Fractional Wigner Crystal in the Helical Luttinger Liquid

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The properties of the strongly interacting edge states of two dimensional topological insulators in the presence of two-particle backscattering are investigated. We find an anomalous behavior of the density-density correlation functions, which show oscillations that are neither of Friedel nor of Wigner type: they, instead, represent a Wigner crystal of fermions of fractional charge $e/2$, with e the electron charge. By studying the Fermi operator, we demonstrate that the state characterized by such fractional oscillations still bears the signatures of spin-momentum locking. Finally, we compare the spin-spin correlation functions and the density-density correlation functions to argue that the fractional Wigner crystal is characterized by a nontrivial spin texture.

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The helical Luttinger liquid (HLL) [1,2] is the state of electronic matter that describes the interacting helical edges of the recently predicted [3–5] and experimentally realized [6,7] two dimensional topological insulators [8–10] (2DTI). Particular emphasis has been devoted to the investigation of the transport properties of 2DTI: topological protection of the edge states facilitates the observation of conductance quantization [6,7,11] in short samples. Long edges on the other hand are characterized by a reduced conductance. Even though an intense effort has been devoted to the investigation of different scattering mechanisms [12–23], a comprehensive theoretical understanding of the scattering sources causing the reduction of the conductance of the edge is still lacking. The main analytical tool employed so far for describing the helical edges is bosonization [24–26], a procedure that enables us to recast the Hamiltonian of the interacting electrons on the edges into a Hamiltonian of free bosonic excitations, representing charge density waves. The physical implications of the bosonization technique can be understood within the framework of the exactly solvable Luttinger model [27–29]. The validity of the Luttinger model as a basis for the description of interacting electrons has a number of experimental demonstrations [30–34]. However, the Luttinger model alone fails in predicting a reasonable behavior of local observables [35–38], such as the electron density and the density-density correlation functions, when electron-electron interactions are strong. In particular, the Luttinger model is not able to capture the transition between a weakly correlated state dominated by Friedel oscillations of the density [39,40], and the strongly correlated one dimenional Wigner crystal [41–48]. To overcome these problems, one has to consider a richer theory: the Luttinger liquid, the universal model describing low energy properties of gapless one dimensional systems [24–26]. The construction by Haldane [26] clearly shows that the Fermi operator $\psi_s(x)$, where $s = \pm$ is the spin projection, of a generic one dimensional electron system can be significantly different with respect to the one of the Luttinger model [26,38]: the standard relation $\psi_s(x) \sim e^{i\theta(x)} \sum_{p=\pm 1} e^{-ipk_F x} e^{ip\phi(x)}$, with k_F the Fermi momentum and $\phi(x)$ and $\theta(x)$ the usual bosonic fields, is replaced by the more general expression $\psi_s \sim e^{i\theta(x)} \sum_{p=-\infty}^{\infty} c_p^{(s)} e^{-ipk_F x} e^{ip\phi(x)}$, with the model dependent coefficients $c_p^{(s)}$. We aim at understanding the properties of the strongly interacting HLL, and hence, we have to build the appropriate Luttinger liquid theory. This formulation presents difficulties: spin-momentum locking breaks the symmetry, usually holding for standard Luttinger liquids, $c_{-p}^{(s)} = c_p^{(s)}$; time reversal symmetry protects from one-particle backscattering off nonmagnetic impurities, which implies that the usual Friedel oscillations of the density are forbidden.

In this Letter, we develop a Luttinger liquid picture of the strongly interacting quantum spin Hall system in the presence of two-particle backscattering extending over the full helical edge. We discover a state characterized by charge oscillations. These oscillations are profoundly different from usual Friedel or Wigner ones in view of their different wavelength: they are characterized by a wavelength that is half of the wavelength of the usual Wigner crystal, suggesting the formation of a correlated state of fermions with charge $e/2$, with e the electron charge. The presence of fractional charges in the system is made evident by the use of refermionization and can be intuitively understood by means of the following argument: Two-particle backscattering brings two-particles from one branch of the dispersion relation to the other one and, hence, is characterized by a momentum transfer of $4k_F$, with k_F the Fermi momentum. Along the lines of Landau's theory of the Fermi liquid, we can postulate that the helical liquid in the presence of two-particle backscattering can be reformulated in terms of new quasiparticles. For weaker interactions (Luttinger parameters $K_L > 1/2$), two-particle backscattering is irrelevant, and the new quasiparticles are not very different from the original electrons. For stronger interactions ($K_L < 1/2$), two-particle backscattering is relevant and determines the nature of the quasiparticles. Interestingly, the two-particle backscattering can be viewed as a single quasiparticle backscattering term. Then, we have to fix the Fermi momentum k'_F of the quasiparticles to $2k_F$. Thus, the number of quasiparticles is twice the number of electrons, and their charge is $e/2$ because of charge conservation. For $K_L \to 0$ the residual interaction among fractional quasiparticles becomes strong; thus, they eventually crystalize. Finally, after demonstrating that signatures of spin-momentum locking still survive in the strongly interacting regime, we address the spin-spin correlations and demonstrate that they have the wavelength of the usual Wigner oscillations, suggesting a complex spin pattern. At the end, we comment on possible experimental realizations of the fractional Wigner crystal.

The Lagrangian density $\mathcal{L}$ in the presence of two-particle backscattering reads

$$\mathcal{L} = \frac{(\partial_t\phi)^2}{2} - \frac{(\partial_x\phi)^2}{2} - \frac{m^2}{\beta^2}\cos(\beta\phi) + \frac{\mu\beta}{4\pi}\partial_x\phi. \quad (1)$$

Here, ϕ is the Luttinger liquid bosonic field, which includes the zero modes, μ is the chemical potential, β is related to the Luttinger parameter K_L by $\beta = 4\sqrt{\pi K_L}$, and m^2 measures the strength of the two-particle backscattering. Periodic boundary conditions, with period L, are imposed. Two-particle backscattering can arise in the presence of anisotropic spin interactions [1] or in generic helical liquids [49]. It is allowed by time reversal symmetry, and its effect is more pronounced when the Fermi level is close to the Dirac point. Our interest in such an interaction term is due to its formal analogy with the umklapp term occurring in usual Luttinger liquids, which is known to lead to Wigner oscillations in the density [24,35,50]. We are interested in analyzing the helical counterpart of the Wigner crystal, and hence, we focus on the regime of strong interaction $\beta \to 0$. Within the framework of semiclassical treatments of the sine-Gordon Hamiltonian [51–57], an effective strategy for dealing with such a regime in the case of a regular spinful Luttinger liquid has been developed in Ref. [56]. We adapt the same strategy to the case of the HLL. The classical solution ϕ_0 of the equation of motion, upon which a low energy theory for fluctuations can be obtained [57], becomes

$$\phi_0(x, X) = \frac{2}{\beta}\mathrm{am}\left[\frac{m(x+X)}{k}, k\right], \quad (2)$$

with $\mathrm{am}[y, k]$ the Jacobi amplitude function with elliptic index k. The quantity $X \in [0, L]$ is a free parameter whose presence is due to translational invariance. Physically, it represents the "center of mass" of the soliton solution. The index k is fixed by the number Q of electrons on the helical edge, as measured from the Dirac point. In fact the density $\rho_0(x)$, corresponding to the classical solution, is given by $\rho_0(x,X) = (\beta/4\pi)\partial_x\phi_0(x,X)$ (consistently with the assumption that we have included the zero modes in the field ϕ), so that we find $(mL/4Q) = kK(k)$, with $K(k)$ the complete elliptic integral of the first kind. The condition relating Q and k, which is different from the one occurring in usual Luttinger liquids [56], has important implications on physical observables. Although a deeper analysis, which will be discussed below, requires the inclusion of quantum fluctuations, a flavor of the physical consequences of two-particle backscattering can be gained already by considering the properties of the classical solution. We can consider the electron density $\rho_0(x, X)$ once a particular choice of X, say $X = 0$, is done. This quantity describes the local electron density if a local perturbation pins it. It is important to note, however, that when the average (integration) over X is carried out, as required for the clean system described in Eq. (1), the average local density $\bar{\rho}_0 = Q/L$ is constant due to translational invariance. Alternatively, we can consider the density-density correlation function $\bar{\xi}(x)$ in the classical regime, after averaging over the variable X. The average over X is physically needed since the center of mass of the classical solution can be anywhere with equal probability. Explicitly, we have

$$\rho_0(x, 0) = \frac{2QK(k)}{\pi L} dn\left[\frac{4QK(k)x}{L}, k\right], \quad (3)$$

$$\bar{\xi}_0(x) = \frac{1}{L}\int_0^L dX \rho_0(x, X)\rho_0(0, X), \quad (4)$$

where $dn[x, k]$ is the Jacobi dn function with elliptic index k. As shown in Fig. 1, the number of peaks of $\rho_0(x, 0)$ and $\bar{\xi}(x)$ is $2Q$, instead of Q, as it is in the usual Luttinger liquid and as would be expected if the system was in the Wigner molecule regime. It will soon be shown that this feature is also present when quantum fluctuations are included. The doubling of the number of peaks in the strong interaction regime is due to the fact that time reversal symmetry does not allow for one-particle backscattering. One-particle

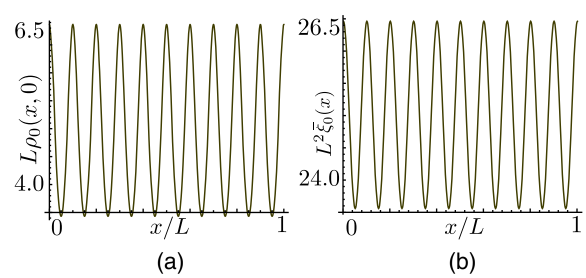

FIG. 1 (color online). Classical electron density $\rho_0(x, 0)$ [panel (a)] and density-density correlation function $\bar{\xi}(x)$ [panel (b)], for $Q = 5$ and $Q/mL = 0.15$.

backscattering, instead, would lead to the expected Q peaks in $\rho_0(x,0)$ and $\bar{\xi}(x)$. The scenario can be interpreted as a Wigner oscillation of quasiparticles with charge $e/2$. In the very same way that strong interaction favors Wigner oscillations of electrons with integer charge in usual Luttinger liquids, here, the interplay of spin-momentum locking, two-particle backscattering, and interactions, lead to the formation of such fractional oscillations. A physical insight in the meaning of this Wigner oscillation of fractional charges can be gained by refermionization. This technique allows us to exactly solve the Lagrangian in Eq. (1) for $K_L = 1/4$, and for that value of the interaction parameter, one has half charged fermions as low energy excitations [49,58]. Since two-particle backscattering becomes relevant in the RG sense for $K_L < 1/2$, and we are considering $K_L \ll 1$, the quasiparticles of our system are also fermions with charge $e/2$. The existence of half-charged quasiparticles in systems described by the Lagrangian in Eq. (1) in the case $Q = 0$ has been predicted [59,60], even in connection to topological insulators (see, e.g., Ref. [61]). However, the region of parameters we inspect is profoundly different: The case $Q = 0$ is gapped and evolves, in the strong interaction regime, toward a Mott insulating phase [1,62], while for $Q \neq 0$ the system is, as we will show below, gapless. In fact it constitutes the fractional Wigner crystal.

In order to refine the classical result, a theory for quantum fluctuations $\eta(x)$ must be addressed. The appropriate framework is the collective coordinate method [57], and the most appropriate way of including fluctuations is to introduce them by the relation [56]

$$\phi(x) = \frac{2}{\beta}\mathrm{am}\left[2K(k)\left(\frac{2Qx}{L} + \frac{\beta\eta(x)}{2\pi}\right), k\right], \tag{5}$$

which reduces to Eq. (2) for $\eta(x) = 0$, up to the variable X. This behavior suggests the existence of a zero energy mode $\eta_0(x)$ which is constant and stems from translational invariance. Note that the zero modes have been isolated from the field $\eta(x)$. The expression for the electron density $\rho(x)$ resulting from Eq. (5) can be written, using a standard series expansion [63], as

$$\rho(x) = \left(\frac{Q}{L} + \frac{\beta}{4\pi}\partial_x\eta(x)\right)\sum_{n=-\infty}^{\infty}\frac{e^{in(4\pi Qx/L+\beta\eta)}}{\cosh(n\tau)}, \tag{6}$$

with $\tau = \pi K(\sqrt{1-k^2})/K(k)$. The expression in Eq. (6) resembles Haldane's expansion for the electron density in the spinless Luttinger liquid [26], though with a striking difference: the harmonics of the density appear with wave vectors which are multiples of $4k_F = 4\pi Q/L$, instead of multiples of $2k_F$. This behavior is consistent with the oscillations in the density corresponding to the classical solution (Fig. 1). Note that, both in the limit $m \to 0$ (no cosine term) and in the limit $Q/(LM) \to \infty$ (significantly away from the Dirac point), only the term with $n = 0$ remains due to the presence of the damping factors $\cosh(n\tau)$. The expected form of the electron density of the HLL, i.e., the long wave density of usual one-channel Luttinger liquids is, hence, recovered in such limits, as expected. The energetics of the fluctuations $\eta(x)$ is encoded, up to quadratic order in $\eta(x)$, in the Lagrangian density [56]

$$\mathcal{L}_\eta(x) = \frac{w^2(x)}{2}[(\partial_t\eta)^2 - (\partial_x\eta)^2], \tag{7}$$

with $w(x) = 4K(k)/\pi dn[4QK(k)x/L, k]$. We can solve the equation of motion, that follows from Eq. (7), for the eigenmodes $\eta(x,t) = \exp[-i\omega_j t]\eta_j(x)$. In fact, this equation becomes

$$\omega_j^2\eta_i(x) = w^{-2}(x)\partial_x[w^2(x)\partial_x\eta_j(x)], \tag{8}$$

and it maps onto a Lamé equation. The lowest energy eigenfunction, which corresponds to $\omega = 0$, is $\eta_0(x) =$ const, as required by translational invariance. When quantizing the theory, this mode can be treated within the collective quantization approach, even though, in the large Q limit, it has been shown to be sufficient to integrate over η_0 [56]. At low energy, still in the large Q limit, the theory for the low energy modes $\eta_j(x)$, with $j \neq 0$, is essentially equivalent to a one channel Luttinger liquid [56], with Fermi velocity $v_F = \sqrt{1-k^2}K(k)/E(k)$ and Luttinger parameter $K_L' = \pi\beta^2/[16\sqrt{1-k^2}K^2(k)]$.

By virtue of Eqs. (6) and (7) in their low energy limit, we can show that the average electron density is, as expected, constant, and the zero temperature density-density correlation function $\bar{\xi}(x)$ is, in the limit $L \to \infty$ and up to the slowest decaying oscillating term,

$$\bar{\xi}(x) = \frac{Q^2}{L^2} + \frac{K_L'}{2\pi^2x^2} + \bar{\xi}_{4k_F}(x), \tag{9}$$

with $\bar{\xi}_{4k_F}(x) \sim \cos(4\pi Qx/L)/x^{2K_L'}$. The absence of the $2k_F$ component, which is the one that indicates the formation of a Wigner crystal, or at least Wigner oscillations, in one channel Luttinger liquids, witnesses that the strongly interacting sector of the HLL is profoundly different from the usual strongly interacting electron gas, and that the HLL does not undergo a transition to a regular one dimensional Wigner crystal. In fact, two-particle backscattering leads to a state characterized by charge fractionalization and the emergent quasiparticles (with fractional charge $e/2$) are, hence, $2Q$ due to charge conservation. They interact among each other to form the fractional Wigner oscillations.

In order to deepen the understanding of the effects of the original helical character of the theory on the strong interaction regime, we investigate how far spin-momentum locking is affected when strong two-particle backscattering is included. Hence, we address the Fermi operator. To do so, one has to build the field $\theta(x)$, conjugated to $\eta(x)$. Leaving aside the discussion of the zero energy component, that is not relevant for the following, we show [56] that the field $\theta(x) = \int^x \tilde{\prod}(x')dx'$, with $\tilde{\prod}(x) = \partial\mathcal{L}_\eta/\partial(\partial_t\eta)$ and with the quantization condition $[\eta(x), \tilde{\prod}(x')] = i\delta(x - x')$, can be evoked to properly define the Fermi operator for right or left moving electrons. By using the series expansion $e^{i\mathrm{am}[2K(k)z]} = \sum_{n=-\infty}^{\infty} C_n e^{2\pi i z(n+1/2)}$, with $C_n = \pi e^{\tau(n+1/2)}/[kK(k)\sinh(2n+1)\tau]$, and the definition of the chiral fields $\phi_{R/L}(x) = (\beta/4)\phi(x) \mp (2\pi/\beta)\theta(x)$, we obtain the expression for the Fermi fields in terms of $\eta(x)$ and $\theta(x)$

$$\psi_{R/L}(x) \propto e^{-2i\pi\beta^{-1}\theta(x)} \sum_{n=-\infty}^{\infty} C_n^{(1)} e^{\pm i(4\pi Qx/L + \beta\eta)(n+1/4)}, \tag{10}$$

where $C_n^{(1)}$ is implicitly given by the relation $C_n = \sum_{n=-\infty}^{\infty} C_{n-n_1}^{(1)} C_{n_1}^{(1)}$. The form of the Fermi spinor $\Psi(x)$ is, hence, $\Psi(x) = (\psi_+(x), \psi_-(x))^T = (\psi_R(x), \psi_L(x))^T$, where $\psi_\pm(x)$ is the spin up or down component of the field operator [64]. The usual form of the HLL can be recovered by setting $C_n^{(1)} \propto \delta_{n,0}$, as can be shown to be the case in the limits $m \to 0$ and $Q/(mL) \to \infty$. For finite m and $Q/(mL)$, the breaking of spin-momentum locking is evident: the terms with $n > 0$ ($n < 0$) represent left (right) moving components in the spin up (down) part of the spinor. However, these terms are suppressed by both the coefficients $C_n^{(1)}$ and the scaling they acquire in the correlation functions. Since it is a direct consequence of the chirality of the Fermi spinor, the strong anisotropy in the spin response function, hallmark of the HLL, is also present in the strong interaction sector in the presence of two-particle backscattering. This finding is in contrast with the behavior of the one dimensional Wigner crystal, which is an almost classical state where spin dynamics plays a rather unimportant role [44]. To support our claim, we address the ground state average $s_{ij}(x)$ of the spin-spin correlation functions $S_{ij}(x)$ given by $S_{ij}(x) = \Psi^\dagger(x)\sigma^i\Psi(x)\Psi^\dagger(0)\sigma^j\Psi(0)$. In the HLL without two-particle backscattering, we find that $s_{11}(x) = s_{22}(x)$ and $s_{12}(x) = -s_{21}(x)$ are purely oscillating functions with wave vector $2k_F$, while $s_{33}(x)$ is a nonoscillating function and the remaining correlations vanish. In our case, $s_{33}(x)$ is still a nonoscillating function, so that the probability of finding two electrons at distance $x \gg \alpha$ from each other is independent of the spin projection and follows the density-density correlations. On the other hand, the oscillations in $s_{11}(x)$, $s_{22}(x)$, and $s_{12}(x)$ have a wavelength which is twice the wavelength of the oscillations of the electron density, and hence, they witness the onset of a spin helix: this

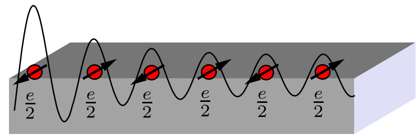

FIG. 2 (color online). Scheme of the results: the fractional quasiparticles form a Wigner crystal [$\bar{\xi}(x)$ is shown]. The arrows indicate the spin correlations.

behavior is in accordance with what happens in the weakly interacting HLL. However, the spin oscillations in the strongly interacting edge with two-particle backscattering are built on oscillating density-density correlations characterized by half of the wavelength. We argue that the spin oscillations can be pinned when translational invariance is broken. A schematic picture is given in Fig. 2. A planar magnetic ordering has also been predicted in the $Q = 0$ case [62], where, however, the ordering competes with the Mott insulator. In our case, instead, both the spin ordering and the fractional Wigner oscillation are enhanced by interactions.

Our theory has been developed on the basis of the edge states of 2DTI, which can be brought into the strongly interacting regime [62,65]. However, we also propose two alternative systems that can potentially host the fractional Wigner crystal: spin-orbit coupled quantum wires in the presence of an external magnetic field and cold atoms in optical traps. As far as the first case is concerned, strong interactions ($K_L \sim 0.26$) have already been reported [66] in gold wires with potentially strong spin orbit coupling. Indeed, it has been demonstrated that the description in terms of a (quasi) helical Luttinger liquid is valid even in the strong interaction regime, for reasonable values of the involved parameters [67]. Referring to the average (spin) density in the case of broken translational symmetry, the fractional (spin) charge oscillations could then be detected by (spin polarized [68]) Coulomb blockade microscopy [69]. The cold atom case is even more favorable: an accessible scheme for realizing interacting helical Luttinger liquids with tunable parameters has been proposed [70]. The possibility for accessing single site quantities could lead to a direct verification of our predictions. Moreover, in the cold atom system proposed in [70], the constraint on the filling makes two-particle backscattering slowly oscillating, thus, recreating the same favorable conditions characterizing the Dirac point in 2DTI.

In conclusion, we have shown that two-particle backscattering, in combination with strong interactions, leads to the formation of a Wigner crystal of quasiparticles with charge $e/2$. This state still bares the signatures of the underlying helicity of the system, both in the form of the Fermi operator and in the anisotropy of the spin-spin correlation functions. Moreover, such a state has a complex spin structure, resembling a spin helix, that suggests an intercalation of particles with opposite spin projection.

Finally, we have commented on the experimental relevance of our findings.

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