydrostatic and thermodynamic equilibrium in a stationary spacetime, TU_t and $(h-\sigma T)U_t$ are uniform [37–39], and Eq. (10) thus becomes

$$-\int d^3x\partial_t(\pi_{\psi_B}\delta\psi_B) = -(h-\sigma T)U_t\delta N - TU_t\delta\Sigma, \tag{11}$$

where $\delta N = \int d^3x\delta J^t$ and $\delta\Sigma = \int d^3x\delta(\sigma J^t)$ are the differences in total baryon number and total entropy between the two solutions. Therefore, the terms in Eq. (7) depending on the matter variables vanish if the two solutions have the same entropy and baryon number [cf. Eq. (1)].

Furthermore, if the two solutions are asymptotically flat, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$, the first term in Eq. (7) also vanishes [17]. This is seen by evaluating the integral at $r\to\infty$, since the integrand decays as $1/r^3$, while $d^2S_r \sim r^2$. Therefore, the mass variation only depends on the non-GR part of the action, i.e.,

$$\delta M = -\int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi_A)}\delta\phi_A - \int d^2S_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})}\delta g_{\mu\nu}. \tag{12}$$

This generalizes similar expressions for scalar-tensor theories [17] and Lorentz-violating gravity [10,11].

Let us now consider an action depending also on *second* derivatives of the metric and extra gravitational degrees of freedom ϕ_A (e.g., Horndeski theories),

$$S = \int \mathcal{L}(Q_A, \partial_\mu Q_A, \partial_\nu\partial_\mu Q_A)d^4x. \tag{13}$$

By introducing new fields $X_{A\mu} \equiv \partial_\mu Q_A$ and enforcing this definition by Lagrangian multipliers, one obtains

$$S = \int[\mathcal{L}(Q_A, X_{A\mu}, \partial_\nu X_{A\mu}) + \lambda^{A\mu}(X_{A\mu} - \partial_\mu Q_A)]d^4x, \tag{14}$$

whose variation relative to $X_{A\mu}$ yields

$$\lambda^{A\mu} = \partial_\alpha\left(\frac{\partial\mathcal{L}}{\partial(\partial_\alpha X_{A\mu})}\right) - \frac{\partial\mathcal{L}}{\partial X_{A\mu}}. \tag{15}$$

Since Eq. (14) is in the form given by Eq. (2), the construction outlined above gives [using also Eq. (15)]

$$\begin{aligned} \delta M = &-\int d^2S_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi_A)}\delta\phi_A \\ &-\int d^2S_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})}\delta g_{\mu\nu} - \int d^2S_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j\phi_A)}\partial_j\delta\phi_A \\ &-\int d^2S_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\partial_j\delta g_{\mu\nu} \\ &+\int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j\phi_A)}\right)\delta\phi_A \\ &+\int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\right)\delta g_{\mu\nu}. \end{aligned} \tag{16}$$

Sensitivities and gravitational radiation in SSHTs.—SSHTs are described by the Lagrangian for the galileon scalar ϕ [19–21],

$$\begin{aligned} \mathcal{L}_\phi = \frac{\sqrt{-g}}{16\pi G}\{&K(X) - G_3(X)\Box\phi + G_4(X)R \\ &+ G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\ &+ G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}[(\Box\phi)^3 \\ &- 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3] + \chi\phi\mathcal{G}\}, \end{aligned} \tag{17}$$

where ∇, R, and $G_{\mu\nu}$ are the Levi-Civita connection, Ricci scalar, and Einstein tensor, K, G_3, G_4, and G_5 are arbitrary functions of $X \equiv -\nabla_\mu\phi\nabla^\mu\phi/2$, $G_{iX} \equiv \partial G_i/\partial X$, $\Box \equiv \nabla^\mu\nabla_\mu$, $(\nabla_\mu\nabla_\nu\phi)^2 \equiv \nabla_\mu\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi$, $(\nabla_\mu\nabla_\nu\phi)^3 \equiv \nabla_\mu\nabla^\rho\phi\nabla_\rho\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi$, χ is a constant, and $\mathcal{G} \equiv R^{\mu\nu\lambda\kappa}R_{\mu\nu\lambda\kappa} - 4R^{\mu\nu}R_{\mu\nu} + R^2$ is the Gauss-Bonnet scalar. [The $\chi\phi\mathcal{G}$ term is shift invariant because $\mathcal{G}$ is (locally) a total divergence. Also, this term can be obtained by choosing $G_5 \propto \ln|X|$ [40].] The total action also includes the Einstein Lagrangian $\mathcal{L}_g$ and the perfect-fluid Lagrangian $\mathcal{L}_m$ described above. Regarding the latter, one may couple the matter fields to a "disformal" metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \xi\nabla_\mu\phi\nabla_\nu\phi$ (ξ being a constant), rather than to $g_{\mu\nu}$ alone. If such a disformal coupling is present, however, one can adopt $\tilde{g}_{\mu\nu}$ as the metric field, which puts the matter Lagrangian in the form $\mathcal{L}_m(\psi_B, \partial_\mu \psi_B, \tilde{g}_{\mu\nu})$ considered above, while the action (17) remains invariant up to redefinitions of the functions K, G_3, G_4, and G_5 [41]. Thus, our results also apply to SSHTs with a (special) disformal coupling to matter. To allow asymptotically flat solutions (see below), we assume that K, G_3, G_4, and G_5 are analytic in X. [This excludes, e.g., $K(X) \sim X^{3/2}$, which reproduces the modified Newtonian dynamics (MOND) [42] in the nonrelativistic limit, and gives $\phi \sim \ln r$ near spatial infinity.] This implies $K(X) = X + \mathcal{O}(X)^2$, since a constant can be absorbed in the matter stress-energy tensor as an effective cosmological constant, and a coefficient for the linear term can be absorbed by redefining ϕ; $G_3 = \mathcal{O}(X)$ and $G_5 = \mathcal{O}(X)$, since a constant produces a total divergence (recall the Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$); and $G_4 = \mathcal{O}(X)$, since a constant in G_4 can be absorbed in the metric Lagrangian $\mathcal{L}_g$ by redefining the "bare" Newton constant G.

Consider now an isolated stationary star, i.e., $\phi = \mathcal{O}(1/r)$, $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$, where the shift symmetry allows one to set ϕ asymptotically to zero. [In theories with a Vainshtein screening, $\phi = \mathcal{O}(1/r)$ and $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$ only for $r \gg r_v$, with r_v the Vainshtein radius within which deviations from GR are screened; we will return to this later]. To determine the sensitivities, recall that Eq. (16) compares two neighboring solutions. If the latter are asymptotically flat, $G_{\mu\nu} \sim R \sim R_{\mu\nu} \sim R_{\mu\nu\alpha\beta} = \mathcal{O}(1/r)^3$, and the differences $\delta\phi$ and $\delta g_{\mu\nu}$ between them scale as $\delta\phi = \mathcal{O}(1/r)$ and $\delta g_{\mu\nu} = \mathcal{O}(1/r)$. With these asymptotics, all the surface integrals in Eq. (16) vanish when evaluated at $r \to \infty$, thus yielding zero sensitivities. For example, by considering the contribution of K to the first term in Eq. (16), we find

$$\delta M \sim \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi)} \delta\phi \sim r^2 |\nabla\phi| \frac{1}{r} \sim \frac{1}{r}, \tag{18}$$

and this surface integral vanishes when evaluated at $r \to \infty$. Similar calculations show that all terms in Eq. (16) vanish.

Note that it is the shift symmetry that makes the sensitivities vanish. In generic Horndeski theories, two solutions ("1" and "2") have in general different asymptotic values of ϕ, i.e., $\phi^{(1)} = \phi_\infty + \alpha/r + \mathcal{O}(1/r^2)$ (α being a constant) and $\phi^{(2)} = \phi_\infty + \delta\phi_\infty + (\alpha + \delta\alpha)/r + \mathcal{O}(1/r^2)$. As such, $\delta\phi = \delta\phi_\infty + \mathcal{O}(1/r)$ [while $\delta\phi = \mathcal{O}(1/r)$ in SSHTs] and Eq. (18) gives $\delta M \propto \alpha\delta\phi_\infty$, hence $s_\phi \propto \partial M/\partial\phi_\infty \propto \alpha$. This is the case for, e.g., FJBD theory [5–7] and Damour-Esposito-Farèse gravity [18], where the sensitivities are proportional to the coefficient α of the $1/r$ term in the scalar field's falloff. Therefore, these theories predict the emission of dipolar radiation; hence, they can be constrained by existing binary-pulsar data [12] and they give testable predictions for upcoming GW interferometers [14–16,43,44]. In general, the sensitivities are also not zero in the presence of a conformal coupling between the galileon and matter, e.g., in massive gravity. This effect is not considered in, e.g., Refs. [45,46].

The vanishing sensitivities imply the absence of dipolar gravitational emission in SSHTs. Consider metric and scalar perturbations $h_{\mu\nu}$ and $\delta\phi$ over a Minkowski background, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon)^2$ and $\phi = \epsilon\delta\phi + \mathcal{O}(\epsilon)^2$, with ϵ a perturbative parameter. To leading order in ϵ, Eq. (17) coincides with the Lagrangian of a minimally coupled scalar field [note that up to boundary terms, $G_4 = \mathcal{O}(X)$ gives $\mathcal{O}(\epsilon)^3$ terms in the action, because $XR + (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2 = G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi + \text{total divergence}$ [40]]. Therefore, the leading-order field equations for a binary become

$$\Box_\eta \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu} + \epsilon\mathcal{O}(\bar{h}\delta\phi, \delta\phi^2, \bar{h}^2), \tag{19}$$

$$\begin{aligned}\Box_\eta \delta\phi &= \mathcal{O}(s_\phi^{(1)}, s_\phi^{(2)}) + \epsilon\mathcal{O}(\bar{h}\delta\phi, \delta\phi^2, \bar{h}^2) \\ &= \epsilon\mathcal{O}(\bar{h}\delta\phi, \delta\phi^2, \bar{h}^2),\end{aligned} \tag{20}$$

where $\Box_\eta = \eta^{\mu\nu}\partial_\mu\partial_\nu$, $\bar{h}^{\mu\nu} = \eta^{\mu\nu} - \sqrt{-g}g^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h^\alpha_\alpha + \mathcal{O}(\epsilon)$, $s_\phi^{(1)} = s_\phi^{(2)} = 0$ are the sensitivities, $T^{\mu\nu}$ is the binary's stress-energy tensor (which is the same as in GR and depends on the stars' masses and velocities), and we have assumed the Lorenz gauge $\partial_\mu \bar{h}^{\mu\nu} = 0$. (These equations can also be obtained from Ref. [17] by noting that the theories studied here coincide at leading order with those of Ref. [17] when the conformal scalar-matter coupling is switched off, i.e., $A(\phi) = 1$ in the notation of Ref. [17]). Equation (20) implies that ϕ is not excited at leading order in ϵ, while Eq. (19) matches its GR counterpart. As such, gravitational emission behaves as in GR at leading PN order; i.e., monopolar and dipolar emission vanish—like in GR, but unlike non-shift-symmetric scalar-tensor theories, where these emission channels are sourced by the sensitivities [17,47]—while the quadrupolar emission matches that of GR. [Note that FJBD theory corrects GR's quadrupole formula even for $s_\phi^{(1)} = s_\phi^{(2)} = 0$, due to the (Einstein-frame) conformal scalar-matter coupling. This coupling (and corresponding corrections) are not present here.]

Let us now replace the leading-order solution in the nonlinear terms of Eqs. (19) and (20). Since $\delta\phi = 0$ at leading order, at next-to-leading order one has

$$\Box_\eta \bar{h}^{\mu\nu} = -16\pi G[(1 - \epsilon\bar{h}^\alpha_\alpha)T^{\mu\nu} + \epsilon\tau^{\mu\nu}] + \mathcal{O}(\epsilon)^2, \tag{21}$$

$$\Box_\eta \delta\phi = -\epsilon\chi\delta\mathcal{G} + \mathcal{O}(\epsilon)^2. \tag{22}$$

Here, $\tau^{\mu\nu} = \mathcal{O}(\bar{h}^2)$ is the GR gravitational stress-energy pseudotensor [48], while $\delta\mathcal{G} = \mathcal{O}(\bar{h})^2$ is the perturbed Gauss-Bonnet invariant. Equations (21) and (22) match those of Einstein-dilaton Gauss-Bonnet gravity [i.e., $K = X$ and $G_3 = G_4 = G_5 = 0$ in Eq. (17)] in the decoupling limit; hence, the leading PN order at which deviations from GR appear is the same as in that theory. Following Ref. [49], we then conclude that non-GR effects only appear at 3PN (2PN) order in the dissipative (conservative) sector.

Our PN formalism is only valid for stellar radii R much smaller than the gravitational wavelength $\lambda_{\rm GW}$, so that the perturbations decay as $1/r$ in both the far zone (i.e., at distances from the binary $r \gg \lambda_{\rm GW}$) and near zone (i.e., for $R \ll r \ll \lambda_{\rm GW}$) [50]. In theories with a Vainshtein mechanism, $\phi = \mathcal{O}(1/r)$ and $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$ only for $r \gg r_v$; i.e., r_v is an "effective" stellar radius. Therefore, our perturbative PN approach requires $r_v \ll \lambda_{\rm GW}$, in which case r_v only causes higher-PN-order "finite-size" effects [50]. Note that for the dominant quadrupole mode, $\lambda_{\rm GW} \sim 10^9$ km for binary pulsars and $\lambda_{\rm GW} \sim 10^3$ km for the late inspiral of neutron-star binaries targeted by upcoming GW detectors. Thus, although the value of r_v (and the very presence of a Vainshtein mechanism) depend on the theory, our approach might break down, especially in the latter case. If the dynamics is not perturbative (i.e., PN), the analysis may be conducted on a case-by-case basis using a WKB approach [45,46]. While simple theories (e.g., cubic galileons) may still provide results in agreement with binary-pulsar data [45] (however, see also Ref. [51]), in more generic theories (e.g., ones including quartic and quintic galileons), the nonperturbative dynamics generally makes the WKB approach also fail unless $r_v \ll \lambda_{\rm GW}$ [46]. If this condition is not satisfied, Ref. [46] concludes that many multipoles radiate with the same strength. This seems difficult to reconcile with binary-pulsar observations, which agree with GR's quadrupole formula. (However, note that, unlike us, Ref. [46] assumes a conformal scalar-matter coupling).

In conclusion, GW emission from stellar binaries is only modified at high PN orders in SSHTs. Therefore, binary-pulsar observations, which agree with GR's quadrupole formula at the percent level and also test the 1PN conservative dynamics, are reproduced in SSHTs, provided that the binary's dynamics is perturbative (i.e., PN) [46] and that cosmological-expansion effects can be neglected [29–32]. Direct detection of GWs from neutron-star binaries, however, may still provide prospects for testing these theories. Moreover, our main result, Eq. (16), is valid for generic gravitational theories, but it applies only to stars and not to black holes, where additional surface integrals at the horizon might be present. In the special case of SSHTs, black holes are the same as in GR if $\chi = 0$ in Eq. (17) [52], but they possess scalar hairs if $\chi \neq 0$ [53–55]. We thus expect dipolar emission from binaries involving black holes in SSHTs with $\chi \neq 0$ [49,56].

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